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Reliability Model Based on Hypergraph for Dependent Failure System

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Abstract

In order to describe the mechanism of system parts occurring dependent failure , extending the definition of stress and strength to wider range which based on the component's failure stress-strength interference theory. Using hypergragh theory to carry on the modeling of related failure system and calculate its component's related failure rate, then obtaining the real failure rate of the system. Using component's failure data of RIAC Automated Databook to calculate system failure rate, then comparing with corresponding system failure data of OREDA to demonstrate the efficiency of this method.

Keywords: dependent failure; hypergraph; stress-stengh;

1. Introduction

During the last decade, the reliability requirements of instrumentation and control (I&C) systems have increased, in particular if the systems working under complex conditions. As a consequence, dependent failure can not longer be neglected in availability analyses. The definition of dependent failure defines in the IEC 61508 standard is that failure whose probability cannot be expressed as the simple product of the unconditional probability of the individual events which cause it, in other words, two events A and B are dependent, where P(Z) is the probability of event Z, only if:

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$P(A\ and\ B) > P(A) \times P(B)$. Compared to common cause failure only occurs in multi-channel redundant system, dependent failure may occur in ordinary I&C system.

Based on static model of stress-strength, a stress is any load that may produce a failure. A failure occurs if the stress exceeds the strength of the system. When some components occurring dependent failure, it means that some forms of stress exceed the strength of components which have similar structures. For example, when components have physical dependencies, loads may be vibration or shock; functional dependencies, loads may be overload when other components are shutdown; location/environmental dependencies, loads may be electrical, thermal or chemical. Furthermore, neighbor components are more likely to be simultaneously affected than others by a given environmental condition or by an attempt at jamming communications; also several components may depend on a common physical resource; or yet the failure of a component may cause a temporary overloading of neighbor ones. Fig. 1 illustrates the relationship between the stress and components with similar strength.

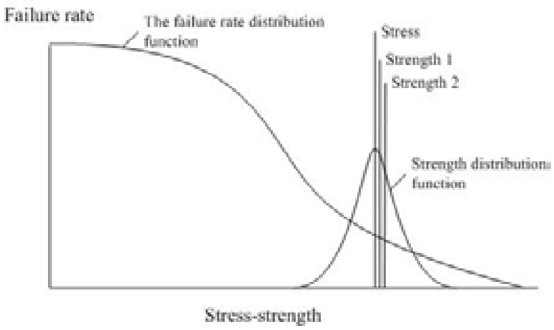

Fig. 1. The relationship between the stress and components with similar strength

2. Overview of hypergraph

Hypergraphs first defined by C.Berge [1] in 1970. In mathematics, a hypergraph is a generalization of a graph, where edges can connect any number of vertices.

Definition 1.

Let $H = (V, E)$ be the hypergraph consisting of vertices

$$V = \{v_m | m \in M\} \tag{1}$$

that is, the vertices are indexed by an index $m \in M$, and the edge set is

$$E = \{e_i | i \in I, e_i \subseteq V\} \tag{2}$$

with the edges e_i indexed by an index $i \in I$

Definition 2.

A dual H^* of H is a hypergraph whose vertices and edges are interchanged, so that the vertices are given by $\{e_i\}$ and whose edges are given by $\{V_m\}$ where

$$V_m = \{e_i | v_m \in e_i\} \tag{3}$$

Definition 3.

Let $V = \{v_1, v_2, ..., v_m\}$ and $E = \{e_1, e_2, ..., e_n\}$. Every hypergraph has an $m \times n$ incidence matrix $A = (a_{ij})$ where

$$a_{ij} = \begin{cases} 1 \text{ if } v_i \in e_j \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

3. The model

The hierarchy of the system with dependent failures in this paper is broken down into three levels starting on top viz.:

- Unit：The system is divided in several units, each with function(s) required for the system to perform its main function.
- Component：These are subsets of each unit and will typically consist of several items.
- Item：It is the lowest level in the hierarchy, and will be repaired or replaced as a whole. Items are independent of each other.

