Physica A 392 (2013) 4997–5005

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Cycle frequency in standard Rock–Paper–Scissors games: Evidence from experimental economics

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HIGHLIGHTS

- Cycles are measured in a standard Rock–Paper–Scissors human experiment.
- The existence of persistent cycles is confirmed from analyzing the evolutionary trajectories.
- The mean frequency of cycles is quantitatively measured.
- The observed highly stochastic but weak cyclic motions are quantitatively understood by a discrete-time logit dynamics model.

ARTICLE INFO

Article history:
Received 31 January 2013
Received in revised form 6 May 2013
Available online 21 June 2013

Keywords:
Rock–Paper–Scissors game
Cycle
Social state
Population dynamics

ABSTRACT

The Rock–Paper–Scissors (RPS) game is a widely used model system in game theory. Evolutionary game theory predicts the existence of persistent cycles in the evolutionary trajectories of the RPS game, but experimental evidence has remained to be rather weak. In this work, we performed laboratory experiments on the RPS game and analyzed the social-state evolutionary trajectories of twelve populations of $N = 6$ players. We found strong evidence supporting the existence of persistent cycles. The mean cycling frequency was measured to be 0.029 ± 0.009 period per experimental round. Our experimental observations can be quantitatively explained by a simple non-equilibrium model, namely the discrete-time logit dynamical process with a noise parameter. Our work therefore favors

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1. Introduction

Evolutionary game theory (EGT) is becoming a general theoretical framework to analysis strategies behaviors [1–3]. EGT is rooted in the classical game theory (CGT) [4] and the theory of evolution [5]. Different from CGT, EGT predicts there could exist persistent cycles in the evolutionary trajectories in the strategy space [6–8].

As an example let us consider the standard Rock–Paper–Scissors (RPS) game. This is a prototype game in textbooks [1,4,6,9,7]. In this game, dynamics equations (e.g., the standard replicator dynamics equations) in EGT predict that the evolutionary trajectory will cycle around the Nash equilibrium persistently. However, the CGT predicts full random behavior: the system

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http://dx.doi.org/10.1016/j.physa.2013.06.039

Table 1
Payoff matrix of the Rock–Paper–Scissors game. The value of each matrix element is the payoff of the row player's strategy given the strategy of the column player.

	R	P	S
R	1	0	2
P	2	1	0
S	0	2	1

is in a mixed-strategy equilibrium with each cyclic motion being balanced completely by its reverse cyclic motion. According to the CGT theory, cycles (also referred to as excess loops) cannot be observed in the evolutionary trajectories in the long run [8,10].

Empirical examples of the RPS cycles are constantly being discovered in nature, e.g., three morphs male lizard [11] and others [12,13]. The environment for animal contests is decentralized, in which the encounter is pairwise, but population strategy shows cyclic behaviors [1,11]. In general, human economic behaviors (e.g., exchanges) are also pairwise and not centralized [14]. To test EGT in human game experiments [15], the traditional setting is decentralized (see review [16]), in which a subject in each round competes with one random-pairwise opponent within a finite population [17–23]. In such traditional setting experiment, whether the trajectories are persistent cycles instead of convergence to a Nash equilibrium remains an open question [16,24,10]. Till now, no persistent cycle has been confirmed in the RPS experiment under such a traditional setting [8,10], and furthermore no dynamics observation has been reported quantitatively.

In this paper, we study the evolutionary trajectories of the Rock–Paper–Scissors game from the perspective of non-equilibrium statistical physics. In non-equilibrium statistical physics studies, formulating a physically meaningful measure of the distance from equilibrium is an area of active research [25]. An equilibrium system satisfies the detailed balance condition, which ensures the time reversal symmetry. However, detailed balance is broken in a non-equilibrium system even in its stationary state, therefore various dynamical patterns may show up in the evolutionary trajectory. Several non-equilibrium order parameters, such as entropy production [26,27] and velocity [28], have been constructed to characterize the distance from equilibrium. In this work we carry out laboratory experiments on the RPS game, and we detect the possible existence of persistent cyclic flows using an angular frequency as the non-equilibrium order parameter. A non-zero angular frequency serves as a quantitative measure of the distance from equilibrium for the evolutionary trajectories. We also compare our experimental observations with the predictions of a simple non-equilibrium model, the discrete-time logit dynamical process with a noise parameter β.

