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The analysis of roundabouts through visibility

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Abstract

Visibility is one of the most important issues in almost all geometrical norms for road and intersection design. This paper deals with the question of visibility on roundabouts but from a different point of view in respect of the norms that have currently been adopted regarding capacity and safety issues. Attention is focussed on the simple consideration that many trajectories, in well designed roundabouts, are curvilinear and therefore two possible measures of distances between vehicles can be calculated along their trajectories and directly (such as the Euclidean distance) between the vehicles themselves. The first measure is associated with the distance in terms of the conflict point and the second one represents the sight distance between the vehicles. These two distances are generally different on a roundabout. The aim of the paper is to demonstrate both analytically (through a mathematical representation) and experimentally (by using trajectories processed from video images shot on a working roundabout) that in many cases the visual distance is less than the distance to the conflict point. This difference changes in time (and space) according to the speeds of vehicles. Comparisons between the two approaches, applied to a real case, are also provided. Three cases are taken into consideration (1- two vehicles both circulating both in the circulating roadway, 2-one is circulating in the circulating roadway and the other is entering, 3-both vehicles are entering) highlighting the effect that visibility can produce.

Keywords: roundabout; visibility; functional analysis; vehicle trajectory; safety; capacity; image processing

1. Introduction

Interest in the design and, above all, in the construction of roundabout intersections has recently increased to the extent that the transformation of signalization or give-way intersections into roundabouts, especially in Northern Italy, has almost swamped the possibilities of intervention (Curti et al., 2008). Despite this saturation, no homogenous and coherent models or design and construction criteria have yet been worked out in Italy. Without losing sight of the situation in the world, these examples and subsequent field observations suggest that visibility plays a role in overall performance which should be taken into consideration in modelling and design.

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The insertion of roundabouts in a road network has certainly entailed a lower use of technology than in the past (e.g. for traffic regulation, flow control) since it is considered no longer necessary thanks to the assumed self-regulation capability of roundabouts. Roundabout design in many cases is often tailored only according to consideration of the class of vehicles, without taking into account that other very different classes of vehicles such as trucks, bicycles and even pedestrians are also allowed to use roundabouts. The functional understanding of roundabouts is often limited to certain qualitative estimates. Among the functional features not properly investigated during design, construction and also maintenance we can single out visibility for a good reason.

In general visibility, or free visual distance, is basic in the design of safe road geometry and to guarantee adequate reaction times. The concept of good norms of practice at (conventional) road intersections has been under consideration for several decades (e.g. Harwood et al., 1995; Bared et al., 1997).

A correlated measure is the visibility distance (also named the free visual distance) that identifies the distance along a road that a driver is able to see and it may be influenced by the presence of other vehicles, weather conditions and road lighting. Accordingly, visibility distance may refer to specific situations or could refer to stopping, overtaking or lane changing manoeuvres.

Visibility distance for stopping, vd_S, which is the main interest of this paper, depends firstly on driver reaction time, t_r (which can be considered the sum of perception, decision making and actuation time) and contributes proportionately to vehicle speed, v; secondly, on the braking distance which is related to vehicle speed, road pavement adherence and other possible contributions which may be positive or negative, such as the presence of ramps, rolling and aerodynamic resistance and it can be simply calculated (without taking into account the aerodynamic and rolling resistances or the longitudinal slope of the road) using the uniformly accelerated linear motion equation with deceleration, a. In this last case it is equal to:

$$vd_S = v * t_r + \frac{v^2}{2a} \tag{1}$$

The way to calculate visibility distance must be set at the level of the driver's eyes and on average it is set at 1-1.10m. Great Britain makes an exception by also considering the drivers of heavy vehicles and sets for them an average of 2.00m. The height (from the road pavement) of the object for which visibility must be guaranteed, ranges within the interval [0, 0.45m]. However it should be noted that the dimension of objects that affect the calculation of visibility are valid only to the fifth decimal digit; but they do take into account the presence of other objects (or obstacles) that limit visibility: the smaller the object to be seen the smaller the obstacles must be.

The main criteria for the calculation of visibility on a roundabout can be found in the international technical papers and norms (Austroads, 1993; Ourston and Doctors, 1995; Setra, 1997; Taekratok, 1998; TRB, 1998; US DOT, 2000; VSS, 1994, 2000, 2001; WS DOT, 2002) and are drawn up in Fig. 1 in which the conflict points taken into consideration in this paper are also shown.

These criteria which are mainly based on the verification of stopping distance are four:

Criterion 1: stopping visibility distance on approach (e.g. vehicle 1, point E);

Criterion 2: visibility distance of the intersection (e.g. vehicle 2, a conflict point in A);

Criterion 3: visibility distance for circulation in the circulating area (e.g. vehicle 3, conflict points in C, B);

Criterion 4: visibility distance for pedestrian crossings (e.g. a conflict point in D).

The analysis of state-of-the-art visibility on a roundabout highlights the prevalence of an elementary statistical point of view about safety (by considering only the number of accidents that are fatal or with injuries) without taking into consideration all the single features that can affect overall performance.

In particular the effect that curvilinear trajectories can produce on the reciprocal perception of circulating vehicles is not considered. Their influence, as a consequence of the geometrical design, have on drive behaviour is limited only to the evaluation of speed diagram in the circulating roadway, the entry and exit links. The relevant differences of the capacity measure of a roundabout that can be obtained by using the numerous methods proposed by international norms and practices could be considered a symptom of an incomplete understanding of the phenomena that occur on a roundabout. Currently in some technical literature (e.g. ACHD, 2011) an awareness of the role that curvilinear trajectories have on safety and on intersection capacity exists although their effects are included within other concepts such as for example the reduction of the number of conflicts.

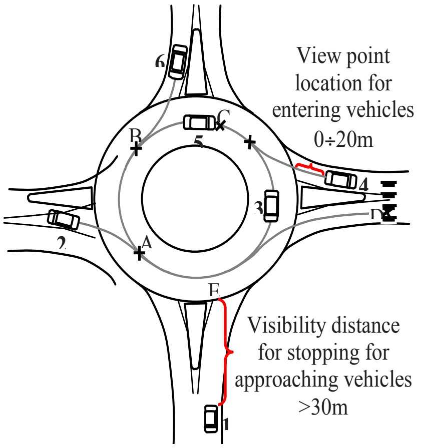


Fig. 1. Conflict points for definition and computation of visibility distances

There are no proposals as to the evaluation of the effect due to different distances between the usable distance for stopping (calculated along the curvilinear trajectory of a roundabout up to the possible conflict point between vehicles) and the visibility (calculated on a straight line between vehicles) which can affect driver perception and which is usually a very complex phenomenon (Ranney, 1994; Farrell, 1999).

In the following sections this problem is considered both from the view point of an analytical geometrical representation and by means of a computational analysis based on real trajectories and it stresses how this peculiar aspect of roundabouts can represent a design parameter for both capacity and safety.

