Theoretical modelling of hot gas ingestion through turbine rim seals

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Abstract   The rim seals of gas turbines are used to prevent or reduce the ingestion of hot mainstream gas into the wheel-space between the turbine rotor and its adjacent stationary casing. The ingestion is caused by local pressure differences between the mainstream and the wheel-space; ingress usually occurs where the mainstream pressure is higher than that in the wheel-space and egress occurs where it is lower. Sealing air, which is supplied to the wheel-space, flows through the seal clearance and joins the mainstream flow. Too much sealing air is inefficient; too little can lead to disastrous consequences. The nozzle guide vanes create three-dimensional (3D) variations in the distribution of pressure in the mainstream annulus and the turbine blades create unsteady effects. Computational fluid dynamics (CFD) is both time-consuming and expensive for these 3D unsteady flows, and engine designers tend to use correlations or simple models to predict ingress. This paper describes the application of simple ‘orifice models’, the analytical solutions of which can be used to calculate the sealing effectiveness of turbine rim seals. The solutions agree well with available data for externally-induced ingress, where the effects of rotation are negligible, for rotationally-induced ingress, where the effects of both external flow and rotation are significant.

1. Introduction

Figure 1 illustrates a typical high-pressure gas-turbine stage showing the rim seal and the wheel-space between the stator and the rotating turbine disc. Sealing air is
supplied to the wheel-space, and it leaves through the clearances in the rim seal.

Figure 2 shows the view looking radially inward into the space between the stationary vanes and rotating blades in the annulus of an experimental rig. The figure, which was adapted from the paper of Zhou et al. [1], shows that the flow past the vanes and blades creates a three-dimensional (3D) variation of pressure radially outward of the rim seal. Ingress and egress occur through those parts of the seal clearance where the external pressure is instantaneously higher and lower, respectively, than that in the wheel-space; this non-axisymmetric type of ingestion is referred to here as externally-induced (EI) ingress. Although the sealing air can reduce ingress, too much air reduces the engine efficiency and too little can cause serious overheating, resulting in damage to the turbine rim and blade roots.

Even when the external flow is axisymmetric, so that there is no circumferential variation of external pressure,
ingress can still occur. The reason for this is that the rotating fluid in the wheel-space creates a radial gradient of pressure, so that the pressure inside the wheel-space can drop below that outside. The so-called ‘disc-pumping effect’ causes a radial outflow of fluid, or egress, near the rotating disc, and the low pressure in the wheel-space causes ingress of external fluid through the rim seal into the wheel-space. This type of ingestion is referred to here as rotationally-induced (RI) ingress.

In gas turbines, EI ingress is usually the dominant type of ingestion. However, in double rim seals (like that shown in Figure 1) the circumferential variation in pressure is attenuated in the annular space between the two seals. If the annular space is large enough to damp out the pressure asymmetry, EI ingress can dominate for the outer seal and RI ingress can dominate for the inner one. The term combined ingress (CI) is used here for the case where the EI ingress is the same order-of-magnitude as the RI ingress.

The designer of internal air systems wants to know the following: the most effective seal geometry; how much sealing air is required to prevent ingress; when ingress occurs, how much hot gas enters the wheel-space; and how does this hot gas affect the temperatures on the rotating and stationary components. Over the years, there have been many experimental and computational studies of the ingress problem. Computational fluid dynamics has had some success in computing the sealing effectiveness, but 3D unsteady CFD is time-consuming and expensive and there are questions about the accuracy of the turbulence models used in the CFD codes. Designers still use simple ‘cheap and cheerful’ models to solve the ingress problem, particularly as this problem is only one part of a much larger air-system network.

Orifice models have been particularly successful in predicting the sealing effectiveness of rim seals. These models treat the seal-clearance as an orifice and use variations of Bernoulli’s equation to relate the sealing flow rate to the pressure drop across the seal. In references [2–5], models recently developed at the University of Bath (referred to below as the Bath orifice models) have had good success in predicting RI, EI and CI ingress, and these models will be described below. Section 2 includes a short overview of ingress research; Section 3 outlines the orifice equations; Section 4 includes the main theoretical solutions; Section 5 shows comparisons between the theoretical solutions and experimental data; and Section 6 summarises the salient conclusions.