Our experiments are the standard RPS games with the experimental setting of discrete time, random pairwise matching and local information. This setting has its reality in biology and economics [16,14,18,19,8,17,20–22,29,30]. We collect a total number of twelve experimental trajectories from our experiments (each trajectory is the result of 300 rounds of the game) and then analyze these trajectories. Like other previous experiments [17,19,21,22,31] and theories [32,33], the evolutionary trajectories are highly stochastic, but using our non-equilibrium order parameter we are able to confirm that cycles exist and do not dissipate. The mean frequency of cycles is about 0.029 ± 0.009 period per experimental round. This mean value is used to evaluate the noise parameter β of the logit dynamics model, and a value of $\beta \approx 0.20$ is obtained.

This paper is organized as the following. In the next section we introduce the standard RPS game in the traditional setting and describe our data analysis protocol. In Section 3 we describe our main experimental results. The experimental results are compared with the predictions of the discrete-time logit dynamics model in Section 4. We conclude this work in the last section.

2. Experimental setup and data analysis

There are three different pure strategies in the Rock–Paper–Scissors game, namely Rock (R), Paper (P) and Scissors (S). These three strategies form a directed circle $R \to S \to P \to R$, namely R beats S, S beats P, and P in turn beats R. In our experiments we use the simple payoff matrix shown in Table 1 to make the RPS game a constant-sum game: In each play between two players, the winning player gets a payoff 2 (i.e., two experimental points) while the losing player gets a payoff 0; if there is a tie then each player gets an equal payoff 1.

2.1. Experimental setting

There were twelve independent and disjoint groups in our laboratory experiments. Each group was formed by six players, therefore the RPS game is a finite population game with population size $N = 6$. Each group played the RPS game 300 rounds (we will explain the motivation to use 300 rounds later on). In each round of the play, the six players of each group were first randomly assigned to three disjoint sub-groups by a computer program, and then the two players of each sub-group played the RPS game once. All players made their own decisions simultaneously and anonymously. After all the players had submitted their choices, each player then got the feedback information through her/his private computer screen. The feedback information included her/his own strategy, her/his opponent's strategy, and her/his own payoff. No other information

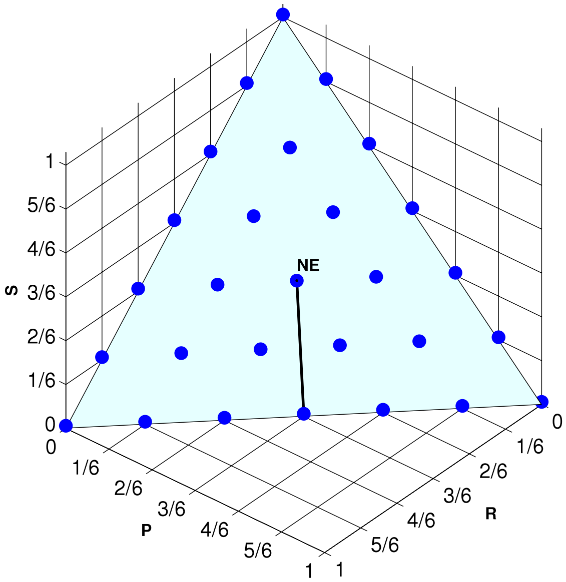

Fig. 1. Social state space and Poincaré section of the RPS game with $N = 6$ players. Each social state is represented by a point (x, y, z) on the plane characterized by $x + y + z = 1$. All the 28 social states are distributed within or on the boundary of an equilateral triangle. The social state $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is the Nash equilibrium (NE) point. To detect possible persistent flow within the social state space, a line segment linking the Nash equilibrium point and the social state $(\frac{1}{2}, \frac{1}{2}, 0)$ is drawn. This line is referred to as the Poincaré section.

was provided to the players. Each player also understood that her/his strategy in each round of the play is only shown to her/his opponent of this round but not shown to the other players.

These 12 experimental sessions were conducted during December 2010 in the experimental social science laboratory of Zhejiang University. The 72 experimental subjects (players) were recruited broadly from the student population of the university. They were sitting in an isolated seat with a computer during the games. Both written and oral instructions were provided for each player before the experiment. During the experiment, the players gained experimental points in each round of the game according to the payoff matrix. The experimental sessions lasted about 1.5–2 h. The players got their earnings in cash privately after the experiment according to the accumulated experimental points over the 300 rounds. The exchange rule is one experimental point equals 0.15 Yuan RMB. In addition, each player got 5 Yuan RMB as show-up fee. The average earning was about 50 Yuan RMB.