The proposed analysis suggests the importance of some new view-points

- roundabout performance should be evaluated also considering interaction between vehicles and not only for isolated vehicles;
- macroscopic parameters used to describe roundabout performance, like follow-up time, gap and lag, could be affected by visibility;
- visibility between interacting vehicles should be checked for all possible combinations of paths and if a certain safety threshold is not reached a new geometry must be considered.

Section 2 presents the problem and the notations used. Section 3 proposes the analytical formulation of possible cases and Section 4 analyses real trajectories for comparisons between theoretical and empirical results. Section 5 draws conclusions and proposes future research developments.

2. Problem definition and notation used

The study of the effects of curvilinear trajectories in wide radius curves (for example on motorways) on visibility distance is the subject of many technical papers (for example AASHTO, 2011).

What this paper aims to analyse instead is the different effect on perception of stopping distance due to curvilinear trajectories with small curvature radii. This occurs in roundabouts where they can range in the interval 7-25 meters for small roundabouts and in the interval 25-50 for large ones; special cases can occur also in larger roundabouts. Curvilinear trajectories in a roundabout involve not only paths of vehicles circulating in the circulatory roadway but also other combinations with entering vehicles.

As in classical (car-following) microscopic models, the treatise considers only the interaction between two vehicles since the interaction with other vehicles is not assumed to be nil but is less important in this context. Interaction between a vehicle and a bicycle or a pedestrian is also postponed for future analysis.

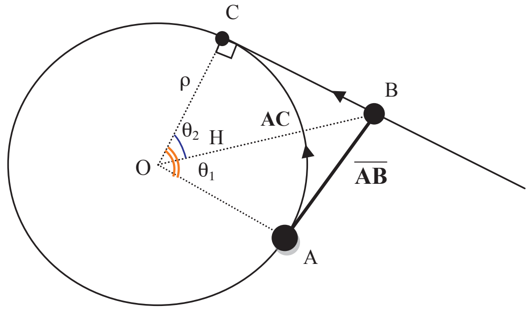


Fig. 2. Geometrical and other notations used and reference distances

As regards the problem treated let us consider Fig. 2 where one possible reference scheme is drawn with a few simplifications from the geometrical point of view.

The larger black circles, named A and B, identify the two vehicles: A is circulating in the circulating roadway and B is entering. The physical dimensions of vehicles are not taken into consideration (because at this stage of the research we are mainly interested in the distance between the drivers' point of view and taking into account the dimensions of the vehicles simply means also taking into account the offset). O is the centre of the roundabout and of the central island (assumed to be perfectly circular) where the trajectory of vehicle A has a radius ρ (implying that the roundabout has a larger radius). For this case the trajectory of vehicle B is assumed rectilinear and tangent to the circulating roadway in C which represents also the first conflict point between the two vehicular trajectories.

Distances on a rectilinear segment from A to B (that is the chord from A to B) have a bar over AB and are denoted as $\overline{AB}$; distances on a curvilinear segment from A to B has no bar and are denoted as AB. Therefore the distance of vehicle A from C, named AC, determines the maximum stopping distance; the visual distance between vehicle A and B is named $\overline{AB}$. In the case of Fig. 2 (as in many other but not all possible cases) $\overline{AB}$ proves to be less than AC. Note that though the relative distance between vehicles A and B is always the same, in other words, $\overline{AB} = \overline{BA}$, the distance of the two vehicles from the conflict point C, AC and $\overline{BC}$, may be different.

This situation, highlighted in the following analyses, can affect the awareness of drivers and therefore reduce the necessary reaction time for a stopping manoeuvre or other manoeuvres to avoid collision. The psycho-technical reaction time, t_r, generally includes all the time needed for a driver to make the stopping manoeuvre, including the time taken to decide to stop and the mechanical delay involved in stopping the engine. The distribution of reaction times for stopping manoeuvres is generally very asymmetric and at the 90° percentile can be set at 1.2s (Gordon et al., 1984). The following analysis assumes this value as the reference value but it must be stressed that a safety design would require higher values (2s or greater) to conform to the norms that many countries require in road design.

The difference AC- $\overline{AB}$ is indicated by the symbol Δ. This is the quantity (with its change in time) on which the analysis is focused. Point C represents the first possible conflict point and the most crucial one since if the conflict occurs downstream along the curvilinear trajectory the safety conditions would increase.

Since vehicles A and B are both moving, it is interesting to calculate how Δ changes in time and therefore the quantity dΔ/dt is also analysed. When dΔ/dt is positive it means that Δ increases in time and this represents a positive situation for safety; if it is negative the reverse is true.

Taking θ_1 and θ_2 be the angles set by vehicle A and B respectively in respect of the point C; v_1 e v_2 are the speeds of vehicle A and B respectively. Let's assume that the speeds of the two vehicles (rectilinear or curvilinear as the case may be) are constant but may be different. This assumption does not hold for the empirical analysis where vehicular speeds are measured. The speed in the circulating roadway is strongly affected by roundabout geometry and values proposed in literature (WS DOT, 2002) are used in the analytical approach; the speed of entering vehicles should be equally affected by the geometry of entry links but since this is not always actually verified it could range within a wider interval.

3. The analytical approach

The possible combinations of the positions of the two vehicles (A and B) on a roundabout can be reduced to three main cases:

- Case 1: the two vehicles are both circulating in the circulating roadway;
- Case 2: one vehicle is entering the circulating roadway and the other is already in the circulating roadway but upstream as regards the entry point of the other vehicle;
- Case 3: both vehicles are entering the circulating roadway in two consecutive links.

Two possible variants of case 2 are disregarded: the first is when the vehicle already circulating in the circulating roadway is downstream as regards entry since, in that case, it can be ascribed to case 1; the second is when vehicle A is downstream as regards the entry of vehicle B and A exits in the next link since the trajectories of two vehicles can cross only if vehicle B also exits in the same link. In case 3 entries from non-consecutive links are disregarded since less significant. In cases 2 and 3 the analysis from both the points of view of vehicle A and vehicle B is developed because although their conflict point is the same, their stopping distance is different.

Analytical formulae for calculating Δ and $d\Delta/dt$ in all considered cases are reported in Tables 1, 3, 5, 6 and 7, respectively. The first part of the table contains the boundary conditions and the way of calculating the angular speed of vehicles; the second one calculates Δ and $d\Delta/dt$ by applying simple trigonometric rules.

3.1. Case 1

Case 1 refers to two vehicles both circulating in the circulating roadway, as drawn up in Fig. 3, where vehicle A follows vehicle B. Their speeds are v_1 and v_2 respectively, θ_1 and θ_2, are the angles that describe their circular motion and have a different meaning in respect of the other cases: since there are no specific reference points they are calculated as starting from a generic arbitrary point Z, therefore, by definition, it always assumes that $\theta_2 > \theta_1$.

Obviously the position of two vehicles depends only on the difference between the two angles and therefore, in theory, only one angle would be necessary to describe it but we are also interested in describing the speeds of the two vehicles that on the contrary depend separately on the two angles; in this case, it must be remembered that the angular speed of each vehicle can be calculated as $\mathbf{d}\theta/dt = \mathbf{v}/\rho$.

