Circulation characteristics of horseshoe vortex in scour region around circular piers

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Abstract: This paper presents an experimental investigation of the circulation of the horseshoe vortex system within the equilibrium scour hole at a circular pier, with the data measured by an acoustic Doppler velocimeter (ADV). Velocity vector plots and vorticity contours of the flow field on the upstream plane of symmetry ($y = 0$ cm) and on the planes ±3 cm away from the plane of symmetry ($y = ±3$ cm) are presented. The vorticity and circulation of the horseshoe vortices were determined using the forward difference technique and Stokes theorem, respectively. The results show that the magnitudes of circulations are similar on the planes $y = 3$ cm and $y = −3$ cm, which are less than those on the plane $y = 0$ cm. The circulation decreases with the increase of flow shallowness, and increases with the densimetric Froude number. It also increases with the pier Reynolds number at a constant densimetric Froude number, or at a constant flow shallowness. The relative vortex strength (dimensionless circulation) decreases with the increase of the pier Reynolds number. Some empirical equations are proposed based on the results. The predicted circulation values with these equations match the measured data, which indicates that these equations can be used to estimate the circulation in future studies.

Key words: experimental investigation; open channel turbulent flow; scour; horseshoe vortex; circulation; circular pier; forward difference technique; Stokes theorem

1 Introduction

Vorticity, or circulation per unit area, reflects the tendency for fluid elements to spin. It is important to know the magnitude of circulation as it implies the strength of the vortex. Circulation or vortex strength ($I$) will increase if the Reynolds number increases and if the viscous effect is negligible. The vortex strength is related to the occurrence of scour around the pier. For this reason it is essential to study the vortex strength around the pier and, moreover, a thorough study of the flow field around the pier is very important for gaining a better understanding of occurrence of scour. Numerous studies have been carried out with the purpose of predicting the scour depth, and various equations have been developed by many researchers, including Laursen and Toch (1956), Liu et al. (1961), Shen et al. (1969), Breusers et al. (1977), Jain and Fischer (1979), Froehlich (1989), Melville (1992), Abed and Gasser (1993), Richardson and Richardson (1994), Barbhuiya and Dey (2004), and Khwairakpam et al. (2012).

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Received May 30, 2012; accepted Oct. 25, 2012
Raudkivi and Ettema (1983) derived an equation for estimating the maximum depth of local scour at circular piers based on laboratory experiments for cohesionless bed sediment. They concluded that the equilibrium depth of local scour \( (d_{eq}) \) decreased as the geometric standard deviation of sediment \( (\sigma_g) \) increased (an exception occurs when \( \sigma_g < 1.5 \)). A similar phenomenon, that the scour depth decreased with the increase of \( \sigma_g \) \( (1.17 < \sigma_g < 2.77) \), was also observed by Pagliara (2007). In the case of a non-uniform material, i.e., \( \sigma_g > 1.3 \), the scour is less compared with that of a uniform material with the same median particle size \( (d_{s0}) \).

In another work, Pagliara et al. (2008) considered sediment material with \( \sigma_g \) up to 1.3 to be uniform. The pier diameter \( (b) \) relative to the median particle size \( (d_{s0}) \) is known as the sediment coarseness \( (b/d_{s0}) \). The equilibrium scour depth decreases with the decreasing sediment coarseness for values less than about 20. It also decreases at a greater rate with the decreasing flow depth for smaller values of the flow shallowness or relative inflow depth \( (h/b) \), where \( h \) is the approaching flow depth), which is one of the main parameters influencing the local scour.

However, these studies mainly focused on the estimation of the maximum scour depth at piers and abutments. Therefore, it is very important to study the horseshoe vortex to gain a clear understanding of scour around circular piers. For a better understanding of horseshoe vortex characteristics, some researchers have focused on the flow field around circular piers.