2.2. Data analysis

There are three pure strategies in the RPS game, therefore we use a vector (x, y, z) to denote a generic *social state* of the population, with x, y and z being respectively the fraction of players using strategy R, P and S. Suppose at the t-th round of the game, $n_R(t)$ players used strategy R, $n_P(t)$ players used strategy P, and $n_S(t) = N - n_R(t) - n_P(t)$ players used strategy S. Then

$$x \equiv \frac{n_R(t)}{N}, \qquad y \equiv \frac{n_P(t)}{N}, \qquad z \equiv \frac{n_S(t)}{N}.$$

Obviously x, y, and z should satisfy $x \geq 0$, $y \geq 0$, $z \geq 0$, and $x + y + z = 1$. The total number of different social states for a population of size N is simply $\frac{(N+1)(N+2)}{2}$. In the studied case of $N = 6$ this number is 28.

The social state (x, y, z) of a population at a given time point is a coarse-grained description about the strategies used by the members of this population [23,8]. The set of all the social states of a population is referred to as the *social state space* of the population. It can be represented graphically by an equilateral triangle in a three-dimensional Euclidean coordinate system; see Fig. 1. Each social state (x, y, z) corresponds to a point in the interior or on the boundary of this triangle. The central point $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ of the triangle is the Nash equilibrium (NE) point of the RPS game.

Generically speaking, the social state of the population is different at different rounds t of the repeated RPS game. The social state (x, y, z) as a function of the discrete time t forms an *evolutionary trajectory* in the social state space [23,8,10,22]; see Fig. 2 for a simple illustration. After an evolution trajectory of T time steps has been collected, we then perform statistical analysis on it. The first quantities of interest are the mean values of x, y and z, namely

$$\bar{x} \equiv \frac{1}{T}\sum_{t=1}^{T} x(t), \qquad \bar{y} \equiv \frac{1}{T}\sum_{t=1}^{T} y(t), \qquad \bar{z} \equiv \frac{1}{T}\sum_{t=1}^{T} z(t). \tag{1}$$

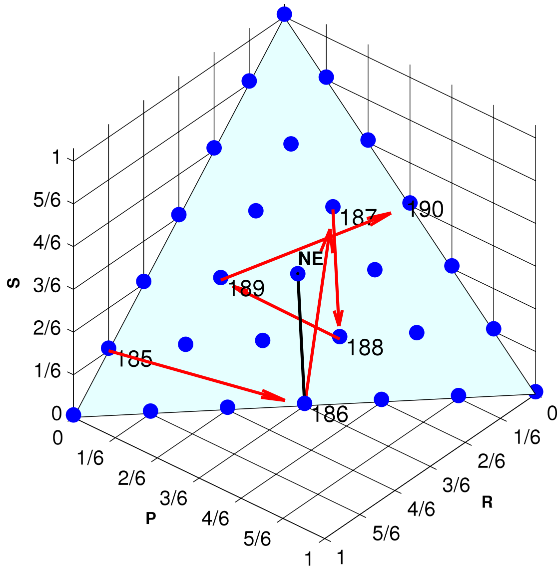

Fig. 2. A pictorial view of a short segment of an experimentally recorded evolutionary trajectory, starting from $t = 185$ and ending at $t = 190$. The counting numbers of the four social state transitions are, respectively, $C_{185} = 0.5$, $C_{186} = 0.5$, $C_{187} = 0$, $C_{188} = -1$ and $C_{189} = 0$. Therefore the accumulated counting number of this trajectory segment is $C_{185,190} = 0$.

To detect weak but persist directional motion in the social state space, we follow Refs. [10,34] and set a line segment between the Nash equilibrium point and a point chosen at the boundary of the triangle (see Fig. 1). Such a line segment is referred to as a Poincaré section. Consider two consecutive social states $\vec{s}(t) \equiv \big(x(t), y(t), z(t)\big)$ and $\vec{s}(t+1) \equiv \big(x(t+1), y(t+1), z(t+1)\big)$. If either $\vec{s}(t)$ or $\vec{s}(t+1)$ is identical to the Nash equilibrium point, or if the line segment from $\vec{s}(t)$ to $\vec{s}(t+1)$ does not cross the Poincaré section, then the transition $\vec{s}(t) \to \vec{s}(t+1)$ is assigned a counting number $C_t = 0$. Otherwise, (1) if the transition $\vec{s}(t) \to \vec{s}(t+1)$ crosses the Poincaré section from left to right (counter-clockwise with respect to the direction axis $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ of the social state plane), then $C_t = +1$; (2) if this transition crosses the Poincaré section from right to left (clockwise), then $C_t = -1$; (3) if $\vec{s}(t)$ is on the Poincaré section but is different from the Nash equilibrium point, then $C_t = 0.5$ ($C_t = -0.5$) if $\vec{s}(t+1)$ is to the right (left) of the Poincaré section; (4) if $\vec{s}(t+1)$ is on the Poincaré section but is different from the Nash equilibrium point, then $C_t = 0.5$ ($C_t = -0.5$) if $\vec{s}(t)$ is to the left (right) of the Poincaré section. We give some concrete examples of computing C_t in Fig. 2.