2. Brief review of ingress research

2.1. Rotationally-induced ingress

In 1970, at the University of Sussex, Bayley and Owen [6] presented experimental results for a simple rotor–stator system with an axial-clearance rim seal in which there was a superposed radial flow of air that discharged through the seal into the atmosphere; there was no external annulus on the rig. Owing to the sub-atmospheric pressure created by the rotating flow in the system, external (atmospheric) air could be drawn into the wheel-space. Increasing the superposed flow rate increased the relative pressure inside the wheel-space and consequently reduced the amount of ingested air. At sufficiently high superposed flow rates, where \( C_{w,0} \geq C_{w,min} \), ingress did not occur. (These terms, and those used below, are defined in the nomenclature, but it should be noted that \( C_{w,0} \) is the nondimensional superposed flow rate and \( C_{w,min} \) is the minimum value needed to prevent ingress.)

Bayley and Owen used a simple ‘orifice model’, in which the seal clearance was treated as an orifice, and they showed that \( C_{w,min} \) was proportional to \( R e_\phi \), the rotational Reynolds number, and to \( G_c \), the seal clearance ratio. Using their measured pressures for \( G_c = 0.0033 \) and 0.0067, and for \( R e_\phi \leq 4 \times 10^6 \), they proposed the following criterion:

\[
C_{w,min} = 0.61G_c R e_\phi
\]  

(1)

(This equation has been widely quoted and has often been used in situations – like EI ingress – where it has no validity.)

In the following years, most of the published research into RI ingress was conducted at Sussex [7–13] and at Hartford in the United States [14,15]. Graber et al. [14] reported extensive concentration measurements in a rotating-disc rig, which was used to determine the effects of seal geometry, rotational Reynolds numbers and the level of swirl in the external annulus on the sealing effectiveness. Their results appeared to show that the external swirl made no significant difference to the effectiveness. Phadke and Owen [8] used flow visualisation, pressure and concentration measurements to determine \( C_{w,min} \) in a rig without external flow – for seven different seal geometries and for a wider range of clearances than that used by Bayley and Owen. The experiments showed that radial-clearance seals were more effective than axial-clearance ones.

It should be pointed out that the term rotationally-induced ingress was not used in any of the above papers. Phadke and Owen [9,10] referred to rotation-dominated and external-flow-dominated regimes; these are the same as the RI and EI ingress referred to in this paper. In the 1980s, interest in RI ingress was overtaken by concern about the more important case of EI ingress.

2.2. Externally-induced ingress

Abe et al. [16], who used a turbine rig with vanes in the annulus upstream of the rim seal, were the first to show that ingress could be dominated by the external flow in the annulus rather than by the rotational speed of the disc. The authors tested several rim-seal geometries and
identified three things that affected ingress: the ratio of the velocities of the sealing air and the flow in the annulus; the rim-seal clearance; the shape of the rim-seal.

Phadke and Owen [9,10] determined $C_{w,\text{min}}$ in a simple rotor–stator system with a number of different rim-seal geometries, with and without an external flow of air. (It should be noted that there were no vanes or blades in the external annulus of their rig; the authors created the circumferential pressure asymmetries referred to below by blocking sections of the annulus with wire mesh.) They observed both RI ingress (where, with no external flow, $C_{w,\text{min}}$ increased with increasing $Re_\phi$) and EI ingress (where, with non-axisymmetric external flow, $C_{w,\text{min}}$ was independent of $Re_\phi$ and increased with increasing $Re_w$, the axial-flow Reynolds number in the external annulus).

Phadke and Owen [10] correlated their results for EI ingress, based on flow visualisation for a number of different seal geometries, by

$$C_{w,\text{min}} = 2\pi KG_c P_{\text{max}}^{1/2}$$

where

$$P_{\text{max}} = \frac{1}{2} C_{p,\text{max}} Re_w^2$$

$C_{p,\text{max}}$ is a nondimensional pressure difference in the external annulus and $K$ is an empirical constant; the data were correlated with $K=0.6$. It should be noted that, although $P_{\text{max}}$ is the controlling parameter, for a given external geometry or in an engine, $C_{p,\text{max}}$ is expected to depend only weakly on either $Re_\phi$ or $Re_w$. Hence $C_{w,\text{min}} \propto Re_w$, as Eqs. (2) and (3) show.