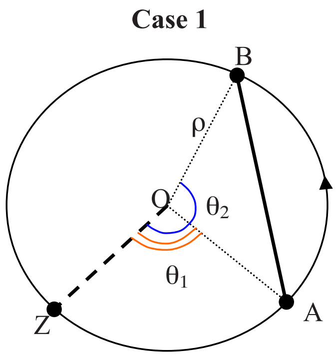


Fig. 3. Reference scheme for computing relative distances between two vehicles (A and B) for case 1 (both vehicles are circulating in the circulating roadway)

In order to limit the analysis to realistic and relevant situations the difference between the two angles ($\theta 2$-$\theta 1$) is kept within the interval [30°, 160°]: smaller angles could be at odds with safety conditions of circulation; higher angles could modify the import of the case (since vehicle B could become the follower).

The formulae to calculate the distances AB (along the circular arc) and $\overline{AB}$ (along the rectilinear line of the chord between A and B, by applying the properties of right triangles), their differences Δ and its derivative dΔ/dt are reported in Table 1. As can be observed $\Delta=\Delta(\rho, \theta_2-\theta_1)$ is an injective function, always positive, linearly dependent on the radius, and increasing with the difference ($\theta_2 - \theta_1$) with a minimum for ($\theta_2 - \theta_1$)=30° and with a maximum for ($\theta_2 - \theta_1$)=160°, the two limits of the assumed interval of feasibility.

The derivative of Δ, $d\Delta/dt=f(v_2-v_1, \theta_2-\theta_1)$, on the other hand, does not depend linearly on the radius (whereas v_1 and v_2 do) and it has a sign depending on the difference between the speeds of two vehicles; it is positive when the speed of vehicle B is higher than that of vehicle A, and it is negative in the opposite situation; it is proportional, but non linearly, to the difference ($\theta_2 - \theta_1$).

Table 1: Summary of parameters, basic relationships and formulae for Δ and dΔ/dt worked out for Case 1

Boundary conditions	$30° \le (\theta_2 - \theta_1) \le 160°$, $v_1, v_2 = const$, $\frac{d\theta_1}{dt} = \frac{v_1}{\rho}$, $\frac{d\theta_2}{dt} = \frac{v_2}{\rho}$
distances	$\overline{AB} = 2\rho \sin\left((\theta_2 - \theta_1)/2\right)$, $AB = \rho(\theta_2 - \theta_1)$
Δ calculation	$\Delta = AB - \overline{AB} = \rho\left[\theta_2 - \theta_1 - 2\sin\left((\theta_2 - \theta_1)/2\right)\right]$
Δ time derivative	$\frac{d\Delta}{dt} = (v_2 - v_1)\left[1 - \cos\left((\theta_2 - \theta_1)/2\right)\right]$

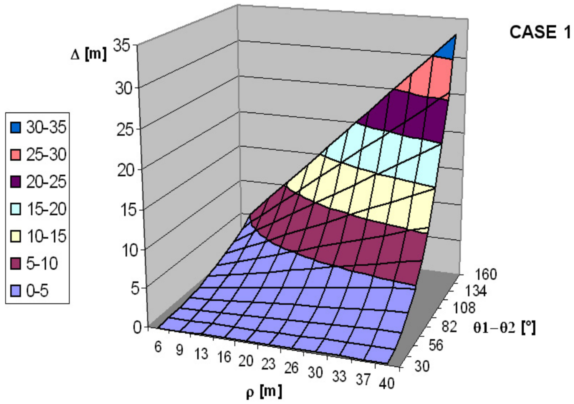


Fig. 4. Trend of Δ by varying ρ and (θ_2 - θ_1) for Case 1

The trend of Δ is reported in Fig. 4. The trend of dΔ/dt is strictly decreasing with θ_2-θ_1.

Since Δ is always positive, the evaluation of the role of visibility is particularly interesting. In order to make this evaluation, the situation when ρ, θ_2-θ_1 and v_1 determine a value for Δ/v_1 equal to the delay due to reaction time is considered.

t_r is assumed equal to 1.2s and hence values

$$\Delta/v_1 = 1.2\,\text{s} \tag{2}$$

must be found. In this calculation the speed of vehicle B does not enter into the formulation since it should be taken into account when calculating the time to collision.

The relationship between ρ and vehicular speed is not easy to write analytically therefore the values proposed by WS DOT (2002) are applied for the different circular trajectory radii. Given ρ and v_1 the value of (θ_2-θ_1) is calculated and hence also the value of Δ; after (θ_2-θ_1) is calculated, the arrow of the arc AB and then the radius of the central island which does not require unobstructed visibility over the central island can be calculated (Fig. 5). The term arrow indicates the maximum distance between the arc AB and the chord AB located variously over the circulating roadway and the central island, and defined by the difference (θ_2-θ_1).

For the sake of example three sample roundabouts are taken into consideration with a radius of circulating vehicles ρ and with a speed of vehicle A consistent with that of the radius (Table 2). As can be observed from Table 2 the sector of the area (central island or circulating roadway, with angle (θ_2-θ_1)) where the visibility must be kept free, decreases linearly when the radius increases. The arrow and the radius of the central island on the other hand, where visibility need not be free, increases. This result has obvious design developments since it imposes visibility as regards the central island (since the circulating roadway is free). Verification of this condition can be linked to the criterion 3 reported in the first section.

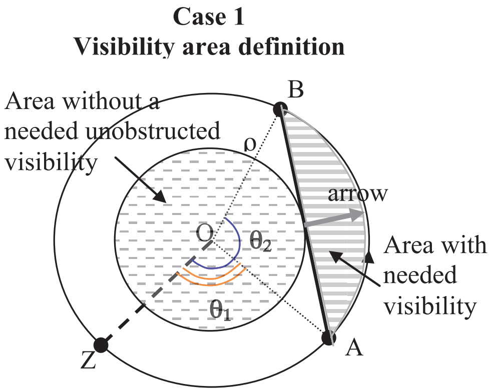


Fig. 5. Visibility arrow (the gray arrow) and area (of central island or circulating roadway) without visibility

Table 2. Some combinations of parameters and angles (θ2−θ1) of visibility needed to compensate a reaction time of 1.2s

ρ [m]	v_1 [km/h]	θ_2 - θ_1 [°]	arrow [m]	Area radius without needed visibility[m]
7	20	170	6.5	0.5
16	30	140	10.5	5.5
25	35	130	14.5	10.5

As concerns the derivative, three situations can occur when the speeds of the two vehicles vary.

- v1>v2 is the more significant case as regards safety; dΔ/dt is negative and decreasing implying that Δ is always decreasing with time, the higher the difference (θ2 - θ1);
- v1<v2; the derivative is positive and Δ increases with time; this case is less interesting since vehicles drive away and their reciprocal distance increases but it can show that safety conditions increase quicker in this situation;
- v1=v2; the derivative is nil since Δ is constant. This condition is quite interesting for safety concerns and corresponds to the many suggestions and norms that aim to increase safety.

