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Chemically reacting ionized fluid flow through a vertical plate with inclined magnetic field in rotating system

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Abstract

Unsteady mixed convective ionized fluid flow through a vertical plate with joule heating, viscous dissipation, thermal diffusion, diffusion thermo, internal heat generation with chemical reaction, thermal radiation and inclined uniform magnetic field has been analyzed in a rotating system. To obtain the non-similar momentum, energy and concentration equations usual non-dimensional variables have been used. An implicit finite difference technique with stability analysis is used to solve the obtained non-similar, coupled, non-linear partial differential non-dimensional equations. The obtained solutions are shown graphically. Finally, a qualitative comparison with published results has been discussed.

Keywords: Ionized Fluid; Chemical reaction; Rotating system

1. Introduction

The ionized fluid flow in a rotating system on MHD boundary layer flow has become important in several industrial, scientific and engineering fields. For ionized fluid two distinct effects has been considered by Cowling [1]. First effect, electric currents can flow in an ionized fluid because of relative diffusion of the ionized gas and electrons, due to agencies of electric forces. The second effect depends wholly on the magnetic field. The convection flow is often encountered in nuclear reactors or in the study of planets and stars. In this flow the phenomenon of mass transfer is also very common in the theories of stellar structure. The studies of MHD incompressible viscous flows with Hall currents have grown considerably because of its engineering applications to the problems of Hall

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 accelerators, MHD generators, constructions of turbines and centrifugal machines, as well as flight magnetohydrodynamics. From the above point of applications, the effects of Hall currents on free convective flow through a porous medium bounded by an infinite vertical plate have been investigated by Ram [2], when a strong magnetic field is imposed in a direction which is perpendicular to the free stream and makes an angle to the vertical direction. The Hall effects on an unsteady MHD free convective heat and mass transfer flow through a porous medium near an infinite vertical porous plate with constant heat flux and variable suction have been studied by Sattar and Alam [3]. The effects of hall and ion-slip current on MHD free convection and mass transfer flow from a vertical plate in a rotating system has been analyzed by Dash et al. [4]. The effects of Soret and Dufour on unsteady MHD flow by mixed convection over a vertical surface in porous media with internal heat generation, chemical reaction and Hall current has been studied by Aurangzaib and Shafie [5]. Ahmed and Alam [6] solved the Aurangzaib and Shafie [5]’s model by implicit finite difference method.

Hence our aim of this work is to extend the work of Ahmed and Alam [6] for the case of rotating system and inclined magnetic field. The problem has been solved by implicit finite difference method. The governing equations involved in this problem have been transformed into non-similar coupled partial differential equation by usual transformations. Finally, the comparison of the present results with the results of Ahmed and Alam [6] has been discussed.

2. Mathematical Model

A flow model of unsteady MHD mixed convective and mass transfer flow of an electrically conducting incompressible viscous fluid past an electrically nonconducting isothermal infinite vertical porous plate with joule heating, viscous dissipation, thermal diffusion, diffusion thermo, internal heat generation with chemical reaction, thermal radiation and inclined uniform magnetic field through a porous medium in a rotating system. The positive $x$ coordinate is measured along the plate in the direction of fluid motion and the positive $y$ coordinate is measured normal to the plate. The leading edge of the plate is taken as coincident with $z$–axis. A normal magnetic field is assumed to be applied in the $y$ – direction and induced magnetic field is negligible. Initially, it is considered that the plate as well as the fluid is at the same temperature $T(=T_{\infty})$ and concentration level $C(=C_{\infty})$. Also it is assumed that the fluid and the plate is at rest after that the plate is to be moving with a constant velocity $U_{\infty}$ in its own plane. Instantaneously at time $t > 0$, the temperature of the plate and spcies concentration are raised to $T_{w}(> T_{\infty})$ and $C_{w}(> C_{\infty})$ respectively, which is there after maintained constant, where $T_{w}$, $C_{w}$ are temperature and spcies concentration at the wall and $T_{\infty}$, $C_{\infty}$ are the temperature and concentration of the species outside the plate respectively. If the plate is infinite in extent and hence all physical quantities depend on $y$ and $t$. The physical configuration of the problem is furnished in Fig. 1.

