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Hidden conformal symmetry of warped AdS_3 black holes

Reza Fareghbal

School of Physics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Islamic Republic of Iran

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ABSTRACT

We show that for a certain low frequency limit, the wave equation of a generic massive scalar field in the background of the spacelike warped AdS_3 black hole can be written as the Casimir of an $SL(2, R)$ symmetry. Two sets of $SL(2, R)$ generators are found which uncover the hidden $SL(2, R) \times SL(2, R)$ symmetry of the solution. This symmetry is only defined locally and is spontaneously broken to $U(1) \times U(1)$ by a periodic identification of the ϕ coordinate. By using the generator of the identification we read the left and right temperatures (T_L, T_R) of the proposed dual conformal field theory which are in complete agreement with the WAdS/CFT conjecture. Moreover, under the above condition of the scalar wave frequency, the absorption cross section of the scalar field is consistent with the two-point function of the dual CFT.

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1. Introduction

It is well known that adding higher derivative terms to three-dimensional gravity (with cosmological constant) yields to novel theories. An interesting example of such theories is Topologically Massive Gravity (TMG) [1,2]. It contains the gravitational Chern–Simons action as a correction to the 3D Einstein gravity with the cosmological constant (see (2.1)). Besides the AdS_3 solution and the BTZ black holes, TMG admits other vacuum known as warped AdS_3 [3,4] and its corresponding black holes [5–7].

Warped AdS_3 solutions can be obtained by fibering the real line over AdS_2 but with a constant warp factor multiplying the fiber metric. As a result, the $SL(2, R) \times SL(2, R)$ isometry of the AdS_3 breaks down to $SL(2, R) \times U(1)$. It was shown in [8] that the warped AdS_3 black holes are discrete quotients of warped AdS_3 by an element of $SL(2, R) \times U(1)$. Motivated by this fact it was conjectured in [8] that the warped AdS_3 gravity has dual conformal field theory with central charges

$$c_L = \frac{4\nu\ell}{G(\nu^2 + 3)}, \quad c_R = \frac{\ell(5\nu^2 + 3)}{G\nu(\nu^2 + 3)}. \quad (1.1)$$

Entropy calculation using Cardy formula and asymptotic symmetry group (ASG) analysis [9–12] support this conjecture. Nevertheless, the warped AdS_3 geometries do not have the full $SL(2, R) \times SL(2, R)$ conformal symmetry in the bulk and this is somehow different from the usual AdS/CFT dictionary. Recently, in an interesting paper [13] a similar problem has been studied in another geometry, namely the 4D Kerr black hole. According to [13] (which

is the continuation of [14–17]), for a AdS/CFT-type duality to work it is not necessary that the conformal symmetry has a realization in terms of the spacetime symmetries. Rather it is sufficient that the solution space of the wave equation for the propagating field has a conformal symmetry. Generalization of this idea to other black holes in 4D and 5D and the study of its various aspects have been recently carried out in [18–26].

In this Letter we show that a similar idea is at work for warped AdS_3 black holes. To do so, similarly to [13], we show that a massive scalar field propagating in the spacelike warped AdS_3 black hole can probe a hidden conformal symmetry. This hidden symmetry is unfolded by reducing the scalar Laplacian to the Casimir of the $SL(2, R)$ symmetry. It is notable that this reduction is done for a particular limit of the scalar field frequency. We find two sets of vector fields which identify the $SL(2, R) \times SL(2, R)$ symmetry of the solution space. We should note that this symmetry is only locally defined and is broken down to $U(1) \times U(1)$ by the periodic identification of the ϕ coordinate. This is somehow similar to AdS_3 case which yields BTZ black hole upon a discrete identification. Hence, using the generator of the identification, we can read the left and right temperatures of the dual CFT which are in agreement with WAdS/CFT conjecture [8]. As a result, massive scalar fields with frequencies less than a particular value do not see the warped aspect of the background. Moreover, the range of frequencies which makes the above reduction of the scalar wave equation possible, brings a significant simplification on the absorption cross section of the scalar field and one can see remarkable similarity between the final result and the two-point function of the conjectured dual conformal field theory.

After a quick review of the warped AdS_3 black hole in Section 2, in Section 3 we identify the hidden $SL(2, R) \times SL(2, R)$ symmetry

E-mail address: fareghbal@theory.ipm.ac.ir.

and calculate the absorption cross section of the scalar field and rewrite it in terms of the CFT quantities. The last section is devoted to conclusions.