Hamabe and Ishida [17] made measurements of the sealing effectiveness in a turbine rig fitted with upstream guide vanes but with no downstream blades. For EI ingress through a simple axial-clearance seal, they correlated the effectiveness with a nondimensional parameter similar to that used by Phadke and Owen, and the results of their orifice model were in reasonable agreement with their measurements. In their model, they approximated the shape of the external distribution of pressure using simplified wave forms, which showed that the prediction of ingress depended on the shape used.

The Sussex group [18–20] conducted several EI ingress studies. Chew et al. [18] included upstream nozzles in the annulus; their orifice model performed less well than that of Hamabe and Ishida in predicting the effectiveness but their steady 3D CFD computations gave encouraging results. They also measured the discharge coefficients, $C_{d,i}$, for the rim seal when there was no disc rotation. The measurements, like those of Hamabe and Ishida, showed that, for outflow, $C_{d,e}$ decreased monotonically with increasing external flow rate. However, for inflow $C_{d,i}$ reached a minimum at a particular ratio of sealing-to-annulus velocity above which it increased with increasing external flow rate.

The first published data for a turbine rig with both vanes and blades was presented by Green and Turner [19]; somewhat surprisingly, they found that at low sealing flow rates the addition of the rotating blades reduced the ingress compared with the case when only vanes were used. Gentilhomme et al. [20] made ingress measurements and carried out computations for a single-stage turbine rig with both vanes and blades. The circumferential pressure in the annulus, obtained by CFD, was used in conjunction with an orifice model to estimate the effectiveness.

The Aachen group [21–25] conducted many ingress studies in turbine rigs with vanes and blades. Of particular relevance here is the paper by Bohn and Wolff [24] who presented a correlation for the sealing effectiveness, $e$, in terms of $C_{w,0}$, $G_c$ and $C_{p,\text{max}}$ and their correlations for the four seal geometries shown in Figure 3 display the linear variation of $C_{w,\text{min}}$ with $C_{p,\text{max}}$ that was found by Phadke and Owen; the value of $K=0.6$ suggested by the latter authors produces a conservative estimate for $C_{w,\text{min}}$.

The Hartland team [26–31] has made a significant contribution to the ingress literature. Johnson et al. [29] used an orifice model to obtain good predictions of the effectiveness measurements in the turbine rig of Bohn et al. [25]. For the external circumferential pressure distribution in their model, they used the values obtained from 2D time-dependent CFD, which allowed the effects of the vanes and blades to be taken into account. A modified version of their orifice model was also successfully applied by Johnson et al. [31] to the ingress measurements made on a turbine rig in Arizona State University.

CFD has been used with some success for the ingress problem, but care must be exercised in computing these unsteady 3D flows. Recently Zhou et al. [1] solved the unsteady 3D equations for a computational model of an experimental ingress rig, and they showed how a ‘thin-seal approximation’ could be used in conjunction with the steady 3D equations to obtain realistic values of the
3. Orifice equations for swirling flow

The derivations given below summarise the more extensive derivations given in references [2,3].

The orifice model is based on an imaginary ‘orifice ring’, as shown in Figure 4 for an axial-clearance seal. Egress and ingress simultaneously cross different parts of the orifice ring through the elemental areas, \( \delta A_e \) and \( \delta A_i \), the sum of which is equal to the clearance area, \( A_c \), of the seal. Egress flows through a stream tube in the wheel-space where the static pressure is \( p_1 \) and emerges in the external annulus where the static pressure is \( p_2 \); conversely, ingress starts in the annulus and emerges in the wheel-space.

For the inviscid equations, it is assumed that there is continuity of mass and energy inside the separate stream tubes for egress and ingress but there is a discontinuity in the pressure across the orifice ring. In addition, angular momentum is conserved, so that free-vortex flow occurs and \( rV_\phi \) is constant. The principal ‘orifice assumptions’ are that \((r_2 - r_1)/r_1 \ll 1\), and that \( V_{r,1}^2 \ll V_{r,2}^2 \) for egress and vice versa for ingress. Although the equations are derived for inviscid flow, discharge coefficients, \( C_d \) (determined empirically and analogous to those used for the ‘standard orifice equations’ for flow from a reservoir into the atmosphere) are introduced to account for losses.