A strong uniform magnetic field $\mathbf{B}$ can be
taken as \( (0, \lambda \beta_0, \sqrt{1 - \lambda^2} \beta_0) \) where \( \lambda = \cos \alpha \) is applied in a direction that makes an angle \( \alpha \) with the normal to the considered plate. Thus if \( \lambda = 1 \) the imposed magnetic field is parallel to the \( y \)-axis and if \( \lambda = 0 \) then the magnetic field is parallel to the plate. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is negligible in comparison with applied magnetic field. Within the framework of the above stated assumptions and using the dimensionless quantities, \( Y = \frac{y U_\infty}{v}, \ U = \frac{u}{U_\infty}, \ W = \frac{w}{U_\infty}, \ \tau = \frac{t U_\infty^2}{v}, \)

\[
\mathcal{T} = \frac{T - T_m}{T_w - T_\infty} \quad \text{and} \quad \mathcal{C} = \frac{C - C_w}{C_w - C_\infty}
\]

in the equations relevant to the problem is governed by the following coupled non-linear non-dimensional partial differential equations under the electromagnetic Boussinesq approximations as;

\[
\frac{\partial U}{\partial \tau} - S \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + G_r \mathcal{T} + G_m \mathcal{C} - \frac{M}{(1 + m^2 \lambda^2)}(U + m \lambda W) + 2R'W - KU \quad (1)
\]

\[
\frac{\partial W}{\partial \tau} - S \frac{\partial W}{\partial Y} = \frac{\partial^2 W}{\partial Y^2} + \frac{M}{(1 + m^2 \lambda^2)}(m \lambda U - W) - 2R'U - KW \quad (2)
\]

\[
\frac{\partial \mathcal{T}}{\partial \tau} - S \frac{\partial \mathcal{T}}{\partial Y} = \left( \frac{1 + R}{P_r} \right) \frac{\partial^2 \mathcal{T}}{\partial Y^2} + D_u \frac{\partial^2 \mathcal{C}}{\partial Y^2} + \frac{M \mathcal{E}_c}{(1 + m^2 \lambda^2)}(U^2 + W^2) + E_c \left( \frac{\partial U}{\partial Y} \right)^2 + (\frac{\partial W}{\partial Y})^2 \right) + \beta \mathcal{T}^p \quad (3)
\]

\[
\frac{\partial \mathcal{C}}{\partial \tau} - S \frac{\partial \mathcal{C}}{\partial Y} = \frac{1}{S_c} \frac{\partial^2 \mathcal{C}}{\partial Y^2} + S_r \frac{\partial^2 \mathcal{T}}{\partial Y^2} - \gamma \mathcal{C}^q \quad (4)
\]

corresponding boundary conditions are;

\[
U = 1, \ W = 0, \ \mathcal{T} = 1, \ \mathcal{C} = 1 \quad \text{at} \ Y = 0
\]

\[
U = 0, \ W = 0, \ \mathcal{T} = 0, \ \mathcal{C} = 0 \quad \text{as} \ Y \to \infty,
\]

where \( S, \ G_r, \ G_m, \ R', \ K, \ M, \ m, \ R, \ P_r, \ D_u, \ E_c, \ \beta, \ S_c, S_r \) and \( \gamma \) are the Suction parameter, Grashof number, Modified Grashof number, Rotational parameter, Permeability of the porous medium, Magnetic Parameter, Hall parameter, Radiation Parameter, Prandtl number, Dufour Number, Eckert number, Heat generation or absorption parameter, Schmidt number, Soret number and Chemical reaction parameter respectively.

