Mechanical analysis of second order helical structure in electrical cable

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Abstract

A new model for calculating the response of electrical cables to bending has been developed accounting for multi-order helical structure and frictional effects. Stresses occurring in wires and the number of slipping wires were suggested as key properties in terms of expectation of cable life time. The model was applied to chosen cables and the magnitudes of the mentioned key properties were calculated. Parametric study was also done to investigate how these values are affected by changing the properties of the cable. It has been shown that increasing the pressure from jacket or insulation material causes larger stress while it prevents the slippage of wires. Applying a tensile force to the cable produces the same result, whereas using larger lay angles for both conductors and wires decreases internal stress and prevents the slippage.

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1. Introduction

Cables used in industrial robots are periodically exposed to external loading which includes torsion and bending. The task of the cables is to supply electricity. Under mechanical loads, the cable has a limited usage period due to internal failures caused by the deformation, fatigue and wear of internal components. From an industrial point of view, it is desired to distinguish the poor designs and good designs of the cables already in the stage of cable design. Good or bad, in this case, refers to short or long life time under given deformations. Existing methodology for evaluating cables is strongly dependent on empirical data. Due to the lack of accurate theoretical models, limited mechanical analysis is performed. Theoretical models which calculate the mechanical response of the cable accurately may strongly improve the design methodology and are of interest for industrial engineering.

A typical cable consists of a jacket, shield, tape, fillers and conductors. The jacket serves several purposes. It is the main protection of the vital parts of the cable from external loads and environmental effects. The cross
sectional layout is preserved mainly by the jacket, i.e. the internal organization of the components remains unchanged during deformation of the cable.

A shield is put between the jacket and the tape to protect the surroundings from the electromagnetic field generated by conductors and protect the conductors from electromagnetic disturbance at the same time. The tape is used to hold the components together during the assembly of the cable and it also plays a role in reducing the friction between the conductors and the shield.

Fillers are introduced in order to make a cable geometrically symmetric and more compressible. Conductors are the essential parts of a cable and could have a complex internal structure. These carriers of electric power or signals are wrapped helically around the core of the cable. The configuration of the center line of a conductor helix is uniquely defined with the lay angle, \(\alpha\), and helical radius, \(R\), as shown in Fig. 1.

Conductors consist of copper wires bundled to form a helical structure and are wrapped with insulation material. This forms a helix-in-helix which is the characteristic structure of the electrical cable and it is referred to as a second level helical structure in this work. Lay angle and helical radius are again defined for center line of the wire.

The imposed motion of the cable produces stress and strain in the internal components which may lead to wear and fatigue. The cable has failed when the failure occurs on a certain number of wires due to above mentioned factors. The basic idea for life time estimation is to find key properties, which are strongly related to these failure mechanisms and develop a model to evaluate these properties. Since these phenomenon occur in wires that have a second order helical structure, a complete description of the mechanical behavior of the cable is important.

The behavior of wire ropes, which consists of metallic wires wrapped helically around a core wire, to external loading was investigated by Costello (1997). The equation of equilibrium was suggested by considering each component as a thin wire by LeClair and Costello (1988) and later generalized by Velinsky (1985) to make it applicable for wire ropes with complex cross sections. A number of studies, which concern the cable behavior for various situations, have followed based on this theory. The response of the cable to tensile loads was a main concern in wire rope and studied by Huang (1978, 1996) or Utting and Jones (1987).

The response to bending was also studied by Knapp (1983, 1988) or Raoof (1991). Above all, Lanteigne (1985) obtained the axial strains in a first order helical structure, consisting of several concentric layers of conductors, through geometrical considerations. A general stiffness matrix was formulated through investigation of the strains and a rudimentary treatment of internal friction and slippage in the structure was included.

It is well known that the cable can be in two different states under bending, i.e., when the applied bending curvature is small, all the conductors are deformed together and the cable behaves as a solid beam whereas...
each conductor begin to change its relative positions with slipping as the curvature increases. These two situations are referred to as sticking and slipping state, respectively. Lanteigne’s model was rather rough in that the frictional force was neglected during the slipping state. The effect of the frictional force was taken into consideration by Papailiou (1996) although his model was limited to single layer and first order helical structures. Then his model was again extended by Hong et al. (2005) to deal with multi-layered structures.

Jolicoeur and Cardou (1996) adopted a different approach. In their research, semi-continuous modeling was used, in which the layer of conductors is mathematically represented by an orthotropic cylinder and bending stiffness of the cable was successfully evaluated. But this model is not adequate in terms of life time estimation because it is difficult to treat frictional effect with this approximation.

It can be said in common that the above models were only treating the first order helical structure. Hobbs and Nabijou (1995) formulated the change of curvature in double helical structure after applying bending deformation, but the analysis was limited to the pure bending case. The first step of a general treatment of multi-level helical structure was done first in this paper, where the strain in the helical structure of any order are formulated using an hierarchical approach, i.e., the strains in the second order helical structure are expressed as functions of strains in the first order helical structure and the same relations are applied to any helical level.

Leech (2002) suggested the hierarchical approach where the highest order of helix, which is the wire in our case, develops into a structure and this structure is then a component which forms the basis for the next lower order helix, which is the conductor in our case. In his model, the highest order helix is assumed to have tensile and torsional stiffnesses and these stiffnesses are then used to find the stiffness of the structure of next hierarchical level. He also studied the splices used in synthetic ropes (2002) by taking friction and contact forces into consideration. Since his interest was concentrated on the fibre ropes, the usage of which is to carry axial loads, the bending curvature is not considered. The model shown in our paper is based on the hierarchical approach similar to Leech with a small difference, that is, the highest order helix is analyzed as a first step in Leech’s model while our model starts from the lowest order helix, where the strains in the first order helix is evaluated from the strains in the cable and then the strains in the second order helix is evaluated. Our model is especially devoted to study the response of the cable to bending deformation.

In a realistic cable, the behavior of the wires are affected not only by applied loading, but also by the pressure from the jacket, insulation material and other components, which result in a frictional force on the surface. The objective of this paper is to theoretically calculate the response of wires to bending taking this frictional effect into account. The main part of the present paper is organized in the two following sections. First the details of the theoretical model used for the mentioned computations and estimations is presented. The results obtained using the method is presented and discussed thereafter. A short conclusion is presented at the end of the paper.