Melville (1975) was the pioneer who measured the turbulent flow field within a scour hole at a circular pier using a hot-film anemometer. He measured the flow field along the upstream axis of symmetry and the near-bed turbulence intensity for the case of a flat bed, intermediate scour, and an equilibrium scour hole. Dey et al. (1995) investigated the vortex flow field in clear-water quasi-equilibrium scour holes around circular piers. They measured velocity vectors on the planes with azimuthal angles of 0°, 15°, 30°, 45°, 60°, and 75° with a five-hole Pitot probe. They also presented the variation of circulation with the pier Reynolds number, \( R_p \) (equal to \( Ub/\nu \), where \( U \) is the depth-averaged approaching flow velocity and \( \nu \) is the kinematic viscosity), on a 0° plane for all eighteen tests. The study showed satisfactory agreement with the observations of Melville (1975). Ahmed and Rajaratnam (1998) attempted to describe the velocity distributions along the upstream axis of symmetry within a scour hole at a circular pier using a Clauser-type defect method. Melville and Coleman (2000) explained that the strength of the horseshoe vortex depended on \( R_p \) and \( h/b \). Thus, the circulation of the horseshoe vortex was also a function of \( R_p \) and \( h/b \). Graf and Istiarto (2002) experimentally investigated the three-dimensional flow field in an equilibrium scour hole. They used an acoustic Doppler velocity profiler (ADVP) to measure the three components of the velocities on the vertical symmetry (stagnation) plane of the flow before and after the circular pier. They also calculated the turbulence intensities, Reynolds stresses, bed-shear stresses, and vorticities of the flow field on different azimuthal planes within the equilibrium scour hole. Results of the study showed that a vortex system was established in front of the circular pier and a trailing wake-vortex system of strong turbulence was formed at the rear of the
Muzzammil and Gangadhariah (2003) confirmed experimentally that the primary horseshoe vortex, formed in front of a circular pier, was the prime agent responsible for scour over the entire process of scouring. They employed a simple and effective method to obtain the time-averaged characteristics of the vortex in terms of parameters relating variables of the flow, pier, and channel bed. They also experimentally showed that the circulation is proportional to $Ub$ for $R_p \geq 10000$ and presented the variation of non-dimensional circulation, $\Gamma_n$ (equal to $\Gamma/(\pi Ub)$), with $R_p$. However their results showed less agreement with those of Unger and Hager (2005). Dey and Raikar (2007) and Raikar and Dey (2008) presented the results of an experimental study on the turbulent horseshoe vortex flow within the intermediate scour hole (having scour depths $d_i$ of 0.25, 0.5, and 0.75 times the equilibrium scour depth $d_e$ and equilibrium scour hole at cylindrical and square piers, with the data measured by an acoustic Doppler velocimeter (ADV). They presented the contours of the time-averaged velocities, turbulence intensities, and Reynolds stresses on the planes with azimuthal angles of 0°, 45°, and 90°, and determined the bed-shear stresses from the Reynolds stress distributions. They also computed vorticity contours and circulations and observed that the flow and turbulence intensities in the horseshoe vortex in a developing scour hole were reasonably similar. Kirkil et al. (2008) investigated the flow field around circular piers with the help of the large-scale particle image velocimetry technique.

However, until now observations by these researchers on the variations of circulations of the horseshoe vortices at circular piers with respect to the flow shallowness and pier Reynolds number are particularly scanty for $10000 \leq R_p \leq 35000$. Based on that, an initiative has been taken in this study to measure the turbulent flow field at circular piers of different sizes within a clear-water equilibrium scour hole. The time-averaged velocity vectors and vorticity contours are presented on the plane $y = 0$ cm (that is, on the upstream plane of symmetry) and the planes $y = \pm 3$ cm (3 cm away from the plane of symmetry). On the vertical planes, the planes $y = \pm 3$ cm were chosen to observe the nature of circulation for two cases: case 1 in which these planes were not obstructed by the pier with a diameter of $b = 5$ cm, and case 2 in which these planes were obstructed by the pier with a diameter of $b = 7.5$ cm or 10 cm. The obtained comprehensive data set demonstrates some important relations between the flow shallowness, circulation, densimetric Froude number, and pier Reynolds number. In addition, some comparative studies were carried out in non-dimensional forms.