2. Warped AdS_3 black holes

Topologically Massive Gravity (TMG) is given by the action

$$I_{TMG} = \frac{1}{16\pi G} \left[\int_M d^3x \sqrt{-g} \left(R + \frac{2}{\ell^2} \right) + \frac{1}{\mu} I_{CS} \right], \quad (2.1)$$

where μ is a positive constant and

$$I_{CS} = \frac{1}{2} \int_M d^3x \sqrt{-g} \epsilon^{\lambda\mu\nu} \Gamma_{\lambda\sigma}^\alpha \left(\partial_\mu \Gamma_{\alpha\nu}^\sigma + \frac{2}{3} \Gamma_{\mu\tau}^\sigma \Gamma_{\nu\alpha}^\tau \right) \quad (2.2)$$

is the gravitational Chern–Simons term [1,2]. The theory admits regular black hole solution for [5–7]

$$\nu \equiv \frac{\mu\ell}{3} > 1. \quad (2.3)$$

These black holes may be obtained by performing discrete identifications in the spacelike $WAdS_3$ given as [8]

$$ds^2 = \frac{\ell^2}{\nu^2 + 3} \left[-\cosh^2 \sigma d\tau^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2 + 3} (du + \sinh \sigma d\tau)^2 \right]. \quad (2.4)$$

In the ADM parametrization, the spacelike warped AdS_3 black holes is given by

$$ds^2 = -N(r)^2 dt^2 + \ell^2 R(r)^2 (d\phi + N^\phi(r) dt)^2 + \frac{\ell^4 dr^2}{4R(r)^2 N(r)^2}, \quad (2.5)$$

where

$$\begin{aligned} R(r)^2 &\equiv \frac{r}{4} (3(\nu^2 - 1)r + (\nu^2 + 3)(r_+ + r_-) \\ &\quad - 4\nu\sqrt{r_+ r_- (\nu^2 + 3)}), \\ N(r)^2 &\equiv \frac{\ell^2 (\nu^2 + 3)(r - r_+)(r - r_-)}{4R(r)^2}, \\ N^\phi(r) &\equiv \frac{2\nu r - \sqrt{r_+ r_- (\nu^2 + 3)}}{2R(r)^2}, \end{aligned} \quad (2.6)$$

and r_+ and r_- are respectively the location of the outer and inner horizons.

For $\nu = 1$ this metric reduces to the metric of BTZ in a rotating frame. Moreover, for $\nu < 1$ the metric (2.5) results in closed timelike curves and hence we only consider the $\nu \geq 1$ range.

The Hawking temperature, T_H , and the angular velocity at the outer horizon, Ω_H , are given by

$$T_H = \frac{(\nu^2 + 3)(r_+ - r_-)}{4\pi\ell(2\nu r_+ - \sqrt{(\nu^2 + 3)r_+ r_-})}, \quad (2.7)$$

$$\Omega_H = -\frac{2}{\ell(2\nu r_+ - \sqrt{(\nu^2 + 3)r_+ r_-})}. \quad (2.8)$$

One can easily verify that black holes (2.5) satisfy the first law of thermodynamics for the values of the Wald entropy, ADT mass and angular momentum given by [27–30]

$$S = \frac{\pi\ell}{24\nu G} [(9\nu^2 + 3)r_+ - (\nu^2 + 3)r_- - 4\nu\sqrt{(\nu^2 + 3)r_+ r_-}], \quad (2.9)$$

$$\mathcal{M}^{ADT} = \frac{(\nu^2 + 3)}{24G} \left(r_+ + r_- - \frac{1}{\nu} \sqrt{r_+ r_- (\nu^2 + 3)} \right), \quad (2.10)$$

$$\begin{aligned} \mathcal{J}^{ADT} &= \frac{\nu\ell(\nu^2 + 3)}{96G} \left[\left(r_+ + r_- - \frac{1}{\nu} \sqrt{r_+ r_- (\nu^2 + 3)} \right)^2 \right. \\ &\quad \left. - \frac{(5\nu^2 + 3)}{4\nu^2} (r_+ - r_-)^2 \right]. \end{aligned} \quad (2.11)$$

3. Hidden conformal symmetry

In this section we consider a massive scalar field propagating in the background of warped AdS_3 black hole (2.5). The classical wave equation describing the dynamics of the scalar field is

$$\left(\frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} \partial^\mu - m^2 \right) \Phi = 0. \quad (3.1)$$