3.2. Case 2

Case 2 can be split into two sub-sets (*a* and *b*, Fig. 6) according to the value assumed by the difference of the angles θ1 and θ2, whether it is greater or less than 90°. Nevertheless the final mathematical formulae representing these sub-sets are identical and therefore in the following discussion no further distinction is taken into account.

As in the following section for Case 3, the two different points of view, from vehicle A (shown as A→B) and from vehicle B (shown as B→A) are developed. In both points of view, in order to limit the analysis to real and relevant situations, the angles, θ_1 and θ_2, range in the interval [35°, 180°] and [35°, 60°] respectively. Higher values of these angles mean a position of vehicles that is very far from the yield line for vehicle B or beyond the middle of the circulatory roadway for vehicle A and therefore not particularly interesting; lower values indicate the position of vehicles that are too close to the conflict point, C, and are, therefore, more interesting but do not lead to any particular advantage.

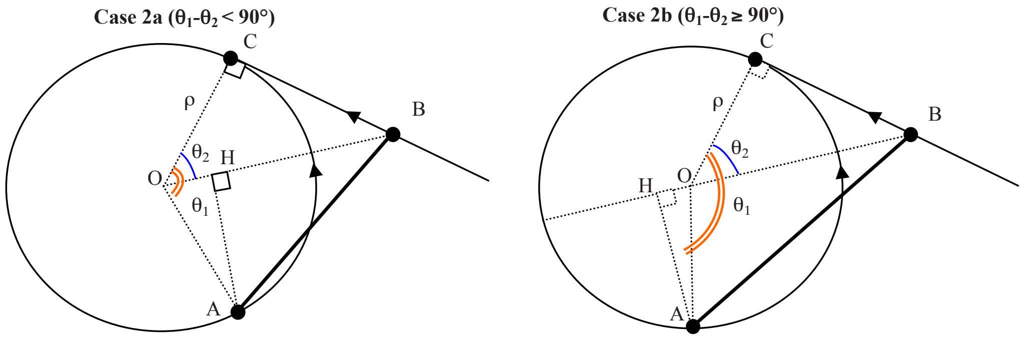


Fig. 6. Reference scheme for computation of relative distance between two vehicles (A e B) in Case 2 with (θ_1-θ_2)<90° (a) and with (θ_1-θ_2)≥90° (b)

Table 3. Summary of parameters, basic relationships and formulae for Δ and dΔ/dt worked out for case 2 (A→B)

Boundary conditions	$(\theta_1-\theta_2)>0°,\ 35°\le\theta_1\le180°,\ 35°\le\theta_2\le60°,\ \frac{d\theta_1}{dt}=\frac{v_1}{\rho},\ \frac{d\theta_2}{dt}=\frac{v_2\cos^2(\theta_2)}{\rho}$
distances	$\overline{AB}=\rho\left[\frac{1}{\cos^2(\theta_2)}+1-2\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_2)}\right]^{\frac{1}{2}},\ AC=\rho\theta_1$
Δ calculation	$\Delta=AC-\overline{AB}=\rho\left\{\theta_1-\left[\frac{1}{\cos^2(\theta_2)}+1-2\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_2)}\right]^{\frac{1}{2}}\right\}$
Δ time derivative	$\frac{d\Delta}{dt}=v_1-\frac{v_2\frac{\sin(\theta_2)}{\cos(\theta_2)}+(v_1-v_2)\cos(\theta_2)\sin(\theta_1-\theta_2)-v_2\cos(\theta_1-\theta_2)\sin(\theta_2)}{\cos^2(\theta_2)\left[\frac{1}{\cos^2(\theta_2)}+1-2\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_2)}\right]^{\frac{1}{2}}}$

3.2.1. Case 2 (A→B)

The calculation of distances AC and $\overline{AB}$, of their difference, Δ, and its derivative, dΔ/dt, is reported in Table 3. The calculus of $\overline{AB}$ is based on the properties of right triangles (OCB and ABH). In this case too $\Delta=\Delta(\rho, \theta_1, \theta_2)$ depends linearly on the radius, increases with θ_1 and depends non linearly on θ_2; it must be underlined that Δ is not always positive and for $\theta_1<65°$ (only when close to the conflict point) it becomes negative.

The derivative $d\Delta/dt=f(v_1,v_2, \theta_1, \theta_2)$ does not depend directly on the radius (while speeds v_1 and v_2 do) and it has a sign that is significantly dependent on the four variables. The trend of Δ and dΔ/dt is reported in Fig. 7 and 9 respectively. Since the relationship between Δ and ρ is linear, only the ratio Δ/ρ is drawn up and it is studied by varying θ_1 and θ_2. Since the derivative also depends on speeds, four more possibly significant combinations are proposed: $v_1>>v_2$, $v_1>v_2$, $v_1=v_2$, $v_1<v_2$. Δ/ρ always increases with θ_1 and decreases with θ_2. The dependence on θ_1 is much more significant especially when θ_1 is in or near 80° and 150°. When θ_1 is less than θ_2, Δ becomes negative. In

Table 4 the calculation of Δ and Δ/ρ is reported for the three sample roundabouts (ρ means the radius of the circulating vehicle) maintaining the hypothesis of compensating a reaction time of 1.2s ($1.2=\Delta/v_1$). In Fig. 7 the

curves with constant Δ/ρ satisfying data in
Table 4 can be easily extracted.

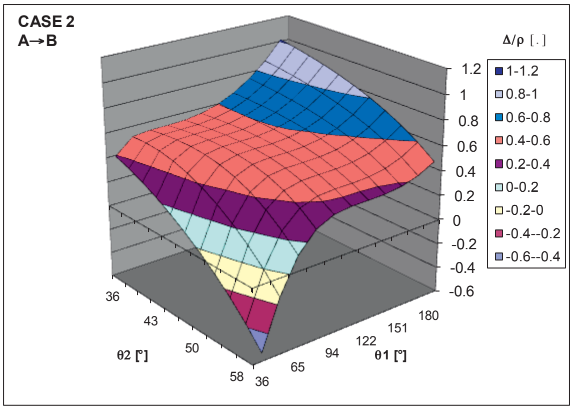


Fig. 7. Trend of Δ/ρ by varying θ2 and θ1 for Case 2 (A→B)

Table 4. Values of Δ and Δ/ρ by varying ρ for Case 2 (A→B)

ρ [m]	v_1 [km/h]	Δ [m]	Δ/ρ [.]
7	20	6.6	0.95
16	30	10	0.63
25	35	11.5	0.46

Only a few combinations of angles satisfies the equality and when ρ decreases the number of combinations decreases notably; when ρ=7 there is a limited interval of combinations ($36° \le \theta_2 \le 41°$ e $\theta_1=180°$). This implies that in this case the reaction time cannot be balanced by the roundabout features and visibility conditions and stopping distance must be guaranteed by the geometrical design of entries. The most favourable situation can be observed for values $\theta_1>90°$ when the impact on safety is less because vehicle A is closer to the conflict point. θ_2 does not appear to play a very decisive role.