2 Experimental setup

In this study, experimental investigation of the scour depth and velocity around a circular pier was carried out with an ADV. The experimental setup and conditions are shown in Fig. 1. All the experiments were conducted in a re-circulating tilting flume with a length of 11 m, a width of 0.81 m, and a depth of 0.60 m in the Fluvial Hydraulics Laboratory of the School of Water Resources Engineering at Jadavpur University in Kolkata, India. The working section of
the flume was filled with sand to a uniform thickness of 0.20 m, the length of the sand bed being 3 m, and the width being 0.81 m. The sand bed was located 2.9 m upstream from the flume inlet. The re-circulating flow system was served by a 10 hp variable-speed centrifugal pump located at the upstream end of the tilting flume. The pump had a rotational speed of 1 430 r/min, a power capacity of 7.5 kW, and a maximum discharge of 25.5 L/s. The water discharge was measured with a flow meter connected to the upstream pipe at the inlet of the flume. Water ran directly into the flume through a 0.2 m-diameter pipe line. A vernier point gauge with an accuracy of 0.1 mm, fixed with a movable trolley, was placed on the flume to measure the water level, initial bed level, and scour depth. A Cartesian coordinate system (Fig. 1) for all the experiments is used to represent the turbulence flow fields where the time-averaged velocity components in the x, y, and z directions are represented by \( u, v, \) and \( w, \) respectively. In Fig. 1, \( i, j, \) and \( k \) denote the direction indices in the x, y, and z directions, respectively, and \( \varphi_x \) is the dynamic angle of response. The ADV readings were taken along several vertical planes \( (y = 0, \text{ and } y = \pm 3 \text{ cm}), \) with the lowest longitudinal, transverse, and vertical resolution, i.e. \( \Delta x, \Delta y, \) and \( \Delta z \) being 1.5 cm, 3 cm, and 2 mm, respectively. Fig. 2 shows the horizontal planes for ADV measurements for different pier diameters: 5 cm, 7.5 cm, and 10 cm.
In the present study, circular piers were placed in the middle of the working section of the tilting flume. The bed slope, $S$, which is equal to 1:2 400, was kept constant for all the tests. The flow depth in the flume was adjusted by a tailgate. The range $0.6 < h/b < 1.6$ was restricted to obtain controlled flow in the flume. Raudkivi and Ettema (1983) suggested that the flume width ($B$) should be more than 6.25$b$ to avoid the wall friction effect. The minimum value of $B/b$ was 7.36 in this study, so that these piers could be used without a measurable effect of the side walls on the local scour at the piers. In this study, one bed material was used with the values of $d_{50}$, $d_{16}$, $d_{84}$, $d_{90}$ being 0.825 mm, 0.5 mm, 1.62 mm, and 1.78 mm, respectively, which were measured using a vibrating shaker and sieve analysis. The geometric standard deviation of the bed material, $\sigma_g = (d_{84}/d_{16})^{0.5}$, was 1.8. The relative density of the sand ($\rho_s$) was measured as 2.582. It can be seen that the bed material used here is not uniform, as $\sigma_g > 1.3$. This indicates that the scour will be less than that with the uniform bed material, eventually affecting the vortex formation inside the equilibrium scour hole. Table 1 shows the experimental conditions for all tests.

### Table 1 Experimental conditions for all tests

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<th>$b$ (cm)</th>
<th>$Q$ (L/s)</th>
<th>$\tau_0$ (N/m²)</th>
<th>$h$ (cm)</th>
<th>$U$ (m/s)</th>
<th>$\phi_\theta$ (°)</th>
<th>$Fr$</th>
<th>$u_c$ (m/s)</th>
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Note: $Q$ is the discharge; $\tau_0$ is the average bed shear stress; $Fr$ is the Froude number, and equal to $U/\sqrt{gh}$; $u_c$ is the critical velocity, and $Re$ is the Reynolds number, and equal to $UR/\nu$.

### 3 Methodology

The critical condition for bed material movement was checked before each test using the following steps:

1. The depth-averaged approaching flow velocity ($U$) was calculated using Manning’s equation and Strickler’s formula. Considering steady uniform flow in a rectangular flume, the
bed shear stress \( (\tau_0) \) can be expressed as \( \tau_0 = \rho_f g R \sin \alpha \), where \( \rho_f \) is the mass density of fluid, \( g \) is the gravitational acceleration, \( R \) is the hydraulic radius, and \( \alpha \) is the angle between the longitudinal sloping bed and the horizontal direction.

(2) The critical bed shear stress \( (\tau_{0c}) \) was determined using the expression \( \tau_{0c} = \Theta_c \Delta \rho g d_{s0} \), where \( \Delta = s - 1 \), and \( \Theta_c \) is the critical Shields parameter and is calculated using the following van Rijn’s empirical equations for the Shields curve (van Rijn 1984):

\[
\Theta_c = \begin{cases} 
0.24/D_c & D_c \leq 4 \\
0.14/D_c^{0.64} & 4 < D_c \leq 10 \\
0.04/D_c^{0.10} & 10 < D_c \leq 20 \\
0.013D_c^{0.29} & 20 < D_c \leq 150 \\
0.055 & 150 < D_c
\end{cases}
\]  

where \( D_c \) is the dimensionless particle parameter, expressed as \( D_c = d_{s0}^{3/2} \Delta g/\nu^2 \). The value of \( D_c \) was equal to 22.076 in this study. The critical bed shear stress \( \tau_{0c} \) was calculated as 0.408 \( 4 \) N/m\(^2\).