2. Theory

The theoretical model described in this paper consists of two different parts. The first part is the geometrical model where the no slipping state is assumed to evaluate the strains in internal components against the cable deformation. Then, based on the geometrical model, the friction model is developed which takes friction into account. This enables to consider the transition from no slipping state to no sticking state. It should be notified here that only the segment of cables which is away from terminations are considered in the following discussion. This assumption is introduced to get rid of the difficulty in taking boundary effect into consideration.

2.1. Geometrical model

In this section a multi-level helical structure, of arbitrary order, made up of a single, linear elastic material is concerned. The helical structure on each level consists of one or several concentric, fully populated layers and all components within a specific layer are identical and have the same lay angle. Size and shape of the cross section, on all helical levels, are assumed to be constant during deformation of the structure, and no relative movement between the internal components is permitted. The structure undergoes deformation that consists
of an arbitrary combination of tension, torsion and bending. It is also assumed that the diameter of the cross section of conductor or wire is small in comparison to the pitch length of the helix.

The objective of the present work is to establish relations between the applied deformations of the cable and strains in all internal components in the structure. First, relations for a first order helical structure are reviewed followed by an extension to a second order structure. The cable consists of a straight core with the conductors wrapped helically around it in one or several concentric layers. The structure of these conductors are referred to as the first order helix. Each component in the conductor also consists of a core and several helical wires which are called second order helix. This continues until the smallest component is reached though only second order helix is treated in this paper. In the following, the values with sub-index of the layer. The structure of the conductor, for example, is specified by determining the lay angle and the helical radius. They are denoted by \( \alpha_{cn} \) and \( R_{cn} \), respectively, where the subscript \( n \) is the number of the layer.

### 2.2. Derivation of strains in the conductor

As a first step, the cable exposed to the deformation is considered. The deformation can be expressed with three strains; elongation (\( \varepsilon \)), twisting (\( \tau \)) and bending curvature (\( \kappa \)). It has been found both theoretically and experimentally by McConnell and Zemke (1981) that the response of the cable to these three deformations interrelate each other. Consequently, there exists three coupling parameters, axial torsion, axial bending and bending torsion. However, it has been shown by Lanteigne (1985) that coupling effects of bending to axial and torsion are little under the condition that the configuration of the conductor is symmetrical. As a result, bending can be separately treated. In the following, the axial and torsional strains are considered first and then the influence of the bending is added. The geometry of the cable which is exposed to elongation and twisting is shown in Fig. 1(a).

The centerline of any element in the conductor is specified with the polar coordinates \( (R_{cn}, \theta_c) \), as illustrated in Fig. 1(b), where \( \theta_c \) is defined so that it becomes zero when the point is on the neutral axis. As a result, the vector \( r \), which defines the center line of the \( i \)th conductor in layer \( n \), is given as a function of \( \theta_{cni} \) according to

\[
\theta_c = \frac{2\pi i}{K_{cn}} + l \tan \alpha_{cn},
\]

\[
r = R_{cn} \cos \theta_c \hat{x} + R_{cn} \sin \theta_c \hat{y} + \frac{R_{cn} \theta_c}{\tan \alpha_{cn}} \hat{z},
\]

where \( i \) is the index of the conductor in \( n \)th layer, \( K_{cn} \) is the total number of conductors in layer \( n \), \( l \) is the axial length of the cable and \( [\hat{x}, \hat{y}, \hat{z}] \) are the unit vectors of the coordinate system which has its origin on the center line of the core as illustrated in Fig. 2. When the cable is subject to loading, the lay angle of a helix changes from the initial value due to the deformation of the cable and the new lay angle, \( \alpha_{cn} \), can be calculated using the compatibility relationship between the helix and the circular arc around the core, which is shown in Fig. 1(a), according to

\[
\alpha_{cn} = \arctan \left( \frac{\tan \bar{\alpha}_{cn} + R_{cn} \tau}{1 + \varepsilon} \right),
\]

where \( \bar{\alpha}_{cn} \) is the initial lay angle. This modified lay angle should be used in further calculations of the mechanical state of the helices.

Elongation, \( \varepsilon_{cn} \), of the conductor in layer \( n \) is expressed as a function of the cable deformation as

\[
\varepsilon_{cn} = \sqrt{(1 + \varepsilon)^2 \cos^2 \alpha_{cn} + (\sin \alpha_{cn} + R_{cn} \cos \alpha_{cn} \tau)^2} - 1,
\]

which is derived from geometrical considerations, see details in Lanteigne’s work (1985). After some simplification with neglecting higher order term, Eq. (4) is reduced to

\[
\varepsilon_{cn} = \cos^2 \alpha_{cn} \varepsilon + R_{cn} \sin \alpha_{cn} \cos \alpha_{cn} \tau.
\]
This equation only considers elongation and twisting of the cable. Hence the influence of the cable bending should be added. Bending of the cable imposes axial strain in the conductor, the magnitude of which is proportional to the distance between the conductor and cable neutral axis. Taking this into account, the elongation of the conductor is finally written as

\[
\epsilon_{cn} = \cos^2 \alpha_c (\epsilon + R_{cn} \sin \theta_c \kappa) + R_{cn} \sin \alpha_c \cos \alpha_c \tau.
\] (6)

To study the actual direction of the conductors, a new local coordinate system is introduced, the origin of which moves along the center line of the conductor as shown in Fig. 2. Unit vectors of this rotating frame are given by

\[
\begin{bmatrix}
\hat{t} \\
\hat{n} \\
\hat{b}
\end{bmatrix} =
\begin{bmatrix}
-\sin \alpha_c \sin \theta_c & \sin \alpha_c \cos \theta_c & \cos \alpha_c \\
-\cos \theta_c & -\sin \theta_c & 0 \\
\cos \alpha_c \sin \theta_c & -\cos \alpha_c \cos \theta_c & \sin \alpha_c
\end{bmatrix}
\begin{bmatrix}
\hat{x} \\
\hat{y} \\
\hat{z}
\end{bmatrix} =
\begin{bmatrix}
\hat{x'} \\
\hat{y'} \\
\hat{z'}
\end{bmatrix},
\] (7)