The derivative of Δ depends significantly on the four parameters (θ_1, θ_2, v_1, v_2) and a highly non monotonous trend can be observed. If v_1 is set to 30km/h (as a reasonable speed for a circulating vehicle in the circulating roadway) we find that the derivative:

- when $v_1>>v_2$, is always positive, meaning that Δ increases in time, for every combination of θ_1 and θ_2;
- when $v_1>v_2$, is negative only when $\theta_1\approx 80°$ (and thus acquires the meaning of the critical angle); Δ increases for all the other combinations of feasible angles;
- when $v_1=v_2$, is negative for $\theta_1<110°$; Δ increases for all the other combinations of feasible angles;
- when $v_1<v_2$ is always negative; in other words Δ decreases in time for every combination of feasible angles.

For high values of θ_1 that is $\theta_1>150°$ and for very low ones that is $\theta_1<60°$ the derivative is always positive unless $v_1>v_2$. It seems obvious that to have cases with a Δ that is always positive the speed relationship should be $v_1>>v_2$ (or, at least, but less effectively $v_1>v_2$); this means that entering vehicles should have a lower speed than that of circulating vehicles. In the other cases a few favourable combinations do exist but only when vehicles are very far from the conflict point ($\theta_1>120°$).

3.2.2. Case 2 (B→A)

Calculation of distances $\overline{BA}$ and $\overline{BC}$, of their difference, Δ, and its derivative, dΔ/dt, is reported in

. Also in this case $\Delta=\Delta(\rho, \theta_1, \theta_2)$ depends linearly on the radius, increases with θ_2 and depends non linearly on θ_2 and θ_1. The derivative $d\Delta/dt=f(v_1,v_2, \theta_1, \theta_2)$ does not depend directly on the radius (while speeds v_1 and v_2 do) and it has a sign that is significantly dependent on the four variables. The trend of Δ and $d\Delta/dt$ is reported in Fig. 8 and 11 respectively. Since in this case the relationship between Δ and ρ is also linear, only the ratio Δ/ρ is drawn up and it is studied by varying θ_1 and θ_2. Since the derivative depends also on speeds, four more possibly significant combinations are proposed: $v_1<<v_2$, $v_1>v_2$, $v_1=v_2$, $v_1>>v_2$. Δ/ρ always decreases both with θ_1 and θ_2.

The values of Δ and Δ/ρ in the hypothesis of compensating a reaction time of 1.2s are the same as in

Table 4. In Fig. 8 the curves for a constant Δ/ρ are drawn up. Not all possible combinations of angles (θ_1 and θ_2) are admissible and when ρ decreases the favourable situations are reduced to the extent that for ρ=7m (Δ/ρ=0.95) no combination exists to satisfy the compensation hypothesis. For ρ=16m (Δ/ρ=0.63) combinations are limited to the values $\theta_1<90°$ and $\theta_2>40°$. For ρ=25m (Δ/ρ=0.46) favourable combinations exist for $\theta_1<110°$; also in this case low values of θ_2 can prevent the fulfilment of the compensation hypothesis. For values of $\theta_1>130°$, Δ is also negative so there is no point in carrying out further analyses.

The derivative depends significantly on the four parameters (θ_1 , θ_2, v_1, v_2) and shows a highly non monotonous trend, similar to the previous case A→B as regards the relationship to speeds: when v_1 increases with respect to v_2 the combinations of angles producing a positive derivative are greatly reduced. When $v_1<<v_2$ the derivative is positive only when $\theta_1>110°$; when $v_1>>v_2$ the derivative is positive only when $\theta_1<70°$. The most interesting combination is when $v_1>>v_2$ where the derivative is positive when θ_1 is low, that is when vehicle A is near the conflict point.

Table 5. Summary of parameters, basic relationships and formulae for Δ and dΔ/dt worked out for case 2 (B→A)

Boundary conditions	Same of Case 2 A→B
distances	$\overline{BA}=\rho\left[\frac{1}{\cos^2(\theta_2)}+1-2\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_2)}\right]^{\frac{1}{2}}=AB,\ \overline{BC}=\rho tg(\theta_2)$
Δ calculation	$\Delta=\overline{BC}-\overline{BA}=\rho\left\{tg(\theta_2)-\left[\frac{1}{\cos^2(\theta_2)}+1-2\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_2)}\right]^{\frac{1}{2}}\right\}$
Δ time derivative	$\frac{d\Delta}{dt}=v_2-\frac{v_2\frac{\sin(\theta_2)}{\cos(\theta_2)}+\left((v_1-v_2)\cos^2(\theta_2)\right)\sin(\theta_1-\theta_2)-v_2\cos(\theta_1-\theta_2)\sin(\theta_2)\cos^2(\theta_2)}{\left[\frac{1}{\cos^2(\theta_2)}+1-2\frac{\cos(\theta_1-\theta_2)}{\cos(\theta_2)}\right]^{\frac{1}{2}}}$

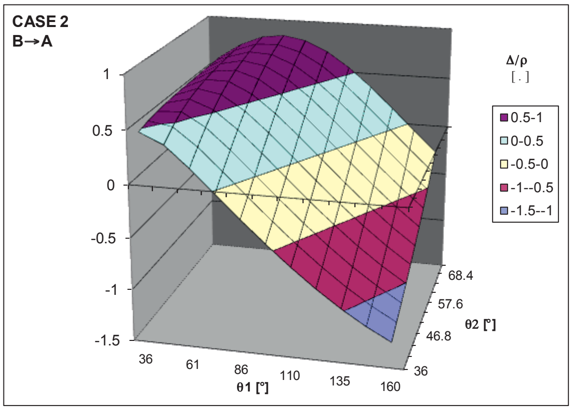


Fig. 8. Trend of Δ/ρ by varying θ2 and θ1 for Case 2 (B→A)

3.3. Case 3

Case 3 is proposed in the scheme of Fig. 9. As for case 2 the two different points of view, from vehicle A (A→B) and from vehicle B (B→A) are developed. The analysis is limited to real and relevant situations and therefore the angles can range in the interval [90°, 150°] and [35°, 60°] for θ1 and θ2 respectively.

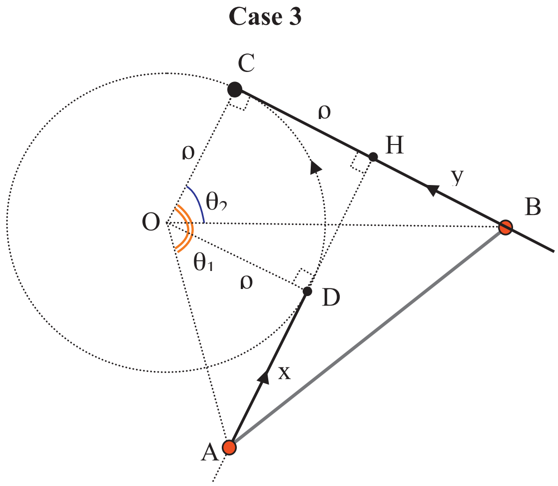


Fig. 9. Reference scheme for computation of the relative distance between vehicles (A and B) for Case 3 (both vehicles are entering)

3.3.1. Case 3 (A→B)

The calculation of distances $\overline{AB}$ and $\overline{ADC}$, of their difference, Δ, and its derivative, dΔ/dt, is reported in Table 6. Having assumed entering trajectories tangent to circulating circular path previous distances are easy calculated by using the properties of right triangles. Also in this case Δ=Δ(ρ, θ1, θ2) depends linearly on the radius, increases with θ1 and is related non linearly with θ2 and θ1. The derivative dΔ/dt=f(v1,v2, θ1, θ2) depends on speeds and on angles as well.