(3) The critical velocity \( (u_c) \) was calculated from the equation \( u_c = u_{c*} \{5.75 \ln[h/(2d_{s0})] + 6\} \), where \( u_{c*} \) is the critical shear velocity, expressed as \( u_{c*} = \sqrt{\tau_{0c}/\rho_f} \). Finally, the critical discharge \( (Q_c) \) was determined using the continuity equation. The critical shear flow Reynolds number, \( R_{c*} \), expressed as \( R_{c*} = u_{c*} R/\nu \), was determined at a water temperature of 20°C.

(4) The threshold Froude number, \( F_t \), expressed as \( F_t = U/u_{c*} \), was maintained at 0.67 to 0.99 to satisfy the clear-water scour conditions as recommended by Oliveto and Hager (2002) and Dey and Raikar (2007). It was determined from the measured vertical profile of the approaching flow velocity 2 m upstream of the pier, where the presence of the pier did not affect the approaching flow.

When a negligible difference (1 mm or less) of scour depth was observed at an interval of 2 hours after the experiment lasted for 60 hours, it was assumed that an equilibrium stage of the scour hole had been attained. The total duration of each experiment of 67 hours was adequate for achieving the equilibrium scour (Dey and Raikar 2007). After the run was stopped, the maximum equilibrium scour depth, observed at the upstream base of the pier, was carefully measured by the vernier point gauge. After carefully draining out the water from the scour hole, when the bed was reasonably dry, a synthetic resin mixed with water (1:3 by volume) was sprayed uniformly over the scoured bed to stabilize and freeze it. The sand bed was sufficiently filled with the resin when it was left to set for a period of 48 hours. Having dried further for up to 72 hour, the scoured bed profile became rock-hard, facilitating the ADV measurements.

A three-beam 5-cm down-looking ADV (16 MHz MicroADV Lab Model), manufactured by Sontek, was used to measure the instantaneous three-dimensional velocity components. A
sampling rate of 50 Hz and cylindrical sampling volume of 0.09 cm$^3$, having a 2 to 5 mm sampling height ($\Delta z$), were set for measurements. Sampling heights of 5 mm and 2 mm were used for measurement of the velocity components above and within the interfacial sub-layer, respectively. Sampling durations varied from 120 to 300 seconds to achieve a statistically time-independent average velocity. The sampling durations were relatively long near the bed. It is impossible to measure the flow field with the ADV probe within the range from 0 to 4.5 mm above the sand bed, because the ADV needs a measuring volume of 0.09 cm$^3$. The output data from the ADV were filtered using the software WinADV32 version 2.027, developed by Wahl (2003). It is important to point out that the ADV sensor had an outer radius of 2.5 cm, and three receiving transducers mounted on short arms around the transmitting transducer at 120° azimuth intervals, which made it possible to measure the flow as close as 2 cm from the pier boundary.

4 Results and discussion

The literature review revealed that, for the ripple-forming sediment having $d_{s0} < 0.7$ mm, it is rarely possible to maintain a plane bed (Breusers and Raudkivi 1991). Ripple formation was also described by Raudkivi and Ettema (1983) for non-cohesive alluvial sediments with the particle size of 0.05 to 0.7 mm, which form distinctive small ripples when bed shear stresses are slightly greater than the threshold value. For flow with uniform ripple-forming sediments, the scour depth is less than that with non-ripple-forming sediments. The reason is that it is impossible to maintain a flat sand bed under the near-threshold conditions. Thus, ripples develop, and a small amount of sediment transport takes place, replenishing some of the sand scoured at the pier. The sand used in this study had a median grain size of 0.825 mm. Thus, the true clear-water scour conditions could be maintained experimentally in this study.

Fig. 3 shows the experimental results plotted on the Shields diagram (Shields 1936). The threshold of sediment motion occurs when $\theta > \Theta_c$, or $\tau_0 > \tau_{0c}$, or $u_* > u_{*c}$, where $\theta$ is the Shields parameter and $u_*$ is the shear velocity. The flow is hydraulically laminar or turbulent when the particle Reynolds number is less than 2 or more than 500, respectively. The region below the solid line in Fig. 3 indicates that no sediment motion occurs in these experimental conditions. Fig. 3 shows that the discharge during each test was lower than the minimum discharge required for the threshold conditions of the bed particles. Therefore, it can be said that all the experiments were carried out under the clear-water scour conditions.