The accumulated counting number C_{t_0,t_1} of the evolutionary trajectory during the time interval $[t_0, t_1]$ is defined as

$$C_{t_0,t_1} \equiv \sum_{t=t_0}^{t_1-1} C_t. \tag{2}$$

The accumulated counting number C_{t_0,t_1} quantifies the *net* number of cycles around the Nash equilibrium point. Such a quantity can help us to detect deterministic behaviors in a stochastic process [34]. Starting from the initial time $t_0 = 1$, if $C_{1,t}$ scales linearly with t during the social-state evolution process, then it indicates the existence of persistent cycles around the Nash equilibrium; if $C_{1,t}$ as a curve of t only fluctuates around 0, then there are no persistent cycles around the Nash equilibrium. The mean frequency of cyclic motion in the time interval $[t_0, t_1]$ is defined as

$$f_{t_0,t_1} \equiv \frac{C_{t_0,t_1}}{t_1 - t_0} = \frac{1}{t_1 - t_0} \sum_{t=t_0}^{t_1-1} C_t. \tag{3}$$

Starting from the initial time $t_0 = 1$, we are interested in the value of $f_{1,t}$ as t becomes large.

3. Experimental results

Table 2 lists the total number of times the three strategies have been used in each of the 12 evolutionary trajectories. Among the twelve evolutionary trajectories of length 300, the total number of times the strategy R, P and S being used is, respectively, 7702, 6937 and 6961. The mean value of x, y and z as defined in Eq. (1), is then $\bar{x} = 0.357 \pm 0.005$, $\bar{y} = 0.321 \pm 0.004$ and $\bar{z} = 0.322 \pm 0.007$ (the standard deviation is estimated over the 12 evolution trajectories; see Table 2). The observed mean point $(\bar{x}, \bar{y}, \bar{z})$ is only slightly different from the theoretical Nash equilibrium point $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

Table 2
Statistics on the strategies. #R denotes the total number of times the strategy R being chosen by members of a given population (#P and #S have similar meanings). $\bar{x}$, $\bar{y}$, $\bar{z}$ are defined by Eq. (1).

Group	#R	#P	#S	$\bar{x}$	$\bar{y}$	$\bar{z}$
1	675	601	524	0.375	0.334	0.291
2	632	533	635	0.351	0.296	0.353
3	584	591	625	0.324	0.328	0.347
4	688	615	497	0.382	0.342	0.276
5	669	568	563	0.372	0.316	0.313
6	642	578	580	0.357	0.321	0.322
7	606	583	611	0.337	0.324	0.339
8	625	558	617	0.347	0.31	0.343
9	675	581	544	0.375	0.323	0.302
10	604	604	592	0.336	0.336	0.329
11	643	567	590	0.357	0.315	0.328
12	659	558	583	0.366	0.31	0.324

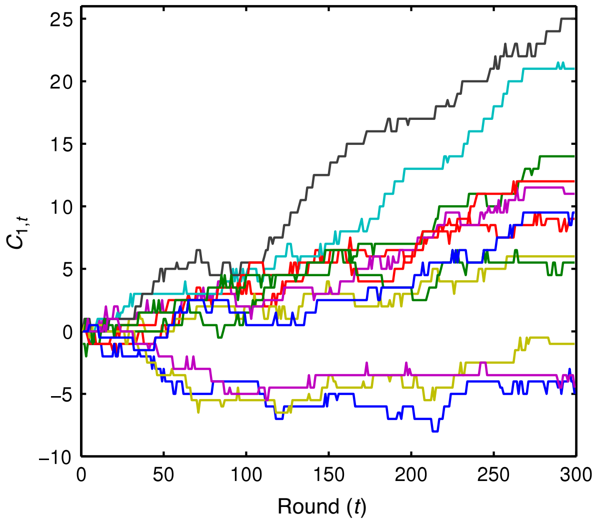

Fig. 3. Accumulated counting number $C_{1,t}$ as a function of evolution time t. Each of the twelve curves corresponds to one evolutionary trajectory involving six players.