where the tangent vector, \( \hat{t} \), is tangent to the axis of the conductor, the normal vector, \( \hat{n} \), is perpendicular to \( \hat{t} \) and always pointing to the center of the core, and the bi-normal vector, \( \hat{b} \), is perpendicular to both \( \hat{t} \) and \( \hat{n} \) as shown in Fig. 2. The tangent vector \( \hat{t} \) is calculated by differentiating the position vector, \( \hat{r} \), with respect to \( \theta_c \). The normal vector \( \hat{n} \) lies in the \( x-y \) plane and it is easily given from Fig. 2. The bi-normal vector \( \hat{b} \) is obtained by \( \hat{t} \times \hat{n} \). Vector properties on the cable level can be projected on the local system \( \{\hat{t}, \hat{n}, \hat{b}\} \) by means of the transformation matrix \( T \). In addition to the axial strain, the conductor is also subjected to local bending in the normal and bi-normal directions and local twisting in its own axial direction. The torsional strain \( \tau_c \) in the conductor is obtained by projecting the bending vector in direction of \( \hat{t} \) as

\[
\tau_c = \kappa \sin \alpha_c \cos \alpha_c \sin \theta_c.
\] (8)

Similarly, the curvature consists of projection of the cable loading according to

\[
\kappa_c^{no} = \kappa \cos \alpha_c \cos \theta_c,
\] (9)

\[
\kappa_c^{bi} = -\kappa \cos^2 \alpha_c \sin \theta_c,
\] (10)

where \( \kappa_c^{no} \) is the normal direction component and \( \kappa_c^{bi} \) is the bi-normal direction component of the curvature which were given by LeClair and Costello (1988) and Witz and Tan (1992). The change of lay angle was not taken into consideration in their papers, this has been modified by using new lay angles calculated in Eq. (3).

It should be notified that only the change of strains are considered here although the conductors must have initial strains to be in a helical structure even in undeformed state. These initial strains are neglected to
2.3. Derivation of strains in the wire

Once the strains acting in the conductor are evaluated, the wires in the conductor are considered. It can be said that the wires are to the conductor what the conductors are to the cable. Applying the same relation, the strains in the wire can be evaluated as functions of strains in the conductor. First, the lay angle of the wire in the conductor and that the effect of cable bending implies a change in torsion and curvature in the wire. The location of any point on the wire is specified with position angle $\theta_w$.

$$
\theta_w = \frac{2\pi j}{K_{wm}} + \frac{R_{\theta_c}}{R_{wm} \tan z_{wm} \cos z_c} + (\pi - \theta_c),
$$

where $m$ is the number of the layer and $j$ is the number of the wire in the layer, as illustrated in Fig. 1(b). Elongation of the wire can be calculated in analogy with Eq. (4) according to:

$$
\epsilon_w = \cos^2 z_{wm} (\epsilon_c + R_{wm} \sin \theta_w \kappa_{c}^{bi} - R_{wm} \cos \theta_w \kappa_{c}^{no}) + R_{wm} \sin z_{wm} \cos z_{wm} \tau_c.
$$

It can be noted that the contribution from curvature now contains two components since the curvature has been divided into two local components, $\kappa_{c}^{no}$ and $\kappa_{c}^{bi}$.

To evaluate the torsion and bending strain, new local coordinate system $[\hat{i}, \hat{n}, \hat{b}]$ is again introduced. This new local coordinate system can be defined by multiplying $[i, n, b]$ with the transformation matrix in which $z_c$ and $\theta_c$ in Eq. (7) are replaced with $z_w$ and $\theta_w$, respectively. Thus, this coordinate system will move along the center line of the wire. The projection of the curvature on the $\hat{i}$-axis defines the torsional contribution on the wire. Thus, the torsion in the wire is given by

$$
\tau_w = (-\kappa_{c}^{no} \cos \theta_w + \kappa_{c}^{bi} \sin \theta_w) \sin z_w \cos z_w.
$$

In analogy with the calculation in the conductor, the curvature in the wire can be calculated by projecting the curvatures in the conductor onto the new local coordinate system $\hat{n}'$ and $\hat{b}'$ which yields

$$
\kappa_{w}^{no} = (\kappa_{c}^{no} \sin \theta_w + \kappa_{c}^{bi} \cos \theta_w) \cos z_w,
$$

$$
\kappa_{w}^{bi} = (-\kappa_{c}^{no} \cos \theta_w + \kappa_{c}^{bi} \sin \theta_w) \cos^2 z_w.
$$

In the explanation so far, the response of the conductors and the wires have been studied. This can be rephrased that strains in the first and second order helix have been calculated. It has been shown that the strains in the second order helix can be expressed as functions of the strains in the first order helix. This implies that the helix of any order can be calculated by repeating the same process hierarchically starting from cable level.

2.4. Friction model

For the model described in this section, several assumptions are introduced.

(1) The change of helical radius due to deformation of the cable is neglected.

(2) The contact between components within the same layer is neglected.

The first assumption is realistic when the cable is exposed to pure bending but not when the cable is twisted. The second assumption is valid when each component in a layer is arranged so that a small tangential space exists between them which is the case for many industrial cables, and also for the specimens used in this paper.
This model is described as an extension of Papailiou’s model (1996) to take several new elements into account. So the basic idea is identical to his model.

In the following, the values with subscript c belong to the conductor, subscript w stands for the wire and the following number corresponds to the index of the layer as already referred in the previous section. For example, \( T_{w2} \) is the tensile force acting in the wire in the second layer. The index of the layer is counted from outside to inner side. A loading condition is considered where the cable which consists of conductors and wires are deformed with pure bending given with curvature \( \kappa \).

To start with, the mechanical response of the conductors to deformation is considered where the second order helical structure, inside the conductors, is not taken into account.

Consider a single conductor located in the outermost layer of conductors, i.e., directly underneath the jacket. The position of the conductors can be specified with position angle \( \theta_c \) as illustrated in Fig. 1(b). The position angle becomes zero on the bending neutral axis.