The time-averaged bed shear stress for the flat bed without a pier was also estimated using the distribution of Reynolds stresses, as was done by Dey and Barbhuiya (2005):

$$\tau_0 = \sqrt{\tau_x^2 + \tau_y^2} \quad \tau_x = -\rho_l \left( u'v' + u'w' \right) \quad \tau_y = -\rho_l \left( v'u' + v'w' \right)$$

(2)

where $\tau_x$ and $\tau_y$ are the bed shear stresses in the x and y directions, respectively, and $u'$, $v'$, and $w'$ are the fluctuations of $u$, $v$, and $w$, respectively.
The maximum value of $\tau_0$ was obtained for Test 15 using Eq. (2), and was equal to 0.3485 N/m$^2$. The time-averaged critical bed shear stress $\tau_{0c}$ on the sloping bed was also estimated using the method proposed by Dey (2003a, 2003b), and was equal to 0.4018 N/m$^2$. This also indicates that a clear-water condition occurred during all the tests.

As an example, the contour lines of the equilibrium scour holes at the circular piers, plotted with the Golden software Surfer 8, for Test 11 and Test 12 are shown in Figs. 4(a) and 4(b), respectively. Here both scours are slightly asymmetric due to some local effects. Table 2 shows the equilibrium scour depth $d_{se}$, equilibrium scour length $l_{se}$, and equilibrium scour width $w_{se}$ for all tests.

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<th>$w_{se}$ (cm)</th>
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Fig. 5 shows the scour-affected zones around the piers. Figs. 6 and 7 show time-averaged velocity vectors on the $xz$ planes ($y = -3$ cm, $y = 0$ cm, and $y = 3$ cm) for equilibrium scour holes, plotted with the OriginLab software, for Test 11 and Test 12, respectively. The magnitude and direction of those vectors are $(u^2 + w^2)^{0.5}$ and $\arctan(w/u)$, respectively. Figs. 6(b) and 7(b) display the characteristics of velocity vectors on the plane $y = 0$ cm. Figs. 6(a), 6(c), 7(a), and 7(c) indicate the characteristics of the flow passage by the side of the
pier. The magnitude of $u$ on the planes $y = \pm 3$ cm is 1.2 to 1.9 times greater than that on the plane $y = 0$ cm at the corresponding location. The magnitude of $u$ increases with $z$. The vertical gradient of $u$ (i.e., $\partial u / \partial z$) within the scour hole (i.e., $z \leq 0$) is greater than that above the scour hole (i.e., $z > 0$), and it is similar to the observation of Dey and Raikar (2007). However, the magnitude of $\partial u / \partial z$ on the plane $y = 0$ cm is 1.1 to 1.5 times greater than those on the planes $y = \pm 3$ cm at the corresponding locations. On the plane $y = 0$ cm, $v$ is almost negligible, as it was

![Fig. 5 Scour-affected zones around piers](image1)

![Fig. 6 Velocity vectors on $xz$ planes for equilibrium scour hole for Test 11](image2)
detected by the ADV. The measured maximum magnitudes of $w$ at $z = -0.015$ m on the plane $y = 0$ cm were $0.43U$ and $0.47U$ for Test 11 and Test 12, respectively. These magnitudes are slightly lower than the $0.6U$ obtained by Istiarto and Graf (2001) and Dey and Raikar (2007). The flow is almost horizontal above the scour hole ($z > 0$), but it is downward close to the pier. The velocity is reversed within the scour hole in the vertical direction, forming a horseshoe vortex. The maximum downflow velocity for a horseshoe vortex was very close to that on the upstream side of the pier, where measurements were impossible due to the limitations of the ADV.

Figs. 8 and 9 show the vorticity contours at equilibrium scour holes on the $xz$ planes ($y = -3$ cm, $y = 0$ cm, and $y = 3$ cm) for Test 11 and Test 12, respectively. The $y$ component of vorticity, $\omega$, which is equal to $\partial u / \partial z - \partial w / \partial x$, was computed for each test by converting the partial differential equation into a finite difference equation with the help of the forward difference technique of computational hydrodynamics. The $y$ component of vorticity at the grid point $(i, j, k)$ can be expressed as

