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Elimination of ambiguities in $\pi\pi$ phase shifts using crossing symmetry

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Abstract

Roy's equations, which incorporate crossing symmetry of the $\pi\pi$ scattering amplitudes, are used to resolve the present ambiguity between two solutions for the scalar–isoscalar phase shifts below 1 GeV. It is shown that the "down-flat" solution satisfies well Roy's equations and consequently crossing symmetry while the other solution called "up-flat" does not and thus should be eliminated.

1. Introduction

A good knowledge of the pion–pion scattering is important in studies of different processes in nuclear and particle physics [1]. Construction of phenomenological $\pi\pi$ amplitudes requires not only experimental input but also use of theoretical constraints such as unitarity, analyticity and crossing symmetry. Direct study of the $\pi\pi$ collisions is beyond present experimental possibilities. Phenomenological phase shifts are obtained through partial wave analyses of final states in which pions are produced. These analyses are often model dependent and can sometimes lead to ambiguous results.

In 1997 a study of the $\pi^- p_\uparrow \to \pi^+\pi^- n$ reaction on a polarized target has been performed in the $m_{\pi\pi}$ effective mass between 600 and 1600 MeV giving four solutions for the $\pi\pi$ scalar–isoscalar phase shifts below 1 GeV [2]. Using the unitarity constraint two "steep" solutions were rejected and the two remaining ones, called "down-flat" and "up-flat", passed this test [3]. Elimination of this remaining ambiguity is necessary for extension of studies done near the $\pi\pi$ threshold and above using chiral perturbation models based on QCD (see, for example, [4,5]).

A steady increase of the $\pi\pi$ scalar–isoscalar phase shifts below 1 GeV can be interpreted as due to the presence of a broad σ-meson [6,7]. Previous elimination of "steep" solutions excludes a narrow σ with a width comparable to that of the ρ-meson [3]. In this channel, near the $K\overline{K}$ threshold and above it, other resonances exist which could be mixed with glueball states predicted by QCD.

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In order to eliminate the above mentioned "up–down" ambiguity one can check if the corresponding amplitudes satisfy crossing symmetry. Roy's equations [8] can serve as a tool to perform this check. They also correlate different experimental phase shifts determined near the $\pi\pi$ threshold and at higher energy, in particular, for the scalar–isoscalar, $\ell = 0$, $I = 0$, the scalar–isotensor, $\ell = 0$, $I = 2$, and the vector–isovector, $\ell = 1$, $I = 1$, $\pi\pi$ partial waves. These equations have been already applied by Pennington and Protopopescu [9] to study different $\pi\pi$ phase shift solutions obtained in the early seventies. They were able to resolve a "steep-flat" type ambiguity [3] present in the phenomenological $\pi\pi$ amplitudes considered at that time. Later two high-statistics experiments were performed [10,11] and, as previously said, the analyses of these experiments lead to two plausible solutions [2,3]. Recently a comprehensive analysis of Roy's equations for the $\pi\pi$ interaction has appeared [12]. There, a special emphasis is put on the $m_{\pi\pi}$ range from the $\pi\pi$ threshold to 0.8 GeV. In the present Letter we pay a particular attention to the range between 0.8 and 1 GeV. In this range the largest differences between the "up-flat" and "down-flat" solutions occur, reaching values up to 45° (see Fig. 4 in [3] and Fig. 2 here).

2. Roy's equations

Assuming analyticity one can write twice subtracted fixed-t dispersion relations [8] for the unitary $\pi\pi$ amplitudes,

$$T^I(s,t,u) = \sum_{I'=0}^{2}\Bigg(C_{st}^{II'}\big[B^{I'}(t) + (s-u)D^{I'}(t)\big] + \int_{4\mu^2}^{\infty} \frac{ds'}{\pi s'^2}\left(\frac{s^2}{s'-s}1^{II'} + \frac{u^2}{s'-u}C_{su}^{II'}\right) \times \operatorname{Im} T^{I'}\big(s',t,u'\big)\Bigg). \tag{1}$$

In this equation s, t and u are the usual Mandelstam variables satisfying $s + t + u = 4\mu^2$, where μ is the pion mass; $C_{st}^{II'}$ and $C_{su}^{II'}$ are the isospin matrix elements and $1^{II'}$ is the unit matrix. The subtraction functions $B^{I'}(t)$ and $D^{I'}(t)$ can be related to the isospin 0 and 2 S-wave scattering lengths a_0^0 and a_0^2, respectively.