Table 6. Summary of parameters, basic relationships and formulae forD and dD/dt worked out for case 3 (A→B)

Boundary conditions	$(\theta_1-\theta_2)>0°,\ 35°\le\theta_1\le 180°,\ 35°\le\theta_2\le 60°,\ \frac{d\theta_1}{dt}=\frac{v_1}{\rho},\ \frac{d\theta_2}{dt}=\frac{v_2\cos^2(\theta_2)}{\rho}$
distances	$\overline{AB}=\rho\left[1+2tg\left(\theta_1-\frac{\pi}{2}\right)+tg^2\left(\theta_1-\frac{\pi}{2}\right)+\left(tg(\theta_2)-1\right)^2\right]^{\frac{1}{2}},\ \overline{ADC}=x+\rho\cdot tg\left(\theta_1-\frac{\pi}{2}\right)+\rho\frac{\pi}{2}$
Δ calculation	$\Delta=\overline{ADC}-\overline{AB}=\rho\left\{tg\left(\theta_1-\frac{\pi}{2}\right)+\frac{\pi}{2}-\left[1+2tg\left(\theta_1-\frac{\pi}{2}\right)+tg^2\left(\theta_1-\frac{\pi}{2}\right)+\left(tg(\theta_2)-1\right)^2\right]^{\frac{1}{2}}\right\}$
Δ time derivative	$\frac{d\Delta}{dt}=v_1-\frac{v_1+v_1\,tg\left(\theta_1-\frac{\pi}{2}\right)+v_2\left(tg(\theta_2)-1\right)}{\left[1+2tg\left(\theta_1-\frac{\pi}{2}\right)+tg^2\left(\theta_1-\frac{\pi}{2}\right)+\left(tg(\theta_2)-1\right)^2\right]^{\frac{1}{2}}}$

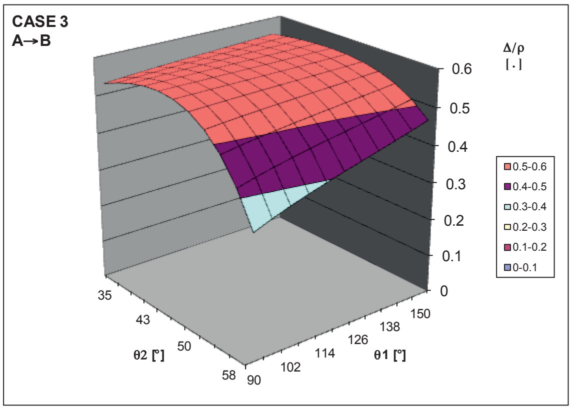


Fig. 10. Trend of Δ/ρ by varying θ2 and θ1 for Case 3 (A→B)

The trend of Δ is reported in Fig. 10. Given the linearity of Δ with respect to ρ, in this case also, the ratio Δ/ρ is drawn up. Since the derivative depends on speeds, the graphics for a few combinations obtained by varying v_2 and keeping v_1=30km/h constant are drawn up. Δ/ρ has a rather flat trend, not very sensitive to θ_1 and θ_2 (for angles less than about 45°). On the graphic of Fig. 10 the curves for a constant Δ/ρ are drawn up. Only for ρ > 18m combinations of angles satisfying the compensation hypotheses for reaction time can be found (Table 4). Only for very high radii (ρ > 35m) these compensation conditions can be found for all combinations of angles.

The derivative is almost always negative both for θ_1 < 120° and for θ_2 < 45°; generally for θ_2 ≥45° it is negative. Some effects exist due to speeds other than those related to angles: the greater v_1 is than v_2 the more the derivative assumes low positive values but on the other hand tends to have a smaller interval of negative values.

In general complete visibility at least for θ_2 < 45° must be guaranteed; for higher values of θ_2 a benefit may exist only for roundabouts with a very large diameter (> 50÷60m). In fact for roundabouts with a small diameter no compensation effects can exist and therefore safety obeys only the rule of a stopping distance that also includes the reaction time.

Conditions of greater advantages occur when $v_1 > v_2$ and increase when the inequality increases. Also in this case as regards safety concerns the utility of reducing as much as possible the speed of the entering vehicle is clear; on the other hand it must be underlined that by reducing speed the time to cross the roundabout increases and, therefore, roundabout capacity is, also reduced.

3.3.2. Case 3 (B→A)

The calculation of distances $\overline{BA}$ and $\overline{BC}$, of their difference, Δ, and its derivative, dΔ/dt, is reported in Table 7. Also in this case $\Delta=\Delta(\rho, \theta_1, \theta_2)$ depends linearly on the radius, increases with θ_1 and is related non linearly with θ_2 and θ_1. The derivative $d\Delta/dt=f(v_1,v_2, \theta_1, \theta_2)$ depends on speeds and angles as well.

The trend of Δ is reported in Fig. 11. Also for this sub-set, given the linearity of the relationship between Δ and ρ, in the figures the ratio Δ/ρ is reported. The derivative depends on speeds therefore four combinations of v_2 are considered assuming v_1=30km/h. Δ/ρ is always decreasing both for θ_1 and θ_2; it is negative for θ_1>120°, and this threshold is lowered when θ_2 decreases. The derivative increases with θ_1 and θ_2 until speed v_2 is much lower than v_1. For situations with $v_1 \geq v_2$ the derivative is almost always negative showing that in this sub-set vehicle B has limited advantages from an anticipated perception of vehicle A.

Table 7. Summary of parameters, basic relationships and formulae for Δ and dΔ/dt worked out for case 3 (B→A)

Boundary conditions	Same of Case 3 (A→B)
distances	$\overline{BA}=\rho\left[1+2tg\left(\theta_1-\frac{\pi}{2}\right)+tg^2\left(\theta_1-\frac{\pi}{2}\right)+\left(tg(\theta_2)-1\right)^2\right]^{\frac{1}{2}}=\overline{AB},\ \overline{BC}=\rho\cdot tg(\theta_2)$
Δ calculation	$\Delta=\overline{BC}-\overline{BA}=\rho\left\{tg(\theta_2)-\left[1+2tg\left(\theta_1-\frac{\pi}{2}\right)+tg^2\left(\theta_1-\frac{\pi}{2}\right)+\left(tg(\theta_2)-1\right)^2\right]^{\frac{1}{2}}\right\}$
Δ time derivative	$\frac{d\Delta}{dt}=v_2-\frac{v_1+v_1\,tg\left(\theta_1-\frac{\pi}{2}\right)+v_2\left(tg(\theta_2)-1\right)}{\left[1+2tg\left(\theta_1-\frac{\pi}{2}\right)+tg^2\left(\theta_1-\frac{\pi}{2}\right)+\left(tg(\theta_2)-1\right)^2\right]^{\frac{1}{2}}}$