3. Shear Stress, Nusselt and Sherwood Number

From the velocity field, the effects of various parameters on the shear stress have been studied. Shear stress in \( x \)-direction, \( \tau_x = \mu_0 \left( \frac{\partial U}{\partial Y} \right)_{Y=0} \) which is proportional to \( \left( \frac{\partial U}{\partial Y} \right)_{Y=0} \). Shear stress in \( z \)-direction, \( \tau_z = \mu_0 \left( \frac{\partial W}{\partial Y} \right)_{Y=0} \) which is proportional to \( \left( \frac{\partial W}{\partial Y} \right)_{Y=0} \). From the temperature field, the effects of various parameters on Nusselt number have been analyzed and the Nusselt number, \( N_u = -\mu_0 \left( \frac{\partial \mathcal{T}}{\partial Y} \right)_{Y=0} \) which is proportional to \( -\left( \frac{\partial \mathcal{T}}{\partial Y} \right)_{Y=0} \). And from the concentration field, the effects of various parameters on Sherwood number have been calculated and the Sherwood number, \( S_h = -\mu_0 \left( \frac{\partial \mathcal{C}}{\partial Y} \right)_{Y=0} \) is proportional to \( -\left( \frac{\partial \mathcal{C}}{\partial Y} \right)_{Y=0} \).
4. Numerical Solutions

To solve the non-dimensional system by the implicit finite difference technique, it is required a set of finite difference equations. In this case, the region within the boundary layer is divided by some perpendicular lines of $Y$ -axis, where $Y$ -axis is normal to the medium as shown in Fig. 2. It is assumed that the maximum length of boundary layer is $Y_{\text{max}} = 25$ as corresponds to $Y \to \infty$ i.e. $Y$ varies from 0 to 25 and the number of grid spacing in $Y$ directions is $\bar{p} (= 400)$, hence the constant mesh size along $Y$ axis becomes $\Delta Y = 0.0625 (0 \leq Y \leq 25)$ with a smaller time-step $\Delta t = 0.001$.

Let $U'$, $W'$, $\bar{T}'$ and $\bar{C}'$ denote the values of $U$, $W$, $\bar{T}$ and $\bar{C}$ at the end of a time-step respectively. Using the implicit finite difference approximation, the following appropriate set of finite difference equations are obtained as;

\[
\frac{U_{i}^{n+1} - U_{i}^{n}}{\Delta \tau} - S \frac{U_{i+1}^{n} - U_{i}^{n}}{\Delta Y} = \frac{U_{i+1}^{n} - 2U_{i}^{n} + U_{i-1}^{n}}{(\Delta Y)^2} + G_{i} \bar{T}_{i}^{n} \\
+ G_{m} \bar{C}_{i}^{n} - \frac{M}{(1 + m^2 \lambda^2)} (m \lambda W_{i}^{n} + U_{i}^{n}) + 2R' W_{i}^{n} - KU_{i}^{n}
\]

(6)

\[
\frac{W_{i}^{n+1} - W_{i}^{n}}{\Delta \tau} - S \frac{W_{i+1}^{n} - W_{i}^{n}}{\Delta Y} = \frac{W_{i+1}^{n} - 2W_{i}^{n} + W_{i-1}^{n}}{(\Delta Y)^2} + \frac{M}{(1 + m^2 \lambda^2)} (m \lambda U_{i}^{n} - W_{i}^{n}) - 2R' U_{i}^{n} - KW_{i}^{n}
\]

(7)

\[
\frac{\bar{T}_{i}^{n+1} - \bar{T}_{i}^{n}}{\Delta \tau} - S \frac{\bar{T}_{i+1}^{n} - \bar{T}_{i}^{n}}{\Delta Y} = \left( \frac{1}{1 + \frac{R}{P_{r}}} \right) \bar{T}_{i+1}^{n} - 2\bar{T}_{i}^{n} + \bar{T}_{i-1}^{n} + \frac{M}{(\Delta Y)^2} \left( \frac{\bar{C}_{i+1}^{n} - 2\bar{C}_{i}^{n} + \bar{C}_{i-1}^{n}}{(\Delta Y)^2} \right)
+ \frac{1}{1 + m^2 \lambda^2} \left( U_{i}^{n} - 2U_{i}^{n} + U_{i-1}^{n} \right) + \frac{1}{1 + m^2 \lambda^2} \left( W_{i}^{n} - 2W_{i}^{n} + W_{i-1}^{n} \right) + \rho(\bar{T}_{i}^{n})^p
\]