Projection of (1) onto partial waves leads to Roy's equations,

$$\begin{aligned} \operatorname{Re} f_\ell^I(s) &= a_0^0\delta_{I0}\delta_{\ell 0} + a_0^2\delta_{I2}\delta_{\ell 0} \\ &\quad + \big(2a_0^0 - 5a_0^2\big)\left(\delta_{I0}\delta_{\ell 0} + \frac{1}{6}\delta_{I1}\delta_{\ell 1} - \frac{1}{2}\delta_{I2}\delta_{\ell 0}\right) \times \frac{s-4\mu^2}{12\mu^2} \\ &\quad + \sum_{I'=0}^{2}\sum_{\ell'=0}^{1} \fint_{4\mu^2}^{s_{\max}} ds'\, K_{\ell\ell'}^{II'}\big(s,s'\big)\operatorname{Im} f_{\ell'}^{I'}\big(s'\big) \\ &\quad + d_\ell^I(s,s_{\max}). \end{aligned} \tag{2}$$

Here the functions $K_{\ell\ell'}^{II'}(s,s')$ are the kernels and $d_\ell^I(s,s_{\max})$ are the so-called driving terms. Detailed expressions of these functions can be found, for instance, in [12,13]. The driving terms contain the low-energy, $s' \leqslant s_{\max}$, contributions from the partial waves $\ell' \geqslant 2$ and the high-energy, $s' \geqslant s_{\max}$, contributions from all the partial waves. If one assumes Mandelstam analyticity, the range of validity of Roy's equations (2) extends to $68\mu^2 = (1.15\ \text{GeV})^2$ [8].

The partial waves amplitudes $f_\ell^I(s)$ are related to the $\pi\pi$ phase shifts δ_ℓ^I and inelasticities η_ℓ^I:

$$f_\ell^I(s) = \sqrt{\frac{s}{s-4\mu^2}}\,\frac{1}{2i}\big(\eta_\ell^I e^{2i\delta_\ell^I} - 1\big). \tag{3}$$

Each set of experimentally determined phase shifts and inelasticities can serve as input to calculate the real and imaginary parts of the partial wave amplitudes. After a suitable parameterization, the imaginary parts can be inserted into Roy's equations (2) from which one obtains the real parts as *output* to be compared with the corresponding real part *input*. The quantitative agreement between output and input will be used to verify how well a given set of phase shifts satisfies crossing symmetry in a particular range of $m_{\pi\pi}$ where Roy's equations can be applied.

3. Input amplitudes

The "down-flat" and "up-flat" data [2] together with the results on the $K\overline{K}$ phase shifts [14] were analysed by us using a separable potential model of three coupled scalar–isoscalar channels ($\pi\pi$, $K\overline{K}$ and an effective 4π system) [15,16]. This model yields particularly good fits to the "down-flat" solution from 600 to 1600 MeV. The "up-flat" data are reasonably well described only above 970 MeV. From 800 to 970 MeV the model values are too low in comparison with the "up-flat" data (see Fig. 1(b) of [15]). In our calculations we need a faithful representation of both "up-flat" and "down-flat" solutions and therefore below 970 MeV we use the following Padé approximation for the δ_0^0 phase shifts:

$$\tan\delta_0^0(s) = \frac{\sum_{i=0}^{4}\alpha_{2i+1}k^{2i+1}}{\Pi_{i=1}^{3}(k^2/\alpha_{2i}-1)}, \tag{4}$$

where $k=\frac{1}{2}\sqrt{s-4\mu^2}$ is the pion momentum and α_j $(j=1,\ldots,7,9)$ are constant parameters. This choice is dictated by the analytical properties of the Jost functions. These constants will be obtained from the best fits to data and to Roy's equations for the "up-flat" and "down-flat" separately. In these fits we also use the near threshold phase-shift differences, $\delta_0^0-\delta_1^1$, recently extracted from the high statistics K_{e4} decay experiment [17]. Using the power expansion of the S-wave amplitudes near the $\pi\pi$ threshold,

$$\operatorname{Re} f_0^I(s) = a_0^I + b_0^I k^2 + \cdots, \tag{5}$$

we can relate the parameters a_0^0 and b_0^0 to the constants α_j. One has: $a_0^0=-\alpha_1\mu$ and $b_0^0=-\alpha_1\mu(0.5\mu^{-2}+\alpha_2^{-1}+\alpha_4^{-1}+\alpha_6^{-1}-\alpha_1^2)-\alpha_3\mu$. The parameters α_7 and α_9 are chosen to match the values of δ_0^0 following from the three-channel fits at 969 and 970 MeV [15]. Above 970 MeV we use the fit A to describe the "down-flat" solution and the fit C for the "up-flat" one. These model amplitudes are used up to the $s_{\max}$ value even if it exceeds 1600 MeV (see (2)).