By making use of the Fourier expansion

$$\Phi(t, r, \phi) = e^{-i\omega t + ik\phi} S(r) \quad (3.2)$$

and using (2.5)–(2.6), Eq. (3.1) can be recast into

$$\begin{aligned} &\partial_r \left((r - r_+)(r - r_-) \partial_r S(r) \right) \\ &\quad + \frac{(2\nu r_+ \omega - \omega \sqrt{r_+ r_- (\nu^2 + 3)} + 2k)^2}{(r - r_+)(r_+ - r_-)(\nu^2 + 3)^2} S(r) \\ &\quad - \frac{(2\nu r_- \omega - \omega \sqrt{r_+ r_- (\nu^2 + 3)} + 2k)^2}{(r - r_-)(r_+ - r_-)(\nu^2 + 3)^2} S(r) \\ &\quad + \frac{3(\nu^2 - 1)}{(\nu^2 + 3)^2} \omega^2 S(r) \\ &= \frac{\ell^2 m^2}{\nu^2 + 3} S(r). \end{aligned} \quad (3.3)$$

Solutions to this equation have been previously studied in literature (see [31–37]). In this section we will observe that for the cases that the last term of (3.3) can be neglected, the scalar Laplacian of the wave equation can be written as an $SL(2, R)$ Casimir. For $\nu = 1$ the last ω^2 term of (3.3) is zero and no extra condition is needed. Since for $\nu = 1$ the warped black holes reduce to the BTZ black holes in the rotating coordinate, this is consistent with the local $SL(2, R) \times SL(2, R)$ symmetry of the BTZ background.

In order to neglect the last term of (3.3) for $\nu > 1$, we require that¹

$$\omega^2 \ll \frac{(\nu^2 + 3)^2}{3(\nu^2 - 1)}. \quad (3.4)$$

Unlike the higher dimensional black holes [13,18–23], it is not necessary to divide the geometry to the near and the far regions. As we will see, condition (3.4) is enough to reduce the scalar Laplacian of Eq. (3.3) to the $SL(2, R)$ Casimir. This means that a scalar field with sufficiently small frequency can probe the $SL(2, R)$ symmetry everywhere in the spacetime.

As has been done in [34] and [36], one may propose that instead of imposing (3.4), it is possible to include the last ω^2 term to the right-hand side of (3.3) and write the remaining part of the LHS of (3.3) as the Casimir of $SL(2, R)$. However, then because

¹ It is clear from (2.5) that t is dimensionless and hence ω is dimensionless too. The natural units for energy is $1/\ell$.

of the new ω^2 term in the RHS, the highest conformal weights will be frequency-dependent. We need to avoid this frequency-dependent weights in the context of hidden symmetry because, as we will see explicitly in the next subsection, ω is the eigenvalue of a non-Casimir combination of $SL(2, R)$ generators. Hence, the $SL(2, R) \times SL(2, R)$ symmetry of (3.3) is only manifest in the low energy limit set by (3.3).

Let us try to understand the condition (3.4) better. Noting that the Warped AdS_3 space (2.4) and Warped AdS_3 black hole (2.5) are related by a local coordinate transformation [8] (see (3.18)), it will prove convenient to translate (3.4) in terms of the frequency and wavelength of the scalar field propagating in the Warped AdS_3 space (2.4). One simple way to do so is using the identification of [36] to relate the (ω, k) of the warped black hole to the $(\tilde{\omega}, \tilde{k})$ of the warped AdS_3 background:

$$\tilde{\omega} = \frac{2}{\nu^2 + 3}k, \quad \tilde{k} = \frac{2\nu}{\nu^2 + 3}\omega. \quad (3.5)$$

In fact, this relations has been introduced in [36] by using the identification of [38] to relate the asymptotic geometry of the warped AdS_3 space and the warped AdS_3 black hole.