(8)

\[
\frac{\bar{C}_{i}^{n+1} - \bar{C}_{i}^{n}}{\Delta \tau} - S \frac{\bar{C}_{i+1}^{n} - \bar{C}_{i}^{n}}{\Delta Y} = \frac{1}{S_{e}} \bar{C}_{i+1}^{n} - 2\bar{C}_{i}^{n} + \bar{C}_{i-1}^{n} + \frac{1}{S_{r}} \bar{T}_{i+1}^{n} - 2\bar{T}_{i}^{n} + \bar{T}_{i-1}^{n} - \gamma(\bar{C}_{i}^{n})^q
\]

(9)

with the boundary conditions,

\[
U_{0}^{n} = 1, \quad W_{0}^{n} = 0, \quad \bar{T}_{0}^{n} = 1, \quad \bar{C}_{0}^{n} = 1
\]

(10)
Here the subscript \( i \) designates the grid points with \( Y \) coordinate and the superscript \( n \) represents a value of time, \( \tau = n \Delta \tau \) where \( n = 0, 1, 2, \ldots \). The primary velocity \( (U) \), secondary velocity \( (W) \), temperature \( (\bar{T}) \) and concentration \( (\bar{C}) \) distributions at all interior nodal points may be computed by successive applications of the above finite difference equations. The numerical values of the shear stresses, Nusselt number and Sherwood are evaluated by Five-point approximate formula. The stability conditions of the method are not shown for brevity.

5. Results and Discussion

To obtain the steady-state solutions, the computations have been carried out up to dimensionless time, \( \tau = 30 \). It is observed that the numerical values of \( U, W, \bar{T} \) and \( \bar{C} \) however, show little changes after \( \tau = 15 \). Hence at \( \tau = 15 \) the solutions of all variables are steady-state solutions.

![Fig. 3. Illustration of (a) shear stress in \( x \) - axis and (b) shear stress in \( z \) - axis for various values of \( \alpha \) when \( p = 2 \) and \( q = 2 \)](image_url)
To observe the physical situation of the problem, the shear stresses, Nusselt number and Sherwood number have been illustrated in Figs. 3-5. The effects of $\alpha$ on shear stress in $x$-axis and $z$-axis respectively in case of cooling plate are presented in Fig. 3. It is observed that the shear stresses decrease with the increase of $\alpha$. The effects of Rotational parameter ($R'$) and Permeability of porous medium ($K$) on shear stress in $x$-axis and $z$-axis respectively in Fig. 4. It is noted that the shear stress in $x$-axis decrease with the increase of Rotational parameter and Permeability of porous medium. While the shear stress in $z$-axis also decrease with the increase of Rotational parameter and increase with the increase of Permeability of porous medium. The Nusselt number for different values of heat generation or absorption parameter $\beta$ and Prandtl number $Pr$ are displayed graphically in Fig. 5(a). The Nusselt number decreases with the rise of $\beta$, where $\beta < 0$ and $\beta > 0$ are treated as heat absorption and generation respectively and increases with the rise of Prandtl number. The Sherwood number for different values of chemical reaction parameter $\gamma$ and Schmidt number $Sc$ are shown in Fig. 5(b). The Sherwood number increases with the increase of $\gamma$, where $\gamma < 0$ and $\gamma > 0$ are treated as generative and destructive chemical reaction respectively and Schmidt number.

6. Conclusion

In this research, the implicit finite difference solution of unsteady MHD mixed convective and mass transfer flow of an electrically conducting incompressible viscous fluid past an electrically nonconducting isothermal infinite vertical porous plate with joule heating, viscous dissipation, thermal diffusion, diffusion thermo, internal heat generation with chemical reaction, thermal radiation and inclined uniform magnetic field through a porous medium in a rotating system for $p \leq 2$ and $q \leq 2$ has been investigated. The physical properties are grafically discused for different values of corresponding parameters and compared our steady-state results with Ahmed and Alam [6]. The accuracy of our results is qualitatively good in case of all the flow parameters. Some important finding of this investigation are given as:

1. The shear stress in $x$ – direction and $z$ – direction decreases with the increase of $\alpha$.
2. The shear stress in $x$ – direction decreases with the increase of $R'$ and $K$. 
3. The shear stress in $z$ direction decreases with the increase of $R'$ while increase with the increase of $K$.
4. The Nusselt number decreases with the increase of $\beta$ and $P_r$.
5. The Sherwood number increases with the increase of $\gamma$ and $S_c$.

References