$$\omega_{i,j,k} = \frac{u_{i,j,k+1} - u_{i,j,k} - w_{i+1,j,k} - w_{i,j,k}}{\Delta z} - \frac{w_{i+1,j,k} - w_{i,j,k}}{\Delta x} + O(\Delta z, \Delta x)$$

where $u_{i,j,k}$ and $w_{i,j,k}$ are the values of $u$ and $w$ at the grid point $(i, j, k)$, and $O(\Delta z, \Delta x)$ is
the term standing for the orders of \( \Delta z \) and \( \Delta x \). The truncation error (TE) is the difference between the partial derivative and its finite difference representation, and is characterized by using the order of \( \Delta z \) and \( \Delta x \). The TE was neglected during the calculation of vorticity. By convention, vorticity was defined to be positive in the anticlockwise direction. The concentration of the vorticity inside the scour hole is shown in Figs. 8 and 9. Examination of the velocity vector plots and the vorticity contours confirms that the horseshoe vortex is a forced vortex. The size of the vortex core decreases with the increase of \(|y|\). The sizes of the vortex cores are almost similar on the planes \( y = -3 \) cm and \( y = 3 \) cm, and these sizes are smaller than that of the vortex core on the plane \( y = 0 \) cm. The vortex cores are larger for Test 12 than Test 11 on the plane \( y = 0 \) cm.

The circulation value (\( \Gamma \)) of the vortex was estimated from the vorticity contours for different Cartesian planes using the following equation:

\[
\Gamma = \oint_c \mathbf{V} \cdot ds = \iint_A \alpha dA
\]

where \( \mathbf{V} \) is the velocity vector, \( s \) is the displacement vector along a closed curve \( c \), and \( A \) is the enclosed area.

The detailed methodology for computing \( \Gamma \) was also described by Dey and Raikar (2007). The anticlockwise direction, by convention, was considered positive for circulation. From Table 3,
we can see that for $h = 8$ cm on the plane $y = 3$ cm, the magnitudes of $\Gamma$ for the piers with diameters of 7.5 cm and 10 cm are approximately 1.5 to 2.5 and 2.7 to 5.1 times those for the pier with a diameter of 5 cm, respectively. Similarly, on the plane $y = -3$ cm, these values are 2.3 to 3.1 and 4.0 to 4.8 times, respectively, whereas, on the plane $y = 0$ cm, these values are only 1.3 to 1.9 and 1.6 to 2.2 times, respectively. Therefore, it is clear that the circulations

## Table 3 Magnitudes of circulation for all tests

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$b$ (cm)</th>
<th>$h$ (cm)</th>
<th>$\Gamma_{y=3}$ ($10^3 \text{ m}^2/\text{s}$)</th>
<th>$\Gamma_{y=0}$ ($10^3 \text{ m}^2/\text{s}$)</th>
<th>$\Gamma_{y=-3}$ ($10^3 \text{ m}^2/\text{s}$)</th>
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increase rapidly on the planes \( y = 3 \text{ cm} \) and \( y = -3 \text{ cm} \) compared with those on the plane \( y = 0 \text{ cm} \) if the pier diameter increases from 5 to 10 cm. It is noticeable from these observations that the circulations increase rapidly on the planes (such as the planes \( y = 3 \text{ cm} \) and \( y = -3 \text{ cm} \) for Tests 4 through 9) which are obstructed by the pier with a diameter of 7.5 or 10 cm, compared with those on the planes (such as the planes \( y = 3 \text{ cm} \) and \( y = -3 \text{ cm} \) for Tests 1 through 3) which are not obstructed by the pier with a diameter of 5 cm.

In Fig. 10, the circulation is plotted against the pier Reynolds number at different pier diameters on the planes \( y = 0 \text{ cm} \), \( y = 3 \text{ cm} \), and \( y = -3 \text{ cm} \). The differences between the values measured in the present study at different pier diameters (\( b = 5 \text{ cm}, 7.5 \text{ cm}, 10 \text{ cm}, \text{ and } 11 \text{ cm} \)) and the values obtained by Dey et al. (1995) on the plane \( y = 0 \text{ cm} \) are shown in Fig. 10. The increasing trend of the circulation of the present study on the plane \( y = 0 \text{ cm} \) corresponds closely with the results of Dey et al. (1995). It is observed that the circulation increases with the pier Reynolds number on the planes \( y = 3 \text{ cm} \) and \( y = -3 \text{ cm} \). Exponential trendlines of circulations on the planes \( y = 0 \text{ cm} \), \( y = 3 \text{ cm} \), and \( y = -3 \text{ cm} \) are also shown in Fig. 10 with solid lines, and can be respectively expressed as

\[
\Gamma = 8.257 \times 10^{-6} e^{54R_p \times 10^{-6}} \quad y = 0 \text{ cm} \tag{5}
\]

\[
\Gamma = 8.82 \times 10^{-6} e^{118R_p \times 10^{-6}} \quad y = 3 \text{ cm} \tag{6}
\]

\[
\Gamma = 1.388 \times 10^{-6} e^{110R_p \times 10^{-6}} \quad y = -3 \text{ cm} \tag{7}
\]