The experimental trajectories are highly stochastic, similar to the observations on other game processes [17,19,21,29,22]. However, if we plot the evolution behavior of the accumulated counting number $C_{1,t}$ with time t in Fig. 3, we find that $C_{1,t}$ increases with t in most of the data sets. The value of $C_{1,300}$ for each of the 12 experimental trajectories is shown in the last column of Table 3. We obtain that the mean value of $C_{1,300}$ to be $\overline{C}_{1,300} = 8.54 \pm 2.66$. Accordingly, the mean cycling frequency of these 12 evolutionary trajectories in 300 steps is $\overline{f}_{1,300} = 0.029 \pm 0.009$. In other words, the empirical frequency of the cycles is 0.029 ± 0.009 period per experimental round. The 95% confidence interval of this frequency is [0.009, 0.048].

Statistical analysis on the 12 sampled values of $C_{1,300}$ suggests that the null hypothesis H_1 that $C_{1,300} = 0$ can be rejected ($p < 0.01$, t-test). Therefore we can say that cycles do exist in the RPS game in our experiments. Statistical analysis also shows that $C_{1,300} > 0$ ($p < 0.01$), i.e., the cycles are counter-clockwise around the Nash equilibrium point. This result is consistent with the theoretical predictions of some evolutionary dynamics models [8,1,7].

According to the last row in Table 3, to confirm the existence of cycles using 12 samples, the trajectory length t should be at least 150. This is because the null hypothesis ($C_{1,t} = 0$) can be rejected ($p < 0.05$) only when $t \geq 150$. That long evolutionary trajectories are needed to confirm the existence of cycles can also be understood from the empirical fact that the mean cycling frequency is very small.

To see the persistence of cycles, setting null hypothesis as $C_{1,150} > C_{151,300}$ which means the cycles are disappearing along time. This hypothesis can be rejected by experimental data ($p = 0.06 < 0.10$). Setting $C_{1,100} > C_{201,300}$, this null hypothesis can be rejected strongly ($p < 0.01$). Concerning the question “Do cycles dissipate when subjects must choose simultaneously?” raised recently by the authors of Ref. [10], our experimental data therefore suggest that cycles do not dissipate.

Table 3
The accumulated counting number $C_{1,t}$ in the twelve evolutionary trajectories.

Group	$C_{1,50}$	$C_{1,100}$	$C_{1,150}$	$C_{1,200}$	$C_{1,250}$	$C_{1,300}$
1	0.50	3.00	5.50	5.50	8.50	9.00
2	0.00	2.00	4.00	3.00	5.00	6.00
3	1.00	1.50	4.50	7.00	10.00	14.00
4	1.50	2.00	3.00	6.50	9.50	11.00
5	−3.00	−4.00	−5.00	−6.00	−4.00	−5.00
6	3.00	4.50	7.00	13.00	18.00	21.00
7	4.00	5.50	12.50	17.00	21.00	25.00
8	1.50	4.50	5.50	6.00	11.00	12.00
9	−3.50	−5.50	−3.50	−4.50	−2.50	−1.00
10	1.50	4.50	6.50	3.50	5.50	5.50
11	−2.00	−5.00	−3.50	−3.50	−3.50	−4.50
12	−0.50	0.50	2.50	3.50	6.50	9.50
Mean	0.33	1.13	3.25	4.25	7.08	8.54
95%L	−1.12	−1.35	0.03	−0.04	2.09	2.68
95%U	1.78	3.60	6.47	8.54	12.07	14.40
p-value	0.62	0.34	0.05	0.05	0.01	0.01

The last four rows are the statistical results of the 12 experimental groups above. The row titled as p-value is t-test result by setting the null hypothesis $C_{1,t} = 0$ for the 12 samples. 95%U(L) means the upper (lower) bound of 95% confidence interval over the 12 samples.

4. Comparison with a simple model

To theoretically understand the experimental observations, we now study a noisy best-response process as a simple model for the RPS game, namely the discrete-time logit dynamics [35]. Multiple equilibria and limit cycles in the logit dynamics have also been studied in a very recent paper by Hommes and Ochea [36] in the continuous-time limit.

Suppose the population of N players is in the social state (x, y, z) after the t-th round of the game. Let us denote by u_i the mean payoff of the strategy $i \in \{R, P, S\}$ for this social state. From the payoff matrix of Table 1 we can easily obtain that

$$u_R = x + 2z, \qquad u_P = y + 2x, \qquad u_S = z + 2y. \tag{4}$$

We assume that at the $(t + 1)$-round of the game, each player of the population will choose a strategy from $\{S, R, P\}$ independently of all the other players. And we further assume that the time-dependent probability p_i for a player to choose strategy i is

$$p_i = \frac{e^{\beta u_i}}{e^{\beta u_S} + e^{\beta u_R} + e^{\beta u_P}}, \quad \forall i \in \{R, P, S\}. \tag{5}$$

The parameter β is referred to as the "inverse temperature" of the logit dynamics, its value quantifies the rationality degree of human agents in strategy interaction [35,37,38,20,39,40,7,41,42]. In the limiting case of $\beta = 0$ each strategy will be chosen with the uniform probability $\frac{1}{3}$.