The steady incompressible equations can be obtained from the inviscid radial momentum equation for a rotating fluid. For the orifice equations, it is assumed that the axial and tangential gradients of velocity are much smaller than the radial gradient, so that

\[
V_r \frac{\partial V_r}{\partial r} - \frac{V_\phi^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r}
\]  

Integrating Eq. (4) between stations 1 and 2 in a stream tube crossing the sealing ring gives

\[
\int_{r_1}^{r_2} \left( V_r \frac{dV_r}{dr} - \frac{V_\phi^2}{r} \right) dr = \frac{1}{\rho} \int_{r_1}^{r_2} \frac{\partial p}{\partial r} dr
\]

As \( rV_\phi \) is constant, it follows that

\[
\left( V_{r,2}^2 - V_{r,1}^2 \right) + \left( V_{\phi,2}^2 - V_{\phi,1}^2 \right) = \frac{2(p_1 - p_2)}{\rho}
\]

For egress, the air flows radially outward through a stream tube from a source in the wheel-space at \( r = r_1 \) into the external annulus via a small orifice of area \( \delta A_e \) (normal to the radial direction) in the orifice ring. For ingress, the air flows radially inward from a source in the annulus at \( r = r_2 \) into the wheel-space via a small orifice of area \( \delta A_i \) in the ring. Consequently for egress, it is assumed that \( V_{r,2}^2 \gg V_{r,1}^2 \), where \( V_{r,2} = V_{r,e} \) is the egress velocity through \( \delta A_e \); for ingress, it is correspondingly assumed that \( V_{r,1}^2 \gg V_{r,2}^2 \), where \( |V_{r,1}| = V_{r,i} \) is the ingress velocity through \( \delta A_i \). As \( r_1 V_{\phi,1} = r_2 V_{\phi,2} \), it follows that for egress Eq. (6) can be expressed as

\[
V_{r,e} = \sqrt{\frac{2(p_1 - p_2)}{\rho} + V_{\phi,1}^2 \left( 1 - \frac{r_1^2}{r_2^2} \right)}
\]

where \( V_{r,2} \) has been replaced by \( V_{r,e} \), and for ingress

\[
V_{r,i} = \sqrt{\frac{2(p_2 - p_1)}{\rho} - V_{\phi,2}^2 \left( \frac{r_2^2}{r_1^2} - 1 \right)}
\]

where \( V_{r,1}^2 \) has been replaced by \( V_{r,i}^2 \).

It can be seen from Eqs. (7) and (8) that swirl in the wheel-space increases egress, which is often referred to as the disc-pumping effect; conversely, swirl in the annulus decreases ingress. It should also be noted that egress can occur even if \( p_2 > p_1 \), which is always the case for RI ingress.

It is convenient to introduce the swirl ratio, \( \beta \), where

\[
\beta = \frac{V_\phi}{Qb}
\]

Hence Eqs. (7) and (8) can be written as

\[
\frac{V_{r,e}}{Qb} = \sqrt{\frac{p_1 - p_2}{1/2\rho \Omega^2 b^2} + \beta_1^2 \left( 1 - \frac{r_1^2}{r_2^2} \right)}
\]

and

\[
\frac{V_{r,i}}{Qb} = \sqrt{\frac{p_2 - p_1}{1/2\rho \Omega^2 b^2} - \beta_2^2 \left( \frac{r_2^2}{r_1^2} - 1 \right)}
\]

It is useful to use separate discharge coefficients for egress and ingress, \( C_{d,e} \) and \( C_{d,i} \), which are – in effect – the ratios of the real to the inviscid mass flow rates in the stream tubes. (The discharge coefficients act as ‘fudge factors’ that take into account the viscous and turbulent shear stresses, which are neglected in the
inviscid equations.) The above two equations, which are valid for both EI and RI ingress, can then be expressed as
\[
\frac{V_{r,e}}{\Omega b} = C_{d,e} \sqrt{C_{\beta_1} - C_p}
\]
when \( C_{\beta_1} \geq C_p \), and
\[
\frac{V_{r,i}}{\Omega b} = C_{d,i} \sqrt{C_p - C_{\beta_2}}
\]
when \( C_p \geq C_{\beta_2} \), where
\[
C_p = \frac{p_2 - p_1}{2\rho \Omega^2 b^2}, \quad C_{\beta_1} = \beta_1^2 \left( 1 - \frac{r_1^2}{r_2^2} \right), \quad C_{\beta_2} = \beta_2^2 \left( \frac{r_2^2}{r_1^2} - 1 \right)
\]

The mass flow rates can be calculated by integrating the velocities across the seal clearance, so that
\[
\dot{m}_e = \rho \int A_e V_{r,e} \, dA_e \quad \text{and} \quad \dot{m}_i = \rho \int A_i V_{r,i} \, dA_i
\]
where
\[
A_e + A_i = A_c = 2\pi h b_c
\]
and where \( s_c \) and \( A_c \) are the effective clearance and area, respectively, of the seal. The superposed mass flow rate, \( \dot{m}_0 \), is given by
\[
\dot{m}_0 = \dot{m}_e - \dot{m}_i
\]

4. Solution of orifice equations

Only the principal solutions are given below, and for details of their derivation the reader is referred to the papers cited in the text.