In terms of the parameters of the warped AdS_3 background, condition (3.4) takes the form

$$\frac{1}{\tilde{k}^2} \gg \frac{3(\nu^2 - 1)}{4\nu^2} = 1 - \alpha, \quad (3.6)$$

where α is the ratio of the radius of the AdS_2 space to the radius of the fiber space in (2.4). For the ordinary AdS_3 space, $\alpha = 1$ and the deviation from this value indicates the amount of warping. Condition (3.6) shows that in order to have hidden $SL(2, R)$ symmetry, the wavelength of the scalar field must be much greater than the difference between the radius of the AdS_2 and the radius of the fiber space in (2.4), so that the warping essentially remains unnoted by the scalar probe.²

3.1. $SL(2, R)_L \times SL(2, R)_R$ symmetry

Let us now define the vector fields

$$\begin{aligned} H_0 &= -\frac{2i\nu}{\nu^2 + 3} \frac{T_L}{T_R} \partial_t + \frac{i}{2\pi \ell T_R} \partial_\phi, \\ H_1 &= ie^{-2\pi \ell T_R \phi} \left[-\frac{\nu}{(\nu^2 + 3)\sqrt{\Delta}} \left((2r - r_+ - r_-) \frac{T_L}{T_R} \right. \right. \\ &\quad \left. \left. + r_+ - r_- \right) \partial_t + \sqrt{\Delta} \partial_r + \frac{2r - r_+ - r_-}{4\pi \ell T_R \sqrt{\Delta}} \partial_\phi \right], \\ H_{-1} &= ie^{2\pi \ell T_R \phi} \left[-\frac{\nu}{(\nu^2 + 3)\sqrt{\Delta}} \left((2r - r_+ - r_-) \frac{T_L}{T_R} \right. \right. \\ &\quad \left. \left. + r_+ - r_- \right) \partial_t - \sqrt{\Delta} \partial_r + \frac{2r - r_+ - r_-}{4\pi \ell T_R \sqrt{\Delta}} \partial_\phi \right], \end{aligned} \quad (3.7)$$

and also

$$\begin{aligned} \bar{H}_0 &= \frac{2i\nu}{\nu^2 + 3} \partial_t, \\ \bar{H}_1 &= ie^{-\frac{\nu^2+3}{2\nu}t - 2\pi \ell T_L \phi} \left[\frac{\nu((\nu^2 + 3)(2r - r_+ - r_-) + 8\pi \ell T_L)}{(\nu^2 + 3)^2 \sqrt{\Delta}} \partial_t \right. \\ &\quad \left. + \sqrt{\Delta} \partial_r - \frac{2}{(\nu^2 + 3)\sqrt{\Delta}} \partial_\phi \right], \end{aligned}$$

$$\begin{aligned} \bar{H}_{-1} &= ie^{\frac{\nu^2+3}{2\nu}t + 2\pi \ell T_L \phi} \left[\frac{\nu((\nu^2 + 3)(2r - r_+ - r_-) + 8\pi \ell T_L)}{(\nu^2 + 3)^2 \sqrt{\Delta}} \partial_t \right. \\ &\quad \left. - \sqrt{\Delta} \partial_r - \frac{2}{(\nu^2 + 3)\sqrt{\Delta}} \partial_\phi \right], \end{aligned} \quad (3.8)$$

where

$$T_R = \frac{(\nu^2 + 3)(r_+ - r_-)}{8\pi \ell}, \quad (3.9)$$

$$T_L = \frac{\nu^2 + 3}{8\pi \ell} \left(r_+ + r_- - \frac{\sqrt{r_+ r_- (\nu^2 + 3)}}{\nu} \right), \quad (3.10)$$

$$\Delta = (r - r_+)(r - r_-). \quad (3.11)$$

It is not difficult to see that they satisfy the $SL(2, R) \times SL(2, R)$ algebra

$$\begin{aligned} [H_n, H_m] &= i(n - m)H_{n+m}, \\ [\bar{H}_n, \bar{H}_m] &= i(n - m)\bar{H}_{n+m} \quad (n, m = -1, 0, 1), \\ [H_n, \bar{H}_m] &= 0. \end{aligned} \quad (3.12)$$

Moreover, one may readily verify that after imposing condition (3.4) the wave equation (3.3) can be written as

$$\mathcal{H}^2 \Phi = \bar{\mathcal{H}}^2 \Phi = \frac{\ell^2 m^2}{\nu^2 + 3} \Phi \quad (3.13)$$

where \mathcal{H}^2 and $\bar{\mathcal{H}}^2$ are the $SL(2, R)$ quadratic Casimir given by

$$\mathcal{H}^2 = -H_0^2 + \frac{1}{2}(H_1 H_{-1} + H_{-1} H_1), \quad (3.14)$$

and similarly for $\bar{\mathcal{H}}^2$. Hence the scalar Laplacian can be written as the $SL(2, R)$ Casimir and the $SL(2, R)_L \times SL(2, R)_R$ weights of the scalar field are

$$(h_L, h_R) = \left(\frac{1}{2} \sqrt{1 + \frac{4\ell^2 m^2}{\nu^2 + 3}} - \frac{1}{2}, \frac{1}{2} \sqrt{1 + \frac{4\ell^2 m^2}{\nu^2 + 3}} - \frac{1}{2} \right). \quad (3.15)$$

It is notable that this conformal weights are in agreement with the results of [34] and [36] in the low frequency limit (3.4).