The force balance in an infinitesimal segment of the conductor is considered. The forces acting on the segment in the radial direction, which is same as \( \theta \) in Fig. 2, are normal force, \( N_0 \), from the jacket and the normal force, \( N_1 \), from the conductors underneath. They are calculated as

\[
\begin{align*}
\text{d}N_0 &= P_c \text{d}\theta_c, \\
\text{d}N_1 &= \text{d}N_0 + T_{c1} \sin \alpha_{c1} \text{d}\theta_c,
\end{align*}
\]

where \( P_c \) is the compressing force from the jacket, \( T_{c1} \) is tensile force acting vertically to the cross section which the same direction as \( \theta \) in Fig. 2, and \( \alpha_{c1} \) is the lay angle of the conductor. The second term of Eq. (17) corresponds to the radial pressure caused by the tensile force acting upon a bent conductor according to Papailiou (1996). The force \( P_c \) caused by the jacket pressure is independent of the position angle \( \theta_c \). It is also constant along the cable radial direction which is implied by the second assumption mentioned above. In the direction of the conductor axis, tensile force, \( T_{c1} \), is acting and also the frictional forces occur due to the normal forces.

The frictional forces are assumed to work in the centerline of the wire so that no moment acts in this plane. The free body diagram of a conductor segment is illustrated in Fig. 3, in which the segment is in slipping state. Here the force acting in the out-of-plane-direction is not considered. This can be rephrased that no slipping state is always assumed for twisting, as is always the case in most of the previous works. This can be valid in this study because the force caused by torsion in conductor, or wire, is small in comparison with the force caused by bending. The force balance in the conductor in a slipping state is given by

\[
\text{d}T_{c1} = \mu_{c0} (\sin \alpha_{c1} T_{c1} + P_c) \text{d}\theta_c + \mu_{c1} P_c \text{d}\theta_c,
\]

where \( \mu_{c0} \) and \( \mu_{c1} \) are the friction coefficients between the conductor and the jacket and between the conductors and inner conductors, respectively. As the tensional force due to bending becomes zero on the neutral axis, the initial condition is given as

\[
T_{c1}(\theta_c = 0) = \tilde{T}_{c1},
\]

where \( \tilde{T}_{c1} \) is the tensile force caused by the extension and twisting of the cable, which is constant along the conductor length. This force can be calculated by multiplying Eq. (5) with conductor axial rigidity \( AE \) which

\[
dN_0 = P \text{d}\theta \\
\mu_{c0} \text{d}N_0 \\
\mu_{c1} \text{d}N_1 \\
T_1 + \text{d}T_1 \\
\text{d}N_1 = (P + T_1 \sin \alpha) \text{d}\theta
\]

Fig. 3. The force balance of a wire segment.
is explained later. Now that the cable is assumed to exposed to pure bending, this value is identically zero. This yields
\[
T_{c1} = B_1 e^{\kappa_1 \sin \theta_i} + A_1,
\]
where
\[
A_1 = -\frac{(\mu_{c1} + \mu_{c2}) P_c}{\mu_{c1} \sin \alpha_{c1}},
\]
\[
B_1 = \tilde{T}_{c1} - A_1.
\]
The force \( T_{c1,f} \) caused by the frictional effect is given by,
\[
T_{c1,f} = \tilde{T}_{c1} - \tilde{T}_{c1}.
\]
The force, \( T_{c1,b} \), is regarded as the maximum frictional force. On the other hand, the force \( T_{c1,b} \) caused by cable bending in sticking state is given by
\[
T_{c1,b} = A E \kappa R_{c1} \sin \theta_c \cos^2 \alpha_{c1},
\]
\[
A E = \sum_m K_{w_m} \pi r^2 \cos^3 \alpha_{w_m},
\]
where \( R_{c1} \) is the helical radius of the conductor, \( \kappa \) is the bending curvature applied in the cable, \( K_{w_m} \) is the number of wires in \( m \)th layer and \( A E \) is the axial rigidity of the conductor which was calculated using the model developed by Lanteigne (1985) taking internal structure into consideration. The tensile force caused by bending has been evaluated in Eq. (6). Therefore, the tensile force can be evaluated by multiplying the axial strain with the axial rigidity \( A E \). When the bending curvature imposed on the cable is small, the force caused by bending is small and slippage is prevented by the friction. Consequently, the conductor is in a sticking state and the tensile force acting in the conductor is \( T_{c1,b} + \tilde{T}_{c1} \). Bending force \( T_{c1,b} \) increases as the curvature increases eventually it exceeds the limiting value, \( T_{c1,f} \). At this point, the frictional force can no longer uphold the force caused by bending and the conductor starts to slip. In this state, the tensile force acting in the conductor becomes \( T_{c1,f} + \tilde{T}_{c1} \) which is independent of the magnitude of the curvature. Hence, the axial force in the conductor will be given by
\[
T_{c1} = T_{c1,b} + \tilde{T}_{c1} : \text{at Sticking state},
\]
\[
T_{c1} = \tilde{T}_{c1,f} + \tilde{T}_{c1} : \text{at Slipping state}.
\]
At the second step, the conductors in the penultimate layer are considered. The force equilibrium is given by
\[
dT_{c2} = \mu_2 \left( (\sin \alpha_{c1} T_{c1} + P_c) \frac{K_{c1}}{K_{c2}} + \sin \alpha_{c2} T_{c2} \right) d\theta_c + \mu_1 (\sin \alpha_{c1} T_{c1} + P_c) \frac{K_{c1}}{K_{c2}} d\theta_c,
\]
where \( T_{c2} \) is the axial force in the conductor in the penultimate layer, \( \mu_2 \) is the friction coefficient between the conductor and the underlying conductor and \( K_{c1} \) and \( K_{c2} \) are the number of conductors in each layer. The first term on the right hand side of Eq. (28) corresponds to the force from the inner side and the second term reflects the influence from the outer side of the conductor. The coefficient \( \frac{K_{c1}}{K_{c2}} \) means that \( K_{c2} \) conductors in inner layer must accept the radial force produced by \( K_{c1} \) conductors in outermost layer. The boundary condition is given by
\[
T_{c2} = \tilde{T}_{c2} \quad \text{at } \theta_c = 0.
\]
From Eq. (28), tensile force \( T_{c2} \) is expected to have the form
\[
T_{c2} = C_2 e^{\kappa_2 \sin \alpha_{c2} \theta_i} + B_2 e^{\kappa_1 \sin \alpha_{c1} \theta_i} + A_2.
\]
It can be said that the first term on the right hand side of Eq. (30) considers the influence of the pressure produced by the tensional force in the conductor itself, the second term corresponds to the pressure from the outer conductors and the third term reflects the influence of the pressure from the jacket. Coefficients \( C_2, B_2, A_2 \) are identified by inserting Eq. (30) into Eq. (28). They are given by
The tensile force caused by bending is then given by
\[
T_{c2,b} = AE\kappa R_{c2} \sin \theta_c \cos^2 \xi_{c2}
\] (34)
and it is compared with Eq. (30) to investigate the occurrence of the slippage. Axial forces in the conductors of any layer are calculated by repeating this process from the outermost layer towards the inner layers.