4.1. Definitions

The flow parameter, \( \Phi \), is usually defined as
\[
\Phi = \frac{C_e}{2\pi G_c Re_c}
\]
(18)

Although the viscous terms cancel, this definition disguises the fact that \( \Phi \) is an inertial parameter. It is more appropriate to use an alternative definition, which is equivalent to Eq. (18), where
\[
\Phi = \frac{U}{\Omega b}
\]
(19)

and \( U \) is the bulk-mean velocity of fluid through the seal clearance. The symbols \( \Phi_e, \Phi_p, \Phi_o \) denote the flow parameters for egress, ingress and the sealing flow, respectively, and \( \Phi_{min} \) is the value of \( \Phi_o \) when the ingress is zero. That is,
\[
\Phi_{min} = \frac{U_{min}}{\Omega b} = \frac{C_{w,\min}}{2\pi G_c Re_c}
\]
(20)

The continuity Eq. (17) becomes
\[
\Phi_o = \Phi_e - \Phi_i
\]
(21)

and, for \( \Phi_o \leq \Phi_{min} \), the sealing effectiveness can be calculated from
\[
\varepsilon = 1 - \frac{\Phi_i}{\Phi_p} = \frac{\Phi_o}{\Phi_p} = \frac{\Phi_o}{\Phi_o + \Phi_i}
\]
(22)

That is, \( \varepsilon = 0 \) when \( \Phi_i = 0 \), and \( \varepsilon = 1 \) when \( \Phi_o = \Phi_{min} \).

Although the effectiveness is a convenient parameter, the designer wants to know how much hot gas enters the wheel-space when \( \Phi_o < \Phi_{min} \). This involves calculating \( \Phi_i \) where, from Eq. (22),
\[
\Phi_i \cdot \Phi_p = \varepsilon^{-1} - 1
\]
(23)

Another parameter that is widely used in the orifice equations is \( \Gamma_c \), the ratio of the discharge coefficients, which is defined by
\[
\Gamma_c = \frac{C_{d,ei}}{C_{d,ei}}
\]
(24)

For EI ingress, \( \Delta C_p \), the nondimensional pressure difference, is the driving force for ingress. This is defined as
\[
\Delta C_p = \frac{\Delta p}{1/2\rho \Omega^2 b^2}
\]
(25)

where \( \Delta p \) is the time-average peak-to-trough difference in static pressure in the annulus.

4.2. Solutions for RI ingress

For RI ingress, neglecting external swirl, it was shown in [2] that
\[
\Phi_{min,RI} = C_{d,ei} C_{\beta_1}^{1/2}
\]
(26)

Implicit solutions of the orifice equations were found in [2] but a more convenient explicit form is given by Sangan et al. [42,43] where, for \( \Phi_o \leq \Phi_{min,RI} \),
\[
\frac{\Phi_o}{\Phi_{min,RI}} = \frac{\varepsilon}{[1 + (1 - \varepsilon)^{1/2}][1 + \Gamma_c^{-2}(1 - \varepsilon)]^{1/2}}
\]
(27)

For \( \Phi_o > \Phi_{min,RI} \), \( \varepsilon = 1 \), and Eq. (27) is referred to as the effectiveness equation for RI ingress. It follows, using Eqs. (23) and (27), that
\[
\frac{\Phi_{i,RI}}{\Phi_{min,RI}} = \frac{1 - \varepsilon}{[1 + (1 - \varepsilon)^{1/2}][1 + \Gamma_c^{-2}(1 - \varepsilon)]^{1/2}}
\]
(28)

Figure 5 shows the variation of \( \varepsilon, \Phi_e \) and \( \Phi_i \) with \( \Phi_o \) according to the above equations with \( \Gamma_c = 1 \). In the limit that \( \Phi_o = 0 \), where \( \varepsilon = 0 \), Eq. (28) reduces to
\[
\frac{\Phi_{i,RI}}{\Phi_{min,RI}} = \frac{1}{2[1 + \Gamma_c^{-2}]^{1/2}}
\]
(29)

\( \Phi_{i,RI}^{\Phi_o} \) denotes the value of \( \Phi_{i,RI} \) when \( \Phi_o = 0 \), and this is the maximum value of the nondimensional ingested flow rate that can enter the wheel-space. For \( \Gamma_c = 1 \),
Figure 5  Theoretical variation of $e$, $\Phi_c$ and $\phi_l$ with $\Phi_o$ for RI ingress with $\Gamma_e = 1$.