Based on stress-strength theory, we assume that stress is random and strength is constant.

The component strength is determined by the failure rate of items which make up it. When random stress exceeds component strength, components will be failed. Components have two kinds of fail model which are independent failure and dependent failure, and the component failure rate is divided in two parts.

The joint probability of components *A* and *B* is

$$P(AB) = P_{\text{ds}} + P_{\text{ind}}(A) \times P_{\text{ind}}(B) \tag{5}$$

Where P_{ds} denotes the probability of dependent failures in both components, and $P_{\text{ind}}(A)$ denotes the probability of component *A*, which are independent of the failures in component *B*. The probability $P_{\text{ind}}(A)$ can be regarded as a portion of the total probability $P(A)$ that component *A* is lost individually.

$$P_{\text{ind}}(A) \le P(A) \tag{6}$$

When the stress is vital, the components which have similar structures (most of items are same) will be more likely occurring dependent failure than others which havedifferent structures. In other words, components making up by same items will have same strength.

Using the characters of hypergraph that each edge is represented as a multi-set of nodes, we propose hypergraph as a natural way to represent the relationship between components which depend on each other. It can be modeled that the system which is impacted by various forms stress as a hypergraph $H = (V, E)$, where each nodes $v \in V$ corresponds to a item which is a part of component and each hyperedge $e \in E$ corresponds to a component which is apart of a unit to realize some functions for it. The system which is made up by *n* components, and every component has *m* items will be modeled by a hypergraph

$$H = \{(v_1, v_2, ..., v_m), (e_1, e_2, ..., e_n)\} \tag{7}$$

The incidence matrix of the hypergraph $H = (V, E)$ for a collection of m items and n components is a matrix *A* with m rows that represent the items (hyperedge of *H*) and n columns corresponding to the components (nodes of *H*) such that

$$\mathrm{A}[i,j]=a_{ij} \tag{8}$$

Where

$$a_{ij}=\begin{cases}\lambda_i \ \text{ if } v_i\in e_j\\ 0 \text{ otherwise}\end{cases} \tag{9}$$

λ_i are the failure rates of the items. The hyperedge of H's dual hypergraph

$$V_m=\{e_i \mid v_m\in e_n\} \tag{10}$$

means that every item is be used by which components.

We use the adjacency matrix A of hypergraph $H=(V,E)$ to define functions corresponding to the notions of the ability of item v_i in component e_j to affect other components. The item influence in the component is modeled by a function α : $\{v_0,\ldots,v_{m\text{-}1}\}\times\{e_0,\ldots e_{n\text{-}1}\}\rightarrow[0,1]$ that maps a item-component pair into a value in the unit interval. It is defined as follows:

$$\alpha(v_i,e_j)=\frac{\mathrm{A}[i,j]}{\sqrt{\sum_{k=0}^{n-1}\mathrm{A}[i,k]^2}} \tag{11}$$

Function α can construct a weighted hypergraph in which the influence of item v_i on different components' strength is used as the weight of node v_i in hyperedge e_j.

The second function

$$\delta:\{e_0,\ldots,e_{n\text{-}1}\}\times\{v_0,\ldots v_{n\text{-}1}\}\rightarrow[0,1] \tag{12}$$

is used to model weight ratios of different items in the same component. If we define $s(\lambda)$, to return 1 if $\lambda>0$ and 0 if $\lambda=0$ (means that the item is not a part of the component), we define δ as follow:

$$\delta(e_j,v_i)=\frac{s(\mathrm{A}^*[i,j])}{\sqrt{\sum_{k=0}^{n-1}s(\mathrm{A}^*[i,j])}} \tag{13}$$

The similarity between components e_i and e_j can be computed using the well-known cosine measure as follows:

$$\beta(a_i,a_j)=\frac{\sum_{k=0}^{n-1}(\alpha(p_i,a_k)\cdot\alpha(p_j,a_k))}{\sqrt{\sum_{k=0}^{n-1}(\alpha(p_i,a_k))^2\cdot\sum_{k=0}^{n-1}(\alpha(p_j,a_k))^2}} \tag{14}$$

Neglecting the relationship and structure between items, the failure rate of the component e_i which is made up by the items $v_1,v_2,\ldots,v_m$ can be defined as follows：

$$F(e_i)=\frac{\sum_{k=1}^{m}\delta_k\lambda_k}{\sum_{k=1}^{m}\delta_k} \tag{15}$$

Where λ_k is the independent failure rate of v_m.