Similar to higher dimensional black holes [13,18–23], the above $SL(2, R) \times SL(2, R)$ symmetry acts on the solution space only locally because the vector fields (3.7) and (3.8) are not periodic under the identification $\phi = \phi + 2\pi$. This identification is generated by the group element

$$e^{\partial_\phi} = e^{-i2\pi \ell (T_R H_0 + T_L \bar{H}_0)} \quad (3.16)$$

which is the same identification used in [8] to produce the warped black holes as a quotient of the warped AdS_3 . Interestingly, H_0 and \bar{H}_0 coincide respectively with $-i\tilde{J}_2/2$ and $i\tilde{J}_2/2$ of [8] and T_R and T_L with the advertised right and left temperatures of the conformal field theory dual to the warped AdS black holes. Thus we obtain another evidence for the conjecture of [8] that warped AdS_3 gravity of TMG has a holographic dual description in terms of a 2-dimensional conformal field theory. The Entropy calculation of [8] using the Cardy formula

$$S = \frac{\pi^2 \ell}{3} (c_L T_L + c_R T_R) \quad (3.17)$$

supports this hypothesis.

² The author would like to thank M.M. Sheikh Jabbari for making this point.

Now let us define a local coordinate transformation as

$$\begin{aligned} \tau &= \tan^{-1} \left(\frac{2\sqrt{\Delta}}{2r - r_+ - r_-} \sinh(2\pi \ell T_R \phi) \right), \\ u &= \frac{v^2 + 3}{2v} t + 2\pi \ell T_L \phi \\ &\quad + \tanh^{-1} \left(\frac{2r - r_+ - r_-}{r_+ - r_-} \coth(2\pi \ell T_R \phi) \right), \\ \sigma &= \sinh^{-1} \left(\frac{2\sqrt{\Delta}}{r_+ - r_-} \cosh(2\pi \ell T_R \phi) \right). \end{aligned} \quad (3.18)$$

In this new coordinate the metric of warped AdS_3 black hole (2.5) takes the form (2.4) i.e. the metric of the spacelike warped AdS_3 geometry [8]. In the coordinate $\{\tau, \sigma, u\}$, the vector fields H_n and \tilde{H}_n ($n = -1, 0, 1$) are written as

$$H_0 = -\frac{i}{2} \tilde{J}_2, \quad H_1 = \frac{i}{2} (\tilde{J}_0 - \tilde{J}_1), \quad H_{-1} = \frac{i}{2} (\tilde{J}_0 + \tilde{J}_1), \quad (3.19)$$

$$\tilde{H}_0 = \frac{i}{2} J_2, \quad \tilde{H}_1 = \frac{i}{2} (J_1 - J_0), \quad \tilde{H}_{-1} = \frac{i}{2} (J_1 + J_0), \quad (3.20)$$

where

$$J_1 = -\frac{2 \sinh u}{\cosh \sigma} \partial_\tau - 2 \cosh u \partial_\sigma + 2 \tanh \sigma \sinh u \partial_u, \quad (3.21)$$

$$J_2 = 2 \partial_u, \quad (3.22)$$

$$J_0 = \frac{2 \cosh u}{\cosh \sigma} \partial_\tau + 2 \sinh u \partial_\sigma - 2 \tanh \sigma \cosh u \partial_u, \quad (3.23)$$

and

$$\tilde{J}_1 = 2 \sin \tau \tanh \sigma \partial_\tau - 2 \cos \tau \partial_\sigma + \frac{2 \sin \tau}{\cosh \sigma} \partial_u, \quad (3.24)$$

$$\tilde{J}_2 = -2 \cos \tau \tanh \sigma \partial_\tau - 2 \sin \tau \partial_\sigma - \frac{2 \cos \tau}{\cosh \sigma} \partial_u, \quad (3.25)$$

$$\tilde{J}_0 = 2 \partial_\tau, \quad (3.26)$$

are the $SL(2, R)_L \times SL(2, R)_R$ Killing vectors of an AdS_3 written in the fibred form

$$ds^2 = \frac{\ell^2}{4} [-\cosh^2 \sigma d\tau^2 + d\sigma^2 + (du + \sinh \sigma d\tau)^2]. \quad (3.27)$$

This is interesting that the hidden conformal symmetry of the spacelike warped AdS_3 black hole is locally the isometry of the AdS_3 geometry. This means that scalar fields with frequency (3.4) do not feel the warped feature of the spacetime.