$\Phi_{o,RI}/\Phi_{min,RI} = 0.35$; that is, for this case the maximum flow that can be ingested is 35% of the flow required to seal the system.

4.3. Solutions for EI ingress

In [3], a linear ‘saw-tooth model’ was used to approximate the time-average circumferential distribution of pressure in the external annulus. This is illustrated in Figure 6, where $\theta$ is the normalised angle between two vanes. Ingress occurs when $p_2$, the pressure in the annulus, is greater than $p_1$, the pressure in the wheel-space, and vice versa for egress.

The saw-tooth model allows the orifice equations to be solved analytically, so that

$$\Phi_{o,EI} = 2/3 C_{d,e} \Delta C_p^{-1/2}$$

(30)

Implicit equations for the effectiveness were derived in [3] but more convenient explicit equations were obtained by Sangan et al. [42,43] where for $\Phi_o \leq \Phi_{min,EI}$

$$\phi_l = \frac{\varepsilon}{\Phi_{min,EI}} [1 + \Gamma_c^{-2/3}(1-e)^{2/3}]^{1/2}$$

(31)

For $\Phi_o > \Phi_{min,EI}$, $\varepsilon = 1$, and Eq. (31) is referred to as the EI effectiveness equation. Figure 7 shows the effect of $\Gamma_c$ on the variation of $\varepsilon$ with $\Phi_o/\Phi_{min}$ according to Eqs. (27) and (31) for RI and EI ingress. The value of $\Gamma_c$ affects the shape of the effectiveness curve, and for most cases of practical interest, $\Gamma_c < 1$.

It can be seen from Eq. (31) that the variation of $\varepsilon$ with $\Phi_o$ depends only on the two parameters $\Phi_{min,EI}$ and $\Gamma_c$. This means that the EI effective equation has uncoupled ingress from its driving force, $\Delta C_p$. However, Eq. (30) shows the relationship between $\Phi_{min,EI}$ and $\Delta C_p$ and the conditions for mathematical consistency in this relationship are explained in [4]. The consistency criterion shows that the locations in the annulus where $\Delta C_p$ should be determined are very restricted, and the criterion is unlikely to be satisfied by experimental measurements of $\Delta C_p$. Consequently, the value of $C_{d,e}$ determined from Eq. (30) will depend on where in the annulus $\Delta C_p$ is measured, and $C_{d,e}$ is therefore of limited practical importance. By contrast, the value of $\Delta C_p$ (wherever it is measured) is of importance in extrapolating the effectiveness measurements from rig to engine, as is discussed in Section 5.

It follows from Eqs. (23) and (31) that

$$\frac{\phi_{l,EI}}{\Phi_{min,EI}} = \frac{1-\varepsilon}{[1 + \Gamma_c^{-2/3}(1-e)^{2/3}]^{1/2}}$$

(32)

In the limit that $\Phi_o = 0$, where $\varepsilon = 0$, Eq. (32) reduces to

$$\frac{\phi_{l,EI}}{\Phi_{min,EI}} = \frac{1}{[1 + \Gamma_c^{-2/3}]^{1/2}}$$

(33)

where $*$ denotes that $\Phi_o = 0$. Eq. (33) gives the value of the maximum nondimensional flow rate that can be ingested into the wheel-space.

4.4. Combined ingress

Combined ingress occurs when the effects of rotation and external pressure are both significant. For example, when a double rim-seal is used the pressure asymmetries are attenuated in the annular space between the inner and outer seals, and the effects of rotation are similar in magnitude to those of the external pressure.