As mentioned above, utilizing Eq. (14) (15) we can obtain the dependent failure rate of components e_i and e_j as follows:

$$F_{\mathrm{ds}}(e_i,e_j)=\mu\cdot\beta_{i,j}\cdot F(e_i)\cdot F(e_j) \tag{16}$$

Where μ is the probability of vital stress .

The dependent failure rate of components e_i e_j and e_k is defined as follows:

$$F_{\text{ds}}(e_i, e_j, e_k) = \mu \cdot \beta_{i,j,k} \cdot F(e_i) \cdot F(e_j) \cdot F(e_j) \tag{17}$$

where $\beta_{i,j,k} = \max\left[\beta_{i,j}, \beta_{i,k}, \beta_{j,k}\right]$ *(18)*

The dependent failure rate of components $e_1, e_2, ..., e_n$ is defined as follows:

$$F_{\text{ds}}(e_1, e_2, ..., e_n) = \mu \cdot \beta_{1,2,...,n} \cdot \prod_{k=1}^{n} F(e_n) \tag{19}$$

Where

$$\beta_{1,2,...,n} = \max\left[\beta_{i,j}\right], (1 < i < C_n^2, 1 < j < C_n^2) \tag{20}$$

4. Example

The fire and gas detector (F&G) system which is mentioned in offshore reliability data handbook (OREDA) is made up by some components like that mounting socket, cover, control card, display, cabinet, and detector head. Except the detector part, the rest parts are shared with different detector heads. The detector head component has two kinds of types that are gas detector and fire detector. For detecting different gas, the gas detector has H2S gas detector and hydrocarbon gas detector. The fire detector has flame detector and infrared detector.

The catalytic gas detector contains a sensor part and a separate electronic unit (interface unit) which combined make up a Wheatstone bridge circuit. The sensor contains items which are sensitive to hydrocarbon gas (catalytic oxidation of the item). The change in electrical resistance produces an out-of-balance current in the Wheatstone bridge circuit. The interface item may be located in control rooms or in the field (transducers). Some vendors of logic control systems have designed separate input cards for gas detectors. Thus, they do not use the interface items provided by the vendor of the gas detector. In the proposed system breakdown, these gas input cards are considered to be part of the gas detector (interface item), and not part of the Control Logic Units (CLUs). The fire detector has similar structure with the gas detector, the only different between them is the sensor part which is used for detecting different targets.
Fig.2. shows the reliability block diagram of the F&G system, and the failure rates of the component are shown in table 1.

Table 1 The failure rate of public system

Mounting socket	Cover	Control card	Display	Cabinet
1.25	0.89	3.22	5.38	12.17

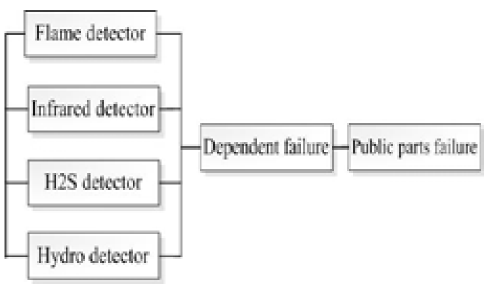

Fig.2. The reliability block diagram of the F&G system

Note: the failure rate is the failure in time, it means that failures per billion hours

The detector head which is a component of F&G system contains several parts (items) that are sensor, A/D converter, interface item and power. The failure rates of these items are found in RIAC Automated Databook (see Fig.3)