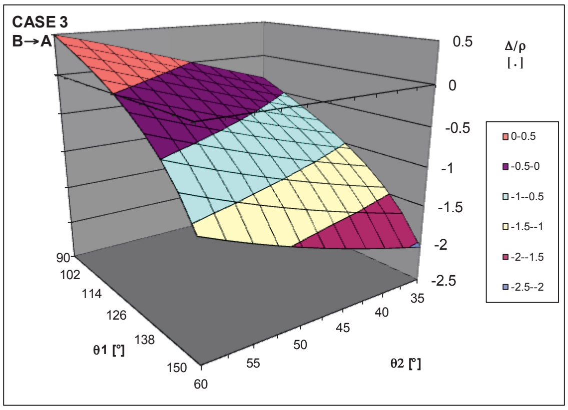


Fig. 11. Trend of Δ/ρ by varying θ_2 and θ_1 for Case 3 (B→A)

For small values of ρ no combinations of angles exist to satisfy the hypotheses on reaction time (see Table 4 for values). Only with high values of ρ (>25m) and with small $\theta 1$ and high $\theta 2$ some favourable combinations exist. Hence the advantages due to a complete visibility of vehicles are limited to roundabouts with a high radius (ρ>25m). An increase in advantages can be achieved when $v_2 > v_1$ but to remain within actual working conditions, advantages can be obtained when speeds are equal ($v_2 = v_1$) and $\theta_2<45°$. This condition identifies the circular sector of the circulating roadway at the left of the entry.

4. Analysis with real trajectories

Trajectories used for the analyses of this section are real and collected by a video-camera survey carried out on a four-leg roundabout located in an Italian urban environment (Fig. 12). The roundabout has an external diameter of about 50m and the circulatory roadway is 13m wide with an apron of 1.5m. With regard to the legs, the average width of the entries with two lanes is 8m, and 6m for the exits.

Fig. 12. The layout of the sample roundabout

The instrumentation used to collect and evaluate data is made up of a vision system and a RTK-GPS system. The vision system consists of a camera with a resolution of 1360x1024 pixels and the RTK-GPS system is composed of a base station and a rover, which is a probe vehicle connected by a radio link to the base station. The vision system provides information on vehicular flow by means of the processing of images recorded by the video camera, while the RTK-GPS system has been used to produce data useful for calibrating and evaluating the vision system (its error is about a few centimetres). A detailed description of this method as to the quality of processed data can be found in (Mussone et al., 2011). Vehicular trajectories on pavement surfaces are calculated from trajectories on the video camera image plane by using a special technique which hinges on homography (a geometrical relationship) between the two planes for each vehicle class. In fact, the trajectories are made up of a set of consecutive points collected every 20ms (if vehicle speed is 10m/s this means a spacing between points of 20cm). Algorithm errors are calculated through comparison with the RTK-GPS system: localization has a median error of 0.375m and speed an average of 0.11km/h.

Then extracted trajectories are worked out by a MATLAB programme that allows us to calculate all information about vehicular motion and distances on the plane of road pavement.

Figure 13 shows the various trajectories considered for the analysis. For each combination of two trajectories the relative position of vehicles is considered and Δ is calculated (in meters); the points on the trajectories of vehicles A and B where Δ is positive are linked by a gray line leading to the gray hatched area in the figures. These combinations refer to case 2 (Fig. 13a and d) and to case 3 (Fig. 13b and d). Combinations for case 1 are not taken into consideration because real trajectories for this case are as circular as in the analytical analysis. It can be observed that the light gray area is the largest one for cases A→B (Fig. 13a, b, d, and f) and much more limited for cases B→A ((Fig. 13c and e). The extension of the light gray area depends on the type of trajectory that crosses the considered A trajectory and can vary significantly according to its length and along the circulating roadway (Fig. 13d, e, and f). It must be stressed that this light gray area could be significantly reduced if only combinations of points on the two trajectories which have similar times to conflict point are considered; but when vehicle A has a higher speed than vehicle B (for example v_1=50km/h and v_2=30km/h) the majority of combinations remains active. The limitations for cases B→A could also be due to the limited extension (due to the video camera's field of view) of trajectories of vehicle B above the entry point though being almost linear further advantages for the trajectories should not be out of the ordinary.

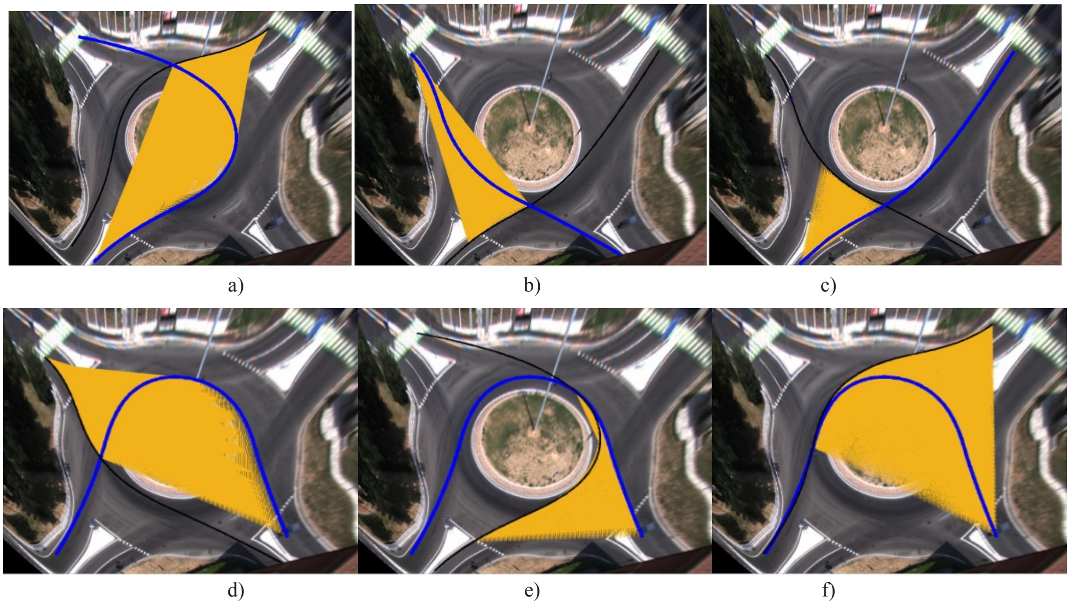
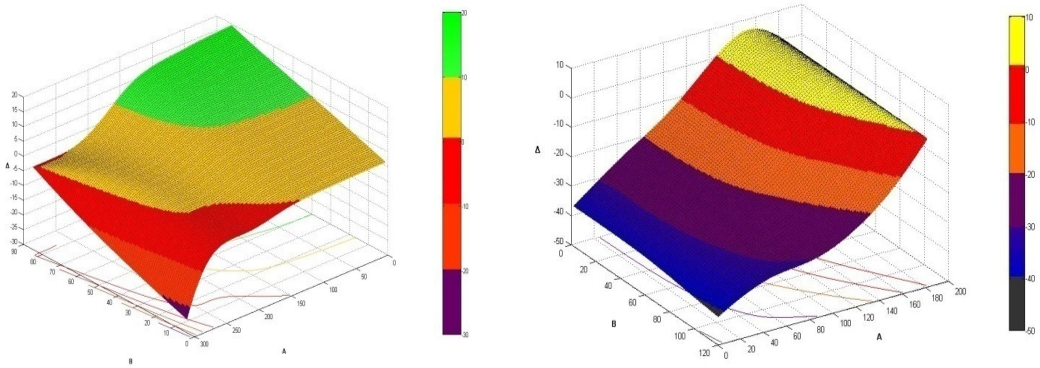