In this case, Eqs. (26) and (30) provide the asymptotes for RI and EI ingress, and Owen [3] showed that

$$\frac{\Phi_{min,CI}}{\Phi_{min,RI}} = \frac{2}{3} C_{d,e}[1 + \Gamma_{Ap}]^{3/2} - 1$$

(34)
where \( C_{d,e,RI} \) is the value of \( C_{d,e} \) when the external flow rate is zero, and

\[
\Gamma_{Ap} = \frac{\Delta C_p}{C_{\beta_1}^{1/2}}
\]

(35)

\( \Gamma_{Ap} \) can be thought of as the ratio of the driving forces for EI and RI ingress.

5. Comparison between theory and experimental data

Zhou et al. [44] used a statistical model, featuring maximum likelihood estimates, to fit the effectiveness equations given in Section 4 to published experimental data. The model produced estimates for \( \Gamma_c \) and \( \Phi_{min} \), together with their confidence intervals, and some of the results for RI and EI ingress are given below.

Effectiveness experiments are usually made using concentration measurements of a tracer gas, typically carbon dioxide. The mass transfer equations include diffusion terms, and the concentration levels are also affected by mixing inside and outside the wheel-space. These effects are not included in the orifice equations used here, but despite (or perhaps because of) that the theoretical results shown below agree well with the experimental data.

5.1. Rotationally-induced (RI) ingress

Owen [2] validated his orifice model for RI ingress using the experimental data of Graber et al. [14]. Graber et al. used concentration measurements to determine the sealing effectiveness \( \varepsilon \) for several rim-seal geometries in a rig in which the axial velocity in the annulus was very small (<33 cm/s) and the swirl ratio in the mainstream flow in the external annulus \( (V_{\phi,2}/\Omega b) \) could be varied. They plotted their measured values of \( \varepsilon \) versus \( \eta_{ri} \), a flow parameter where

\[
\eta_{ri} = 1/2G_e Re_b^{-1/2}\phi_c
\]

(36)

For ease of comparison with the theoretical curves, the experimental data shown in Figures 8 and 9 were obtained by replotting the data shown in [14] versus \( \Phi_{ri} \) the data for \( \Phi_{RI} \) – which were not included in [14] – were obtained using Eq. (23).

For the cases discussed here, the statistical model of Zhou et al. was used to estimate the values of \( \Phi_{min,RI} \) and \( \Gamma_c \). By contrast, Owen assumed values for these two parameters: for \( \Phi_{min,RI} \), he used the Bayley–Owen correlation given in Eq. (1) for an axial-clearance seal, which is equivalent to \( \Phi_{\Omega,RI}=0.097 \), and for simplicity he assumed that \( \Gamma_c=1 \). Owen’s theoretical curves for \( \varepsilon \) based on these two assumed values, are shown in Figures 8 and 9, and the values of \( \Gamma_c \) and \( \Phi_{min,RI} \) estimated by Zhou et al. are included in the legends.

Figure 8 shows a comparison between the theoretical curves and the experimental data for the case of an axial-clearance seal. A thumb-nail sketch of the seal configuration is shown, and it should be noted that the external flow is from right to left (i.e. from the rotor towards the stator). Two levels of external swirl were used in the experiment, \( V_{\phi,2}/\Omega b=1 \) and 2, and it can be seen that there is no systematic effect of external swirl on the effectiveness. For this axial-clearance seal, the theoretical curve for \( \varepsilon \) is only marginally better than Owen’s curve. This is not surprising as the estimated value of \( \Phi_{\Omega,RI}=0.105 \) is relatively close to the Bayley–Owen value of 0.097.

Figure 9 shows the results for a radial-clearance seal for two rotational Reynolds numbers where the fitted curve for \( \varepsilon \) is significantly better than Owen’s curve. (For this case, the estimated value of \( \Phi_{\Omega,RI}=0.157 \) is significantly larger than the Bayley–Owen value.)
As predicted by the theory, the experimental data for the two different values of $Re_{\phi}$ collapse onto single curves.

5.2. Externally-induced (EI) ingress

The effectiveness data used for the correlations in Figures 10 and 11 were respectively based on the CFD (computational fluid dynamics) data given by Owen et al. [4] and on the concentration measurements presented by Johnson et al. [31]. The $\Phi_{i,\text{EI}}$ data – which were not determined in either of these papers – were calculated here using Eq. (23).

The CFD effectiveness data in [4] were obtained using a steady 3D code for an axial-clearance seal in which there were stationary vanes in the annulus upstream of the seal but no rotating blades downstream. Both figures show very good agreement between the theoretical curves and the data, but there are insufficient data points to provide accurate estimates of $\Phi_{\text{min,EI}}$ and $\Gamma_c$ (Zhou et al. [44] suggest that at least 16 data points are needed.)