At the third step, the response of the wires is considered, where each conductor is treated as an object of calculation which has a helical structure inside it. Here, the conductor is regarded as not a helical structure but an originally straight component to which the tension, twisting and bending load has been applied. This is the hierarchical approach to multi-order helical structures.

The result of the calculation above is used to calculate the elongation the conductor \(\epsilon_c\) as
\[
\epsilon_{cn} = \frac{T_{cn}}{AE}
\] (35)
where \(i\) is the index of the layer. It is known that a certain amount of curvature and torsion might exist in conductors before applying external loading since the conductors are deformed to shape a single helical structure. It is assumed here that these initial strains are relaxed and only the variation of torsion \(\tau_{cn}\) and curvature \(\kappa_{cn}\) are considered. They are calculated by geometrical model according to Eqs. (8)–(10) as
\[
\tau_{cn} = \kappa \sin \xi_{cn} \cos \xi_{cn} \sin \theta_c,
\] (36)
\[
\kappa_{cn}^{no} = \kappa \cos \xi_{cn} \cos \theta_c,
\] (37)
\[
\kappa_{cn}^{bi} = -\kappa \cos^2 \xi_{cn} \sin \theta_c.
\] (38)

The elongation and the twisting of the conductor cause tensile forces on wires. The relation between conductor strains, \(\epsilon_{cn}\) and \(\tau_{cn}\), and wire elongation \(\epsilon_{wm}\) are given by Lanteigne (1985) as
\[
\epsilon_{wm} = \cos^2 \xi_{wm} \epsilon_{cn} + \sin \xi_{wm} \cos \xi_{wm} R_{wm} \tau_{cn},
\] (39)
where \(R_{wm}\) is the helical radius of wires, which is the distance from the center of the conductor, not the cable as shown in Fig. 1(b). Consequently, tensile forces, denoted by \(\widehat{T}_{wm}\), acting at the point where the effect of bending becomes zero, are given by
\[
\widehat{T}_{wm} = \pi r^2 E \epsilon_{wm},
\] (40)
where \(r\) is the radius of the wire cross section and \(E\) is the Young’s modulus of copper. Then the boundary condition is described as
\[
T(\theta_w) = \widehat{T}_{wm} \quad \text{at} \quad \theta_w = 0.
\] (41)
The same force balance is working as in Eq. (18), where all the parameters included are in the wire level in this case. The force given by Eq. (40) is not constant and varies with the position angle \(\theta_w\) because \(\tau_c\) is dependent on \(\theta_c\). However, the effect of the variation of \(\tau_c\) is relatively small compared to the magnitude of \(\epsilon_w\) and the assumption of constant force over the small piece of the wire can be valid. With an identical approach as with the conductors, the maximum frictional force in the wire can be calculated starting from the outermost layer.

The tensile force caused by bending is calculated from the variation of the curvature \(\kappa_{c}^{bi}\) and \(\kappa_{c}^{no}\) for conductor and is given by
\[
T_{wm,b} = \pi r^2 E (\kappa_{c}^{no} R_{wm} \sin \theta_w - \kappa_{c}^{bi} R_{wm} \cos \theta_w) \cos^2 \xi_{wm}.
\] (42)
This equation is similar to Eq. (24) but different in some points. Firstly, \(AE\) in Eq. (24) was replaced with \(\pi r^2 E\) because the wire is a simple component, not a structure, and its axial rigidity can be evaluated by multiplying
Young’s modulus with the cross sectional area. The second difference is that the parameters used are of wire level, not of conductor level. The third difference is that it consists of two components of the curvature. This is due to the fact that the bending curvature in the conductor was estimated by projecting it into normal and bi-normal directions. The occurrence of the slippage can be verified by comparing the force caused by friction and Eq. (42), i.e., if \( T_{w,m,b} \) is larger than \( T_{w,m,f} \), the wire is in slipping state and otherwise the wire is in sticking state.

To this point, tensile forces acting in the cross sections of all wires have been calculated. Bending moment is calculated because it can be a representative property of the cable to compare the theoretical and the experimental results. The tensile force \( T_{w,m} \) in the cross section of wire contributes to the bending moment as

\[
M_{n,m} = T_{w,m} \cos \alpha_{cn} \cos \alpha_{wm} (R_{cn} \sin \theta_c + R_{wm} \sin \theta_w).
\]

(43)

Total bending moment acting in the cable is obtained by summing up the contributions from all wires in all conductors. In the process of calculating bending stiffness using this model, some properties of the cable, which are thought to be related to the life time estimations, are also calculated. Firstly, the magnitude of the maximum stress in the cable and where it occurs is computed. Here, the linear elasticity is assumed and the stress is evaluated by multiplying strain with Young’s modulus. The stress is related to the fatigue life and causes cracks in the copper. The break of only one wire in the cable does not make the cable useless but the electrical performance of the cable is affected when a large number of wires are broken. Therefore, in the evaluating method of the cable in industry, the distribution of internal stress is used as one of the important parameters, where the cable is regarded to become out of usage period when the average tensile stress in the internal components overcomes certain value. In this paper, the average stress is shown to estimate the quality of the cable. Another key property is the number of the slipping wires as the slippage causes wear damage inside the cable. The number of slipping wire itself is not directly related to the wear damage, but we can see how the slippage is likely to occur by observing it. These quantities are suggested as important properties and they are compared in the following section.

To summarize, first, the mechanical response of the conductors in the cable, due to cable deformation is considered. Given the configuration of the cable cross section, the position angle \( \theta_c \) is specified for each conductor. Then, maximum friction force in each conductor is calculated using Eq. (20) with the boundary value \( T_c \) found from cable deformation. In the present computation, the value of \( T_c \) is identically zero because no cable deformation is assumed except bending. After this step, the maximum frictional force in known.

For a known bending curvature applied to the cable the tensile force on the conductor can be computed from Eq. (24).