The correlation coefficients (\( r \)) between Eq. (5), Eq. (6), and Eq. (7) and their corresponding observations are 0.954, 0.975, and 0.970, respectively, which also imply an almost perfect positive correlation.

Fig. 11 shows a comparison of observed and predicted values of circulation on the plane \( y = 0 \text{ cm} \). The predicted values of circulation were calculated with Eq. (5). It is clear from Fig. 11 that the predicted data match the measured data with a deviation ranging from −25% to 25%.

The most important observation is that 72% of the measured data of Dey et al. (1995) are also within this range when \( 10000 \leq R_p \leq 35000 \). The only remaining five numbers (28%) of
measured data are lying just below the line with a deviation of –25%. This may occur because of a change in the median particle size of sand \( (d_{50}) \). In the present study, \( d_{50} \) was considered to be 0.825 mm, whereas it was considered only 0.26 and 0.58 mm by Dey et al. (1995). It is well known that circulation increases as the scour increases and an increase of \( d_{50} \) implies an increase of scour. Eq. (5) also corresponds closely with the result of Melville (1975).

Figs. 10 and 11 show that the results of the present study on the \( y = 0 \) cm agree with the observations of Dey et al. (1995) and Melville (1975). Based on that similarity, an attempt was also made to introduce empirical Eqs. (6) and (7) for prediction of circulation when \( 10000 \leq R_p \leq 35000 \) on the planes \( y = 3 \) cm and \( y = –3 \) cm, respectively, as shown in Fig. 10. The magnitudes of \( \Gamma \) on the planes \( y = 3 \) cm and \( y = –3 \) cm were always found to be lower than those on the plane \( y = 0 \) cm for \( 10000 \leq R_p \leq 35000 \). This may be due to a decrease of the scour area on the planes \( y = \pm 3 \) cm, compared with the scour area on the plane \( y = 0 \) cm.

Fig. 12 shows the variation of circulation with the pier Reynolds number on the planes \( y = 3 \) cm and \( y = –3 \) cm. The magnitudes of circulation should be similar on the planes \( y = 3 \) cm and \( y = –3 \) cm, as the two planes are symmetric. Therefore, based on the experimental data on the planes \( y = 3 \) cm and \( y = –3 \) cm, a single exponential trendline, as shown in Fig. 12, was introduced and expressed as

\[
\Gamma = 1106 \times 10^{-6} e^{114 R_p \times 10^{-6}} \quad y = \pm 3 \text{ cm} \quad (8)
\]

Here, the correlation coefficients \( r \) between Eq. (8) and the observations is 0.959, which implies an almost perfect positive correlation. The predicted values of circulation were calculated from Eq. (8). Fig. 13 shows a comparison of observed and predicted values of circulation for the data considered on the planes \( y = \pm 3 \) cm for \( 10000 \leq R_p \leq 35000 \). The \( \pm 25\% \) deviation intervals are added as dashed lines. It can be seen from Fig. 13 that the deviation between the predicted and measured data is mostly in a range of –25% to 25%.

![Fig. 12 Variation of \( \Gamma \) with \( R_p \) on planes \( y = \pm 3 \) cm](image1)

![Fig. 13 Comparison of observed and predicted values of \( \Gamma \) on planes \( y = \pm 3 \) cm](image2)

The non-dimensional circulations \( (\Gamma_n) \) on the planes \( y = 0 \) cm, \( y = 3 \) cm, and \( y = –3 \) cm are plotted against the flow shallowness \( (h/b) \) with different pier Reynolds numbers in Fig. 14.
Fig. 14(a) shows that $I^*_n$ ranges between 0.3 and 0.5. Figs. 14(b) and 14(c) indicate that the magnitudes of $I^*_n$ on the planes $y = 3 \text{ cm}$ and $y = -3 \text{ cm}$ are 0.6 to 0.8 times that on the plane $y = 0$.