The four types of detector heads with four items combining can be modeled by a hypergraph as follows:

$$H = \{(v_{s1}, v_{s2}, v_{s3}, v_{s4}, v_{A/D}, v_{interface}, v_{power}),\\ (e_{flame}, e_{infrared}, e_{H2S}, e_{hydro})\} \quad (21)$$

Hyperedges are defined as follows:

$$e_{flame} = \{v_{s1}, v_{A/D}, v_{interface}, v_{power}\} \quad (22)$$

$$e_{infrared} = \{v_{s2}, v_{A/D}, v_{interface}, v_{power}\} \quad (23)$$

$$e_{H2S} = \{v_{s3}, v_{A/D}, v_{interface}, v_{power}\} \quad (24)$$

$$e_{hydro} = \{v_{s4}, v_{A/D}, v_{interface}, v_{power}\} \quad (25)$$

The failure rate of F&G system is calculated as follows:

$$F_{F\&G} = F_{ind} + F_{ds} + F_{public} \quad (26)$$

$$F_{ind} = F_{flame} \times F_{infrared} \times F_{H2S} \times F_{hydro} \quad (27)$$

where $F_{flame}, F_{infrared}, F_{H2S}, F_{hydro}$ are the independent failure rates of the detector sensors, and calculated by Eq. (15). F_{ds} is the dependent failure rate of the detector sensors, and calculated by Eq. (16), and the probability of vital stress $\mu = 0.1$. F_{common} is the failure rate of the rest part of the F&G system.

The similarity factor β and weight factor δ are shown in table 2. The failure rates comparison between OREDA and this paper are shown in table 3.

- This chapter demonstrates how an F&G system can be modeled by a hypergraph, and how to calculate the failure rate of the system with dependent failure rate. Analyzing table 1 and table 2, we can obtain that:
- For the flame detector and the infrared detector have same structure except the senor part, they have highly similarity, and the similarity factor is $\beta_{1,2} = 0.99$. They will be likely failed in the same time when vital stress is coming.
- Although the flame detector and the hydrocarbon gas detector have similar structure, but because of detect targets are not same, the items which make up them are not same, therefore the similarity factor is $\beta_{1,4} = 0.72$, and the flame has more power strength than the hydrocarbon gas detector.
- The failure rates which are obtained using this paper method are more conservative than the failure rate in the OREDA. It will be safer using the methodology which presents this article than using the data in the OREDA.

Conclusions

It this article, attention is drawn to the treatment of dependent failures in a I&C-system important to safety. Based on stress-strength theory, we develop general principles about component with dependent failures, and the concept of component strength similarity has been build up to indicate the contributions of component strength to the dependent failures. Using the similarity factors we can obtain the system failure rates without assuming that components are independent with each other. In the last section, we give a example to illustrate the application of the result.

Table 2 The failure rate of items

No.	Component	Items and failure rates			
		v_s	$v_{A/D}$	$v_{interface}$	v_{power}
1	e_{flame}	8.2	2.2	5.3	0.5
2	$e_{infrared}$	7.9	2.2	5.3	0.5
3	e_{H2S}	12.5	3.1	4.7	0.3
4	e_{hydro}	43.2	4.3	6.2	0.6

Table 3 The similarity factor and weight factor

$\beta(a_i, a_j)$	$\delta(p_i, a_j)$
$\beta_{1,2} = 0.99$ $\beta_{1,3} = 0.90$ $\lambda_{1,4} = 0.72$	1.0 0.5 0.5 0.5
$\beta_{2,1} = 0.99$ $\beta_{2,3} = 0.87$ $\lambda_{2,4} = 0.69$	1.0 0.5 0.5 0.5
$\beta_{3,1} = 0.90$ $\beta_{3,2} = 0.87$ $\beta_{3,4} = 0.77$	1.0 0.5 0.5 0.5
$\beta_{4,1} = 0.72$ $\beta_{4,2} = 0.69$ $\beta_{4,3} = 0.77$	1.0 0.5 0.5 0.5

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