If the computed tensile force is larger that the frictional force, the conductor is assumed to be in the slipping state and the tensile force acting on each conductor is given by Eq. (20). Otherwise, the conductor is in the sticking state and the tensile forces acting on it are found from Eq. (24).

The torsion and the curvature of each conductor is evaluated using Eqs. (36) and (37), respectively. Note that the right hand sides of these equations contain only known parameters and variables. The same process is repeated for all conductors in the cable.

As the next step, the wires in the conductor are considered. The positions of all wires are defined with position angle \( \theta_w \). The maximum friction force is calculated using the boundary value \( T_w \) which is given by Eq. (40). The force caused by bending is calculated according to Eq. (42). Tensile force in the wire cross section is specified with Eq. (40) for slipping state or Eq. (42) for sticking state. This process is repeated for all wires in each conductor.

As the last step, all the tensile forces acting on the cross sections of all wires are calculated. From these values, the bending moment for the cable can be evaluated according to Eq. (44). Hence, the behavior of the cable can be estimated when all the material properties, geometrical configuration and loading condition are known.

3. Experiment

To compare with the theoretical values, the bending stiffness of some cables are measured with simple experiments. The procedure of this test is described in this section. The experimental procedure is designed
in order to identify the mechanical properties of cables. The results are evaluated in such a way that the mechanical response of the whole cable can be quantified against the imposed deformation.

The jacket contributes to the stiffness by itself and by maintaining an internal pressure in the cable which generates frictional forces as the cable is deformed. Since the primary interest was to investigate the response of the helical structure, not the outer jacket, the cables were tested both with the jacket and with the jacket peeled off. Also, an approximate value of Young’s modulus for the jacket, undergoing deformation at relevant rate, was measured in a separate test. It was found that the effect of viscoelasticity of the jacket is small enough to be neglected. Using this, the direct influence of the jacket on the stiffness of the cable could be extracted and the effect of the internal pressure, exerted by the jacket and possibly the shield, on the response of the helical structure could be studied.

The bending response of the cable is characterized by the relation between applied moment and the resulting curvature of the cable. The test rig is illustrated in Fig. 4. The cable is first located vertically along the center line, where no external force is applied. Then the load cell moves down at a constant velocity applying the compressive force so that the cable is bent to shape a circular arc. The force acting on the cell and the time are recorded. The results presented here are obtained using moving velocity of 100 mm/min. Slower movements of 20 mm/min and 50 mm/min yielded in the same results and therefore the faster movement was chosen to reduce the experimentation time. In order to calculate the curvature of the cable, photos were taken from the front side, as sketched in Fig. 4, at several times during deformation. The curvature of the cable is calculated by analyzing the images in the following way. First, the edge of the cable was identified and thereafter it was approximated with polynomial fitting from which the curvature could be computed at each arbitrary point, $P$, see Fig. 4. At $P$, the distance to the center line is denoted by $d$, $l$ is the arc length of the cable above and $F$ is the force acting on the cable. The moment $M$ acting on point $P$ is then approximated by

$$M = Fd + \rho g l \frac{d^2}{2},$$

(44)

where $g$ (m/s$^2$) is the acceleration of gravity and $\rho$ (kg/m) is the weight per length of the cable. It is observed that the cable is almost straight near the end and that the curvature increases continuously along the cable and reaches maximum at the middle point. Therefore, the data from the first quarter of the cable is not used to reduce the influence of end effects. Also, the lower half of the cable is completely neglected to limit the errors induced by gravitational forces. This results in the lack of experimental data for sticking state. But this is not considered to be important because it has been found that the range of the curvature for sticking state is quite small and it is ignorable. From one picture, the static response of the cable can be obtained over a certain range of curvature. Then, the data from all images are used together to obtain data for the whole range.

![Fig. 4. Illustration of the bending test rig.](image-url)
investigated in the experiments. This test was repeated for several times for one cable. Then, their average values are given a experimental moment–curvature dependence line.

4. Results

The model developed in this paper is applied to two different cables to examine the quantities which are thought to be of importance for the lifetime of the cables. Both cables are of identical structure while the dimensions of the internal components are different. They consist of two layers which include 7 and 14 conductors, respectively, surrounded by a polyurethane (PUR) jacket. The conductors used in a cable are all identical. They consist of 260 copper wires bundled to form a helical structure, which is wrapped with insulation material, fluorinated ethylene propylene (FEP).

The developed model is valid for concentric helical structures of arbitrary order. However, the structure of the real cables used in this work is concentric on the conductor level but not on the wire level. In order to use the model, it was assumed that the wires were organized concentrically as presented in Table 1. The number of wires and their dimensions were slightly changed to generate a concentric conductor with fully populated layers under the constraint that the cross sectional area of the copper was preserved. The assumed geometrical properties of the cables are shown in Table 1 and the cross sections of the cables are illustrated in Fig. 5.

Table 1
Mechanical properties of Cable-A

<table>
<thead>
<tr>
<th>Layer n</th>
<th>$K_{cn}$</th>
<th>$z_{cn}$ (°)</th>
<th>$R_{cn}$ (mm) (Cable-A)</th>
<th>$R_{cn}$ (mm) (Cable-B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>15</td>
<td>0.0039</td>
<td>0.0054</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>12</td>
<td>0.0023</td>
<td>0.0032</td>
</tr>
</tbody>
</table>

Layer m

<table>
<thead>
<tr>
<th>Layer m</th>
<th>$K_{wm}$</th>
<th>$z_{wm}$ (°)</th>
<th>$R_{wm}$ (mm) (Cable-A)</th>
<th>$R_{wm}$ (mm) (Cable-B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>112</td>
<td>5</td>
<td>0.00025</td>
<td>0.0035</td>
</tr>
<tr>
<td>2</td>
<td>56</td>
<td>4</td>
<td>0.0002</td>
<td>0.0028</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>3</td>
<td>0.00015</td>
<td>0.0021</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
<td>2</td>
<td>0.0001</td>
<td>0.0014</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>1</td>
<td>0.0005</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

Fig. 5. Cross sections of cables.
It can be seen that the only difference between two cables is that Cable-B is approximately 1.4 times larger than Cable-A in size.