For this simple Markovian process, the transition probability $T_{(x',y',z')}^{(x,y,z)}$ from a social state (x, y, z) at time t to another social state (x', y', z') at time $(t + 1)$ is expressed as (noticing that $z' = 1 - x' - y'$)

$$\begin{aligned} T_{(x',y',z')}^{(x,y,z)} &= \frac{N!}{(Nx')!(Ny')!(Nz')!} p_R^{Nx'} p_P^{Ny'} p_S^{Nz'} \\ &= \frac{N!}{(Nx')!(Ny')!(Nz')!} \frac{e^{N\beta[xx'+yy'+zz'+2(xy'+yz'+zx')]}}{(e^{\beta(x+2z)} + e^{\beta(y+2x)} + e^{\beta(z+2y)})^N}. \end{aligned} \tag{6}$$

The steady-state probability $W^*_{(x,y,z)}$ that the system is in the social state (x, y, z) at $t = \infty$ can be obtained by solving the following fixed-point equation

$$W^*_{(x,y,z)} = \sum_{(x',y',z')} T_{(x,y,z)}^{(x',y',z')} W^*_{(x',y',z')}. \tag{7}$$

Because the transition probability from any social state (x, y, z) to any another social state (x', y', z') is positive, Eq. (7) has a unique solution with the normalization property $\sum_{(x,y,z)} W^*_{(x,y,z)} = 1$ [43]. It is not difficult to prove that the steady-state probability distribution has the following rotational symmetry

$$W^*_{x,y,z} = W^*_{y,z,x} = W^*_{z,x,y}. \tag{8}$$

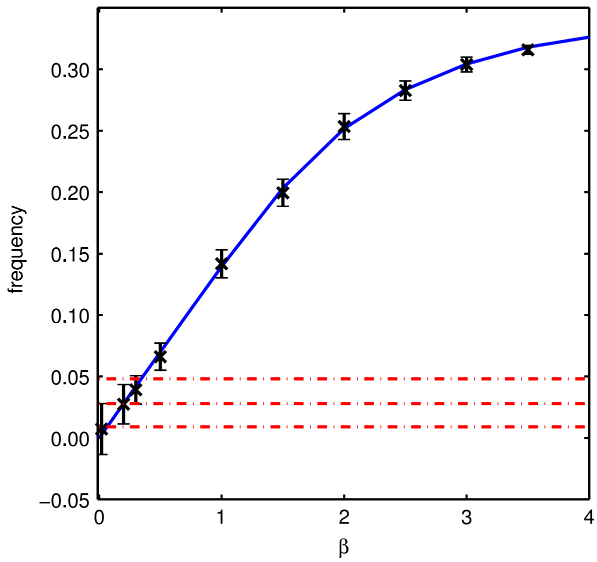

Fig. 4. The steady-state mean cycling frequency f^* of the discrete-time logit dynamical process with population size $N = 6$. The solid line is theoretical result obtained with Eq. (10); the cross ($\times$) symbols with error bars are obtained by averaging over many simulated evolutionary trajectories of length 300. The mean experimental frequency of $\bar{f}_{1,300} \approx 0.029$ and its 95% confidence upper and lower bound are marked by the dashed lines.

This rotational symmetry ensures that

$$\sum_{(x,y,z)} xW^*_{(x,y,z)} = \sum_{(x,y,z)} yW^*_{(x,y,z)} = \sum_{(x,y,z)} zW^*_{(x,y,z)} = \frac{1}{3}, \tag{9}$$

namely the logit dynamics will reach the Nash equilibrium $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ at $t \to \infty$.

It can be checked numerically and analytically that, for any $\beta > 0$, the detailed balance condition is violated at the steady-state of the logit dynamics. For two different social states (x, y, z) and (x', y', z'), in general we will find that

$$T^{(x,y,z)}_{(x',y',z')} W^*_{(x,y,z)} \neq T^{(x',y',z')}_{(x,y,z)} W^*_{(x',y',z')}.$$

Because of the violation of detailed balance, directional flows may persist in the system even at $t \to \infty$.