The circulations on the planes $y = 0 \text{ cm}$, $y = 3 \text{ cm}$, and $y = -3 \text{ cm}$ for $10,000 \leq R_p \leq 35,000$ are plotted against pier Reynolds numbers with different values of flow shallowness or non-dimensional inflow depth ($h/b$) in Figs. 15(a), 15(b), and 15(c), respectively. All the figures clearly indicate that the circulation increases if the pier Reynolds number increases at a constant non-dimensional inflow depth. It is observed that both the circulation and pier Reynolds number decrease with the increase of the non-dimensional inflow depth. This implies that, at a constant approaching flow depth, if the pier diameter increases, then the circulation and pier Reynolds number will both increase, and vice versa.

The circulations on the planes $y = 0 \text{ cm}$, $y = 3 \text{ cm}$, and $y = -3 \text{ cm}$ are plotted against pier Reynolds numbers with different densimetric Froude numbers, $\text{F}_d$, which is equal to $U/\sqrt{\Delta g d}$, in Figs. 16(a), 16(b), and 16(c), respectively. All the figures clearly indicate that the circulation increases with the pier Reynolds number when the densimetric Froude number is constant. It is also shown that both the circulation and pier Reynolds number increase with the densimetric Froude number.
The circulations of the horseshoe vortex inside the scour hole are shown in Fig. 17. The trendline shown in Fig. 17 was proposed by Muzzammil and Gangadhariah (2003). Fig. 17 shows that the results of the present study agree with those of Melville and Raudkivi (1977) and Muzzammil and Gangadhariah (2003). The circulation or vortex strength in dimensionless form is plotted against the pier Reynolds number in log-log scale in Fig. 18. The variation of $I_n$ with $R_p$ was also compared with the results obtained by Dey et al. (1995), Baker (1979), Qadar (1981), Devenport and Simpson (1990), Eckerle and Awad (1991), Srivastava (1982), Muzzammil and Gangadhariah (2003), and Unger and Hager (2005), which shows a good agreement with the results of these researchers. However, as shown in Fig. 18, the trendline of the present study is more similar to the results of Dey et al. (1995), and Muzzammil and Gangadhariah (2003). Here, the value of $r$ is 0.825, which implies a good positive correlation. An overall trend of the data considered herein indicates that $I_n$ decreases with the increase of $R_p$. Fig. 18 reveals that the dimensionless circulation or relative vortex strength depends on the pier Reynolds number and is almost inversely proportional to the pier Reynolds number for $10,000 \leq R_p \leq 35,000$. Therefore, an increase in the pier Reynolds number causes a decrease of relative vortex strength or dimensionless circulation.
5 Conclusions

Clear-water scour tests were performed on a single circular pier with varying inflow depths, pier Reynolds numbers, and densimetric Froude numbers. All sixteen experiments satisfy the clear-water scour conditions. The turbulent flow field was measured with an ADV. The time-averaged velocity vectors and vorticity contours on different Cartesian planes were presented. The vorticity was calculated using the forward difference technique of computational hydrodynamics. The strength of the horseshoe vortex, i.e., the circulation, was computed using the Stokes theorem. Some empirical equations are proposed based on the results. The main conclusions drawn from the present study for $10{,}000 \leq R_p \leq 35{,}000$ are summarized below:

1. The flow is almost horizontal above the scour hole ($z > 0$), but it is downward close to the pier. The velocity is reversed within the scour hole in the vertical direction, forming a horseshoe vortex.

2. The predicted circulation values with the proposed empirical equations match the measured data, which indicates that these equations can be used to estimate the circulation in future studies.

3. Magnitudes of circulations are similar on the planes $y = 3$ cm and $y = -3$ cm, which are less than those on the plane $y = 0$ cm.

4. The circulation decreases with the increase of flow shallowness, and increases with the densimetric Froude number. It also increases with the pier Reynolds number at a constant densimetric Froude number, or at a constant flow shallowness. The relative vortex strength (dimensionless circulation) decreases with the increase of the pier Reynolds number.

Acknowledgements

The helpful suggestions from Professor (Dr.) Subhasish Dey, Brahmaputra Chair Professor for Water Resources of the Department of Civil Engineering, at the Indian Institute of Technology in Kharagpur, India are gratefully acknowledged. The authors also appreciate the help provided by Mr. Ranajit Midya and Mr. Ranadeep Ghosh, M.E. students of the School of Water Resources and Engineering, at Jadavpur University in Kolkata, India, during the investigation.

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(Edited by Yan LEI)