For friction coefficients, representative values are used from literatures as shown in Table 2. It is well known that the cable during bending follows the following equation:

\[ M = IE\kappa, \]

where \( M \) is the total bending moment and \( IE \) is the bending stiffness of the cable. Calculated values of bending stiffness as a function of curvature are shown in Fig. 6. These results are calculated with assumed values for the pressure as the pressure from both jacket and insulation material are not measured. Both cables are showing constant, high stiffness within a certain range of curvature with small values, which is because all the conductors are in a sticking state. When the curvature reaches a certain value, some of the conductors enter the slipping state and consequently the value of the bending stiffness decreases.

It is clear from Fig. 6 that the behavior of the cables drastically changes over a range of small values for the curvature, which is dominated by the behavior of conductors. Slippage of the wires begin in a region of larger curvature and it becomes the most important factor there. In robot applications, the applied bending curvature reaches approximately 10 m\(^{-1}\) at maximum and the interest is concentrated on this region. Based on this fact, only results from curvatures between 0.5 and 10 is extracted and studied in the following.

The theoretical values are compared with experimental data to validate the model. The assumed values for the compressing force \( P_c \) which is caused by pressure from the jacket and \( P_w \), from insulation material, are adjusted so that the theoretical cable response meet the experimental result. From a geometrical consideration, the tangential strains \( \tau_{\phi_c} \) and \( \tau_{\phi_w} \) occurring in the jacket and the insulation material, respectively, are given by.

<table>
<thead>
<tr>
<th>Friction coefficients between components</th>
<th>Copper–copper</th>
<th>Copper–FEP</th>
<th>FEP–FEP</th>
<th>FEP–PUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper–copper</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Fig. 6. Bending stiffness versus curvature.
where $t_c$ and $t_w$ are the thicknesses of the jacket and the insulation material, respectively, and $E_{jac}$ and $E_{ins}$ are Young’s moduli of the jacket and the insulation material, respectively. These values are identical for both cables and given in Table 3.

It is assumed here that the tangential strains in the jacket and the insulation are identical in value, i.e.,

$$\tau_{\phi c} = \tau_{\phi w} = \tau_{\phi},$$

resulting in a constant ratio between $P_c$ and $P_w$. The reason why this strong assumption is used is that the magnitude of these inherent strains is dependent on the manufacturing process and very difficult to measure. But this assumption can be applicable in our case because the effect of the pressure caused by these strains are quite small due to compressibility of the cable. Using that, the magnitudes of the forces and tangential strains are estimated and presented in Table 4 and corresponding pressures are shown in Table 5.

The theoretical and experimental cable responses are shown in Fig. 7. Two thicker lines in Fig. 7 are the hypothetical lines which are gained from experimental data, where the discrete measured points were approximated with the continuous lines. The experimental results are not shown in small curvature range because the measured data is strongly affected by the end condition.

Since the cables are compressible in the radial direction, the pressure from the jacket becomes very small as shown in Table 5. In other words, the compressibility of the cable is accounted for by using a reduced value for the internal pressure.

Fig. 8 shows the theoretical response of wires in Cable-A against pure bending. In Fig. 8(a), (c) and (e) the wires in slipping state are depicted with black dots and the wires in sticking state are expressed with gray dots for different bending curvatures. When applied bending curvature is 0.001 (Fig. 8(a)), all the wires are in sticking state. Then the slippage begins to occur in the outermost layer of conductor and it starts from the neutral axis as shown in Fig. 8(c). This feature conforms to the experimental result which has been reported by Raoof (1989). After all the wires in the first layer falls into the slipping state, the wires in the penultimate layer begins to slip. Wires in the penultimate layer are all in slipping state when the curvature reaches five as shown in Fig. 8(e). It has been found that the wires in third, fourth and fifth layers do not fall in slipping state in the realistic range of the bending curvature.

### Table 3
The values for surrounding material properties

<table>
<thead>
<tr>
<th>$t_c$ (mm)</th>
<th>$t_w$ (mm)</th>
<th>$E_{jac}$ (Pa)</th>
<th>$E_{ins}$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.27</td>
<td>$5 \times 10^7$</td>
<td>$5 \times 10^8$</td>
</tr>
</tbody>
</table>

### Table 4
Assumed values for compressing force (N) and tangential strain (%)

<table>
<thead>
<tr>
<th></th>
<th>$P_c$ (N)</th>
<th>$P_w$ (N)</th>
<th>$\tau_{\phi}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cable-A</td>
<td>0.16</td>
<td>0.05</td>
<td>0.00067</td>
</tr>
<tr>
<td>Cable-B</td>
<td>0.5</td>
<td>0.15</td>
<td>0.002</td>
</tr>
</tbody>
</table>

### Table 5
Corresponding pressure from jacket and insulation (Pa)

<table>
<thead>
<tr>
<th></th>
<th>Jacket</th>
<th>Insulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cable-A</td>
<td>97</td>
<td>3600</td>
</tr>
<tr>
<td>Cable-B</td>
<td>290</td>
<td>11000</td>
</tr>
</tbody>
</table>
The axial stress working in the cross section of each wire is illustrated in Fig. 8 (b), (d) and (f). The maximum and minimum stress are shown by the color map, the unit of which is MPa. In the region of small curvature shown in Fig. 8(b), the wires in the upper half of the cross section are exposed to tensile stress while the wires in the lower half are compressed. And the magnitude of the stress is almost proportional to the distance from the cable neutral axis. This is because all the conductors are also in sticking state, although the state of the conductor is not shown here, and all the components are behaving as a solid beam. When applied curvature is 0.5 as shown in Fig. 8 (d), all the conductors are in slipping state and they are bent around their own axis. Consequently, the appearance of the distribution of axial force is similar for all the conductors. In Fig. 8(f) where the applied curvature is five, it can be seen that the maximum stress is not occurring in the outermost layer of conductor. These phenomenon are explained in the following by showing the profile of these properties.

Maximum stress occurring in the conductor located at $h = \frac{p}{2}$ is presented in Fig. 9, the maximum stress which occurs in the wire cross section is shown. It is observed that the maximum stress in Cable-B is around 1.4 times larger than that of Cable-A. This is because the stress caused by bending is linearly dependent on the distance between wire center line and the neutral axis of bending.

One important feature is that the maximum stress does not necessarily occur in the outermost wires in the outermost conductors as shown in Fig. 8. The reason is that the inner wire is radially compressed not only by the pressure from insulation material but also by the outer conductors. Consequently, the effect of the friction can be larger in an inner layer and they remain in sticking state for a larger applied curvature, which results in a larger stress.