3.2. Absorption cross section

In order to elaborate on the hidden aspect of the conformal symmetry, we consider scattering of the massive scalar field in the background of the warped AdS_3 black hole. It is notable that this issue has recently been studied in [31,33,35]. In this section we only clarify the effect of the condition (3.4) in their calculations.

After imposing condition (3.4) on the scalar wave equation (3.3), the absorption cross section of the scalar field takes the following form (see formula (5.1) of [35])

$$\begin{aligned} \sigma_{abs} &\sim \sinh[2\pi(\Omega_+ \omega + Uk)] \left| \Gamma \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4m^2 \ell^2}{v^2 + 3}} \right. \right. \\ &\quad \left. \left. - i(\omega(\Omega_+ + \Omega_-) + 2Uk) \right) \right|^2 \\ &\quad \times \left| \Gamma \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4m^2 \ell^2}{v^2 + 3}} - i\omega(\Omega_+ - \Omega_-) \right) \right|^2 \end{aligned} \quad (3.28)$$

where

$$U = \frac{2}{(r_+ - r_-)(v^2 + 3)}, \quad (3.29)$$

$$\Omega_+ = \frac{2vr_+ - \sqrt{r_+ r_- (v^2 + 3)}}{(r_+ - r_-)(v^2 + 3)}, \quad (3.30)$$

$$\Omega_- = \frac{2vr_- - \sqrt{r_+ r_- (v^2 + 3)}}{(r_+ - r_-)(v^2 + 3)}. \quad (3.31)$$

Now we want to rewrite (3.28) in terms of the dual CFT parameters such as left and right temperatures, their conjugate charges and the conformal weights (h_L, h_R) . The conjugate charges δE_L and δE_R are determined through the relation

$$\delta S = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R} \quad (3.32)$$

where S is the entropy. Using the first law of thermodynamics

$$\delta S = \frac{\delta M}{T_H} - \frac{\Omega_H}{T_H} \delta J \quad (3.33)$$

and identifying $\delta M = \omega/\ell$ and $\delta J = k$, we can find the left and right conjugate charges as

$$\omega_L \equiv \delta E_L = \frac{1}{2\ell} (v(r_+ + r_-) - \sqrt{r_+ r_- (v^2 + 3)}) \omega, \quad (3.34)$$

$$\omega_R \equiv \delta E_R = \frac{1}{2\ell} (v(r_+ + r_-) - \sqrt{r_+ r_- (v^2 + 3)}) \omega + \frac{k}{\ell}. \quad (3.35)$$

Finally we can rewrite (3.28) as

$$\begin{aligned} \sigma_{abs} &\sim T_L^{2h_L - 1} T_R^{2h_R - 1} \sinh \left(\frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R} \right) \left| \Gamma \left(h_L + i \frac{\omega_L}{2\pi T_L} \right) \right|^2 \\ &\quad \times \left| \Gamma \left(h_R + i \frac{\omega_R}{2\pi T_R} \right) \right|^2, \end{aligned} \quad (3.36)$$

which is the well-known absorption cross section for a 2d CFT.

4. Conclusion

In this Letter we have studied the hidden conformal symmetry of warped AdS_3 black holes. We have shown that although the warped AdS_3 background does not have a $SL(2, R)_L \times SL(2, R)_R$ isometry, it is still the hidden symmetry of the solution space. The context is very similar to that in higher dimensional black holes and seems to open a new window into the BH/CFT duality. Although we have only considered scattering of a scalar field, it is plausible that the analysis works for any other fields propagating in the warped AdS_3 background. Our result may be thought of as a supporting evidence in favor of the WAdS/CFT conjecture.

An important question at this stage is how this hidden symmetry can be enhanced to the Virasoro algebra. In fact this question is in the light of Brown and Henneaux's work [39] that the local symmetry of the background is enhanced to the Virasoro algebra at the boundary by making use of the asymptotic symmetry group. Generalizing the ASG method for the hidden symmetry of the bulk and using it to identify somehow the central charges of the dual CFT seem interesting questions.

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