We are especially interested in the directional flow around the Nash equilibrium point. Consider two social states (x, y, z) and (x', y', z') on the evolutionary trajectory at two consecutive time points t and $t + 1$. If either (x, y, z) or (x', y', z') is identical to the Nash equilibrium point, the transition $(x, y, z) \to (x', y', z')$ is *not* a rotational motion around the Nash equilibrium, and we set the corresponding rotational angle $\theta^{(x,y,z)}_{(x',y',z')}$ to be zero. The Nash equilibrium point may be sitting on the rectilinear line that passing through the social states (x, y, z) and (x', x', z'). If this is the case, the transition $(x, y, z) \to (x', y', z')$ is also *not* a rotational motion around the Nash equilibrium, and its rotational angle $\theta^{(x,y,z)}_{(x',y',z')}$ is again set to be zero. In all the remaining cases, the social states (x, y, z), (x', y', z') and the Nash equilibrium point form a triangle in the social state plane of Fig. 1. The magnitude of the rotational angle $\theta^{(x,y,z)}_{(x',y',z')}$ is just the angle of this triangle at vertex point $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, it must be less than π. The rotational angle $\theta^{(x,y,z)}_{(x',y',z')}$ is defined as positive if the rotation from (x, y, z) to (x', y', z') with respect to the Nash equilibrium point is counter-clockwise, otherwise it is defined as negative.

At the steady-state of the discrete-time logit dynamics, the mean frequency f^* that the evolution trajectory rotates around the Nash equilibrium point can then be computed by the following formula

$$f^* \equiv \frac{1}{2\pi} \sum_{(x,y,z)} W^*_{(x,y,z)} \sum_{(x',y',z')} T^{(x,y,z)}_{(x',y',z')} \theta^{(x,y,z)}_{(x',y',z')}. \tag{10}$$

For the population size $N = 6$, we show in Fig. 4 the steady-state mean frequency f^* as a function of the inverse temperature β. To check the correctness of the theoretical calculations, we also perform computer simulations based on the discrete-time logit dynamics model to generate a set of simulated evolutionary trajectories of length 300. The mean cycling frequencies of these simulated evolutionary trajectories are also shown in Fig. 4. The agreement between analytical calculations and computer simulation results are very good. We find that f^* increases almost linearly with the inverse temperature β when $\beta < 1.5$. Comparing the theoretical results with the mean frequency value of $\bar{f}_{1,300} = 0.029$, we infer the inverse parameter should be set to $\beta = 0.20$.

At $\beta = 0.20$, we also perform computer simulations based on the discrete-time logic dynamics model to generate a set of independent evolution trajectories of length $T = 300$. We then perform the same analysis on these trajectories and

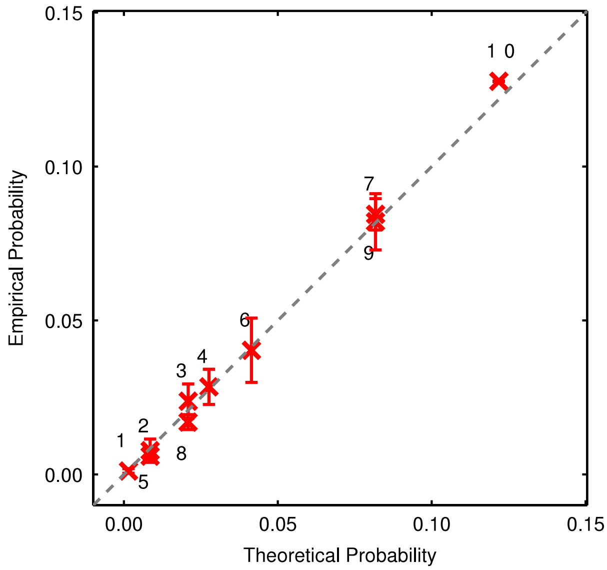

Fig. 5. Probability of staying in the social state (x, y, z) for a population of size $N = 6$. The horizontal axis is the predicted probability by the discrete-time logit dynamics model with inverse temperature $\beta = 0.20$, while the vertical axis is the empirical probability measured from the 12 experimental trajectories. Because of the rotational symmetry (8), the 28 social states can be coarse-grained into ten groups: (1), $\{(0, 0, 1), (0, 1, 0), (1, 0, 0)\}$; (2), $\{(0, \frac{1}{6}, \frac{5}{6}), (\frac{1}{6}, \frac{5}{6}, 0), (\frac{5}{6}, 0, \frac{1}{6})\}$; (3), $\{(0, \frac{1}{3}, \frac{2}{3}), (\frac{1}{3}, \frac{2}{3}, 0), (\frac{2}{3}, 0, \frac{1}{3})\}$; (4), $\{(0, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{2}, 0, \frac{1}{2})\}$; (5), $\{(\frac{1}{6}, 0, \frac{5}{6}), (\frac{5}{6}, \frac{1}{6}, 0), (0, \frac{5}{6}, \frac{1}{6})\}$; (6), $\{(\frac{1}{6}, \frac{1}{6}, \frac{2}{3}), (\frac{1}{6}, \frac{2}{3}, \frac{1}{6}), (\frac{2}{3}, \frac{1}{6}, \frac{1}{6})\}$; (7), $\{(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}), (\frac{1}{3}, \frac{1}{2}, \frac{1}{6}), (\frac{1}{2}, \frac{1}{6}, \frac{1}{3})\}$; (8), $\{(\frac{1}{3}, 0, \frac{2}{3}), (\frac{2}{3}, \frac{1}{3}, 0), (0, \frac{2}{3}, \frac{1}{3})\}$; (9), $\{(\frac{1}{3}, \frac{1}{6}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{3}, \frac{1}{6}), (\frac{1}{6}, \frac{1}{2}, \frac{1}{3})\}$; (10), $\{(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})\}$. All the social states of a given group have the same stationary probability (the same horizontal-axis value) according to the theoretical model, but their measured probabilities might be different (the mean vertical-axis value and the standard error).