5.3. Combined ingress

Owen et al. [5] applied the solution given in Eq. (35) to the experimental data of Phadke and Owen [10]. In the experimental rig, an axial flow of air was created in the external annulus, which contained no vanes or blades. The circumferential asymmetry in the external flow was obtained using honeycomb and gauzes in the annulus, and ingress was determined by concentration, flow visualisation and pressure measurements.

Figure 12 shows the measured variation of $C_{w,\text{min}}$ with $Re_w$, the axial Reynolds number in the external annulus. For $Re_w = 0$, RI ingress occurs and $C_{w,\text{min}} \propto Re_{\phi}$; for large values of $Re_w$, the effects of rotation become small and $C_{w,\text{min}} \propto Re_w$; for small values of $Re_w$, an ‘undershoot’ occurs and it is possible for $C_{w,\text{min}}$ to be less than the RI value.

As shown in Section 4.4, it is $\Gamma_{\Delta p}$ and not $Re_w$ that controls ingress, and Owen et al. [5] showed that $G_{\text{avg}} \propto (Re_w/Re_{\phi})^2$. (The viscous terms in the ratio $Re_w/Re_{\phi}$ cancel, and only the inertial terms are important. This can be readily seen by writing

$$\frac{Re_w}{Re_{\phi}} = \frac{W}{\Omega b}$$

which is the ratio of the axial velocity in the annulus to the tip speed of the rotating disc.)

Figure 13 shows a comparison between Eq. (35) and the experimental data shown in Figure 12. The following empirical correlation was used for the discharge coefficients:

$$\frac{C_{d,e}}{C_{d,e,\text{RI}}} = \exp\left(-A \frac{Re_w}{Re_{\phi}} \frac{\Phi_{\text{min,RI}}}{\Phi_{\text{min,CI}}} \right)$$

(38)

where the constant $A$ controls the decrease of $C_{d,e}$ with increasing axial flow rate. It can be seen that the experimental data collapse onto a single curve, as the CI theory predicts, and the RI and EI asymptotes capture the limiting cases, although the theoretical curve under-predicts the experimental undershoot of the RI asymptote at small values of $Re_w/Re_{\phi}$.

5.4. Extrapolation of effectiveness data from experimental rigs to engines

As demonstrated above, concentration measurements made on an experimental rig can be used to determine $\Phi_{\text{min}}$ and $\Gamma_c$ for a particular value of $\Delta C_p$ or, more generally, $\Gamma_{\Delta p}$. From dimensional similitude, these values should apply to a geometrically-similar engine at the same operating conditions. It is often the case that, even if the geometric conditions are satisfied, the operating conditions (particularly $Re_{\phi}$) for the engine will differ from those for the rig.
Eq. (30) shows that, for EI ingress, $F_{\text{min}}; EI = \frac{2}{p}a$, and it is tentatively suggested that this relationship could be used to extrapolate the results from a rig to an engine. Assuming that $G_c$ is unchanged, Eqs. (31) and (32) could then be used for design purposes. A new rig has been built at the University of Bath to test this hypothesis.

6. Conclusions

The Bath orifice models are based on inviscid swirling flow, with separate discharge coefficients for ingress and egress to account for viscous effects. Solutions of the orifice equations for rotationally-induced (RI) and externally-induced (EI) ingress show the relationship between $\Phi_o$, the sealing flow parameter, and $\varepsilon$, the sealing effectiveness. In addition, when ingress occurs, the flow rate of ingested fluid can be calculated.

Using a statistical model, featuring maximum likelihood estimates, the theoretical effectiveness equations can be fitted to experimental data, from which $\Phi_{\text{min}}$, the minimum value of $\Phi_o$ that prevents ingress, and $\Gamma_{r}$, the ratio of the discharge coefficients, can be estimated. There is good agreement between the effectiveness equations and published data for CI, EI and RI ingress.

As predicted for the combined ingress case, the experimental CI data collapse onto a single curve that spans the asymptotes for RI and EI ingress.

It is tentatively suggested that the orifice models could be used to extrapolate effectiveness data obtained from experimental-rig conditions to engine-operating conditions. A new rig has been built at the University of Bath to test this hypothesis.

References


Theoretical modelling of hot gas ingestion through turbine rim seals