The data presented in Fig. 9 consists of five distinguishable parts. This characteristic can be well explained by taking the number of slipping wires, shown in Fig. 10, into account.

In the first part, all the wires are sticking to conductor and the magnitude of the stress caused by bending is proportional to the bending curvature. The maximum stress occurs in the outermost wire in the outermost conductor. After that wires have shifted to a slipping state, the stress in these wires does not grow and the maximum stress becomes constant for a certain range of curvature. The stress in wires in the penultimate layers exceeds the maximum stress at approximately $\kappa_0 = 1.3$, but they also start to slip when the curvature reaches approximately $\kappa_0 = 2.0$. In the region where the curvature is more than 3.0, the maximum stress occurs
in the wires in the third layer of conductors. The wires in third, fourth and fifth layer never slips in the realistic

Fig. 8. Illustration of the behavior of wires shown in the cross section. (a) Slippage distribution. (b) Stress distribution. (c) Slippage distribution. (d) Stress distribution. (e) Slippage distribution. (f) Stress distribution.
According to Fig. 10, the number of slipping wires increases with curvature, and it stops growing after that a first limit is reached. Then it grows again and becomes a constant after a second limit is passed.
As was mentioned earlier, the critical value of curvature for slippage is smaller in the outer layer because of the effect of the pressure. This means that the slippage of wires proceeds from the outermost layer towards inner layers. The outermost wires in the conductors shifts to slipping mode within the range $0.2 < \kappa < 0.6$, and the wires in the penultimate layers start to slip within the range $1.3 < \kappa < 2$. The conditions for slipping of layers 3, 4 and 5 are never satisfied within the realistic range of the curvature.

The average value of tensile stress in the wires is also shown in Fig. 11. It is clear that the slope decreases as the curvature increases due to the effect of wire slipping.

The model developed in this paper is also used to study the importance of various parameters on helical structures of any order. Here the value of the curvature is set to 0.5, where all the conductors have a layer of slipping wires.

The impact of the pressure from the jacket on the internal stress is illustrated in Fig. 12. The plotting range of the pressure is selected so that the elasticity of the jacket is preserved. The increase of the force caused by the jacket builds up the frictional forces between conductors and it results in larger elongation of conductors and wires. Consequently, the internal stresses are in positive relation with the pressure. It should be noted that the pressure also affects the slipping condition of the wires very little. Here it was found that the number of the slipping wires is constant over the whole range considered. Therefore, the results were not presented explicitly.

Stress and the number of slipping wires are plotted against the pressure from the insulation material in Figs. 13 and 14, respectively. It is shown that the pressure from the insulation material severely affects the wire state. Larger value of the pressure causes larger friction force between wires and slippage is prevented. This, in turn, results in an increase of maximum frictional force. It is also shown that the stress becomes constant in the range where the pressure is large enough so that no wires are allowed to slip. This means that the effect of the insulation pressure is implicit to the internal stress, for example, the increase of the pressure does not affect the internal behavior when the cable is exposed to small bending and the wires are in stick state. On the other hand the internal stress is directly increased by larger pressure when the applied bending curvature is large enough so that the wires can be in a slipping state for lower pressures.

In Fig. 15, the stress is shown against either $\alpha_{c1}$ or $\alpha_{w1}$. Lay angles of inner layers are also changed so that the ratio between lay angles in different layers are kept constant though only the value for the outermost layer is shown here. As a general tendency, the internal stress is gradually reduced by using larger lay angles for
conductors and wires. The relation between lay angles and stress is not simple, but the most dominating factor is that the length of the component included in a unit length of the cable is increased with larger lay angles and it consequently diffuses internal strains.

Increase of the lay angle, on the other hand, prevents the slippage as shown in Fig. 16 since tensile force caused by bending is also reduced according to the same logic, which is indicated by Eq. (24). Another factor

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**Fig. 12. Stress versus pressure from the jacket.**

**Fig. 13. Stress versus pressure from the insulation material.**
is that the radial pressure caused by each layer on the underlying components increases with larger lay angles, which is apparent from Eq. (17). It can also be seen from both figures that the effect of wire lay angle is larger than that of conductor lay angle.
The influence of the tension applied in the cable is investigated and shown in Figs. 17 and 18. This is the case where the cable is preloaded with tensile force before applying bending curvature. It is shown in Fig. 17 that the elongation of the cable directly increases the tensile force on all components while it prevents the slip-page as shown in Fig. 18 due to the fact that increasing the tensile force also increases the radial pressure. This
is explained as the tensile force in the conductor increases with the applied elongation according to Eq. (6) and the normal force \( dN_1 \) in Eq. (17) is also increased. As a result of this, the force working in the radial direction is proportionally increased. It should be noted here that the influence of the radial pressure from the jacket or the insulation material is not taken into account in this calculation in order to extract the pure response of helical structures. It can be expected that the magnitude of the radial pressure from outer material increases with the axial elongation due to the poisson’s ratio effect. Hence the actual response of the cable is emphasized, or, the internal forces increase more and the more slippage is prevented.

5. Discussion

It is now well established that the fretting fatigue is the main reason for cable fatigue. Raoof (1989) suggested that the stress-based analysis for fatigue performance be resolved and new parameter was suggested, which is capable of predicting fatigue life of strands. In our case, though, the cable is not necessarily exposed to axial load and wear-related fatigue is also dominant. Hence we consider that knowing the behavior of internal stress is meaningful when the life time is discussed.

This model developed in this paper is rather rough to analyze the internal behavior of electrical cables precisely and quantitatively. But this still gives us a good rule to compare the different designs of cables in terms of life time expectation. At the same time, this model becomes a basis for the complete modeling of multi-order helical structures. It is now being extended to calculate other possible key properties such as the distance of the slippage or the energy dissipated by the slippage.

6. Conclusion

A new model was developed to calculate the stress in multi-order helical structures taking frictional effect into account. The stresses in the conductors are calculated first and then the same formulae are applied for wires reflecting the results of calculation for conductors. The theoretical response of the cable to bending was obtained by summing up the stresses acting in each wires and compared with experimental result to estimate the unknown pressures from jacket and insulation material.
Internal stress and the number of slipping wires were suggested as key properties for life time expectation and their behavior was studied. It is also investigated how these key properties affected by changing the cable configuration or loading conditions.

References