find that the direction of the cycles is counter-clockwise and the mean cycling frequency is $\bar{f} \approx 0.029$, consistent with the experimental result.

At $\beta = 0.20$, the steady-state probability $W^*_{(x,y,z)}$ of visiting each social state (x, y, z) as predicted by the logit dynamics is compared with the empirically observed probability of visiting (x, y, z); see Fig. 5. The agreement between theory and experiment is again very good.

Although the discrete-time noisy-response logit dynamic model can describe our experimental observations excellently, we should point out an important difference between the model assumption and the experimental setting. In our experiments, after each round of the game, each player only knows the strategy of her/his opponent but not the social state of the whole population. However in the logit dynamics model, we assume that each player chooses a strategy based on the knowledge of the current social state of the population; see Eq. (5). In this sense, the logit dynamics model is still a phenomenological model. It is of interest to quantitatively describe the RPS evolutionary dynamics by a more microscopic model. We hope to return to this issue in a future study.

5. Conclusion and discussions

As a brief summary, in this work we studied the Rock–Paper–Scissors game both experimentally and analytically. Our experimental data gave strong evidence that counter-clockwise cycles around the Nash equilibrium point exist in the social-state evolutionary trajectory of a finite population. We demonstrated that our experimental observations can be quantitatively understood by a simple theoretical model of noisy-response logit dynamics.

RPS game experiments on EGT were also reported quite recently by Cason and co-authors and by Hoffman and co-authors [10,8]. The backgrounds and cutting edges of the experiment research are well documented in these two Refs. [10,8]. Compared with the decentralized setting of our present work, the experimental environments of the RPS game in these two recent works [10,8] are all centralized: Instead of pairwise meetings, in all of the experiments reported in Refs. [10,8], each subject competes against the choices of all other subjects. However, the decentralized setting (especially the random matching pairwise setting) is more closer to the natural environments in biology and economics (e.g., Refs. [16,14]). For example, the encounters of male lizards are pairwise meetings [11]. For decentralized population RPS games, according to our knowledge, the existence of persistent cyclic motions was not confirmed by any previous laboratory experiments.

Going back to traditional (decentralized) setting experiments of the simplest RPS game, the present work added strong evidence in favor of the existence of persistent cycles. As a fundamental observation on cycle, the mean frequency of cycles was quantitatively measured. There are tens of dynamics models which have been build to interpret cyclic behavior in RPS game, however there are rare quantitative observations from real experiments. Quantitative measurements from experiments are important, without which to evaluate a dynamics equation precisely is almost impossible (or plausible).

As demonstrated, our experimental observations can be quantitatively understood by a simple theoretical model of noisy-response logit dynamics.

We wish to emphasize two major points of our experimental approach. First, by recording sufficiently long evolutionary trajectories, we were able to detect weak deterministic motion in a highly stochastic process. We noticed that cycles can only be confirmed ($p < 0.05$) when the trajectories are longer than 150 rounds in twelve samples. Second, we focus on time asymmetry of social state transitions. Importance of time asymmetry has been well emphasized in non-equilibrium statistical physics [34,25,44]. The frequency is observed from the loops out of detailed balance.

Acknowledgments

We thank Ken Binmore for helpful discussion and Zunfeng Wang for technical assistance. The work of B.X. and Z.W. was supported by a grant from the 985 Project at Zhejiang University and by SKLTP of ITP-CAS (No. Y3KF261CJ1). The work of H.J.Z. was supported by the Knowledge Innovation Program of Chinese Academy of Sciences (No. KJCX2-EW-J02) and the National Science Foundation of China (grant Nos. 11121403 and 11225526).

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