Fig. 13. Crossing of two trajectories (A has the gray line and B the black line) and the area (gray hatched) where Δ is positive: a) Case 2, A→B; b and c) The effect on Δ by changing the reciprocal position of A and B for the same trajectories Case 3, A→B (b), and Case 3, B→A (c); d, e, and f) The effect on Δ by changing the crossing trajectory of vehicle B, Case 2, A→B (d), Case 3, B→A (e), and Case 3, A→B (f)

From all the figures and especially from those in Fig. 13a, d and f it can be gathered how wide the visibility over the central island should be. It seems much wider than when assessed by the analytical approach but it depends however on the assumed starting point along the trajectory of vehicle A: in fact in the analytical approach it depends on the choice of angles θ (which are assumed to be quite limited) while in image processing it coincides with the maximum field of view of the video-camera.

Fig. 14 shows how Δ varies when changing the position along trajectories (from the entry to the conflict point) of the two vehicles (A and B) for some sample trajectories. This figure is in a 3D form with the positions of vehicle A on the x-axis, vehicle B on the y-axis and Δ on the Z-axis. The light gray and dark gray layers are the two layers around the zero, positive and negative respectively; these positions along trajectories can be likened to the values of θ_1 and θ_2, but since these vehicular trajectories are not exactly regular curves (as in the analytical approach) this comparison cannot be considered immediate. The closer to zero the position number the closer to entry (on the give way line) the vehicle; therefore the relationship between angles and position numbers is the reverse. It must be underlined also that taking into account the whole trajectory generally means considering angles ranging in wider intervals than was the case in the theoretical analysis. Generally results are quite similar to those obtained by the analytical approach; in particular Case 2 A→B is almost identical.

Considering the A→B cases Δ increases monotonically when vehicle A is near the entry point and it is much less sensitive to vehicle B; this proves that vehicle B should be entirely visible for vehicle A. Case 2 makes it possible to obtain the highest values of Δ; this maximum is reached when vehicle A is at the beginning of the trajectory and vehicle B is close to the conflict point. Effective advantages depend on the relative speed of vehicles and these benefits can all actually be achieved when vehicle A has a higher speed than vehicle B.



a) **b)**

Fig. 14. Trend of Δ by changing the positions of the two vehicles: a) Case 2, A→B; b) Case 2, B→A

Considering the B→A cases Δ increases almost monotonically when vehicle A comes close to the conflict point; vehicle B affects results only in Case 3 when Δ increases when vehicle B comes close to the conflict point.

The derivative is calculated here by means of the incremental ratio between the two points present (on the two trajectories of vehicles A and B) and the two new points estimated by a time step of 0.5 seconds (25 frames). Thus the values of dΔ/dt so calculated are an approximation that can lead to some (negligible for us) errors of discretization.

Results show that the derivative changes with the relative speed of the two vehicles in the same way for all cases in the A→B scenarios. It can be observed that situations with both the highest values of dΔ/dt and the largest number of combinations with a positive dΔ/dt are those with v_1 higher than v_2; the higher the speed difference the higher the derivative. This means that there is always a benefit in reducing the speed of the entry vehicles, as predicted in the analytical approach.

When on the other hand v_2 is higher than v_1 the combinations with a positive derivative are limited to small intervals far from the conflict point for vehicle A and close to it for vehicle B. When speeds are equal, both for high and for low values, the surface of dΔ/dt does not change much and positive values are concentrated when the two vehicles are close to the conflict point.

Comparisons between Cases are also analysed assuming speeds v_1 and v_2 equal to 50km/h. The significant difference between the A→B cases and the B→A ones can be observed; while there is no advantage in the A→B cases having the same speed; the B→A cases find themselves in the most favourable conditions. For our purposes this happens just when vehicles are approaching the conflict point.

5. Conclusions

The paper has investigated the role of curvilinear trajectories of vehicles circulating on a roundabout with a particular concern for the effect of the reciprocal sight on reaction time and finally on the capacity and safety of two vehicles.

The proposed approach is both mathematical (analytical) and empirical (based on real data). Though some simplified hypotheses on trajectories and vehicular kinematics are assumed for the mathematical part, the proposed cases show a remarkable similarity between the approaches.

Effects on visibility and circular trajectories and on the potential reduction of the stopping distance, by anticipating the sight of a conflicting vehicle in respect of the real conflict point, are shown in many cases.

This anticipation effect is obviously potential and related to the perception of drivers; for these reasons it should be tested also in the field according to a specific research project. Compensation for reaction time (so as a driver could have a personal reaction time that is longer than necessary) in the calculation of braking distance creates a safer situation and also increases capacity since it allows a better fit of vehicles (with shorter gaps) especially when they are travelling at different speeds.

The influence on visibility is very significant especially when both vehicles are circulating in the circulating roadway (Case 1). In this case a sector on the central island with a total overtaking sight (free from obstacles) can be singled out in order to improve this aspect.

In the case when a vehicle is circulating in the circulating roadway and the other is entering the roundabout (Case 2) the advantages of a roundabout are somewhat less relevant and linked to a certain combination of angles, θ_1 and θ_2. The situations with vehicles going at the same speed present some advantages but these are greater when the vehicle running in the circulating roadway is going faster than the one entering. This confirms the common rules of design to reduce the speed of entering vehicles.

In the case when both vehicles are entering (Case 3) there are fewer advantages from having free visibility when θ_1 is very high (>120°), vehicle A is very far from the conflict point and this calls into question the criterion imposing free visibility on the lateral entry link as laid down by Australian and some USA norms.

Similar considerations could also be made for the effects of visibility on pedestrian crossings where the expected benefits (in order to avoid actively collision) should be mainly for the vehicle.

Future research consists also in the extension of the study to cases with more than two vehicles for building a model of driver behaviour. The definition of a behavioural model is not negligible but as in all situations related to the driver-vehicle-road interaction it can help us to understand phenomena related to the geometrical characteristics and to visibility. This behavioural model should represent a module of a simulating tool by which to quantify the effect of visibility on capacity and safety. In particular, it is the author's opinion that simply visibility could explain some of the many differences present in all known roundabout capacity models and the differences between these and measures taken in the field.

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