The isotensor wave is parameterized within the separable potential model of [7,15]. We use a rank-two potential:

$$V_{\pi\pi}^{I=2}(p,p') = \sum_{i=1}^{2}\lambda_i g_i(p) g_i(p'), \tag{6}$$

where

$$g_i(p) = \sqrt{\frac{4\pi}{\mu}}\,\frac{1}{p^2+\beta_i^2} \tag{7}$$

are form factors with range parameters β_i, p and p' are the pion center of mass momenta in the initial and final states, respectively. The threshold parameters a_0^2 and b_0^2 can be related to the strength parameters $\Lambda_i=\lambda_i/(2\beta_i^3)$. The $I=2$ $\pi\pi$ phase shifts from [18], obtained with their method A, serve as input in our fitting procedure. Here we assume the isotensor wave to be elastic ($\eta_0^2\equiv 1$) from the $\pi\pi$ threshold to $s_{\max}$.

For the P-wave, from threshold to 970 MeV, we use an extended Schenk parameterization as defined in [12]

$$\tan\delta_1^1(s) = \frac{2}{\sqrt{s}}k^3\left(A+Bk^2+Ck^4+Dk^6\right) \times\left(\frac{4\mu^2-s_\rho}{s-s_\rho}\right). \tag{8}$$

The parameter A is equal to the P-wave scattering length a_1^1 and the parameter B is related to the slope parameter $b_1^1=B+4A/(s_\rho-4\mu^2)$. The parameter s_ρ is equal to the ρ-mass squared. Above 970 MeV the P-wave amplitude is represented by the K-matrix parameterization of Hyams et al. [19]. The parameters C and D of (8) are chosen to match both parameterizations at 969 and 970 MeV. We have checked that the near threshold scalar–isoscalar phase shifts determined from the differences $\delta_0^0-\delta_1^1$ in [17] are insensitive to different P-wave parameterizations. Differences between parameterizations of [12] and [19] are smaller than the errors of $\delta_0^0-\delta_1^1$.

The driving terms $d_\ell^I(s,s_{\max})$ in (2) are calculated including the contributions of $f_2(1270)$ and $\rho_3(1690)$ and the Regge contributions from the Pomeron, ρ- and f-exchanges. We use the Breit–Wigner parameterization of $f_2(1270)$ and $\rho_3(1690)$ as described in [2] with masses, widths and $\pi\pi$ branching ratios taken from [20]. The range parameters are chosen to be 5.3 and 6.4 GeV^{-1} for the $f_2(1270)$ and $\rho_3(1690)$ resonances, respectively (see Table 4 of [19]). The Regge contributions are parameterized as in [12] without inclusion of the small u-crossed terms. The $s_{\max}$ limit is set to $(2\ \text{GeV})^2$ and our results are fairly close to those of [12]. The most important contribution in the $\ell=I=0$ channel comes from the

$f_2(1270)$ resonance. It roughly agrees with the result of Basdevant et al. [13] where only the $f_2(1270)$ contribution was considered and $s_{\max}$ was set equal to $(1.46\ \text{GeV})^2$. The $d_0^2(s, s_{\max})$ results are much smaller than those of [13] due to the difference in $s_{\max}$ and to the lack of the $\rho_3(1690)$ contribution in [13]. In the P-wave there is a strong cancellation between the contributions of $f_2(1270)$ and $\rho_3(1690)$ leading to very small values, again much smaller than in [13]. We have considered the different parameterizations of $f_2(1270)$ and $\rho_3(1690)$ resonances used by [12]. The changes from the Breit–Wigner form, that we used, do not affect the phase shifts in all three partial waves by more than one degree below 970 MeV. In the $\ell = 0$, $I = 0$ case the Regge contributions are of the order of a few percent of the resonance contributions. For the isospin 1 and 2 they are of the same order as the resonance contributions but the corresponding overall driving terms are by an order of magnitude smaller than the isospin 0 term. Alternative Regge contributions were considered following papers [9,21] and [22]. These do not change significantly the driving terms, the changes of phase shifts being much smaller than one degree in the effective mass range up to 1 GeV.

Our thirteen free parameters, six for the isoscalar S-wave, four for the isotensor one and three for the P-wave are determined through a least square fit to the data together with the minimization of the squares of the differences between the input ("in") and output ("out") of Roy's equations for the three waves. We define

$$\chi^2_{\text{total}} = \sum_{I=0,1,2} \left[\chi^2_{\exp}(I) + \chi^2_{\text{Roy}}(I)\right], \tag{9}$$

where

$$\chi^2_{\exp}(I) = \sum_{i=1}^{N_I} \left[\frac{\sin(\delta^I_\ell(s_i) - \varphi^I_\ell(s_i))}{\Delta\varphi^I_\ell(s_i)}\right]^2. \tag{10}$$

In (10) $\varphi^I_\ell(s_i)$ and $\Delta\varphi^I_\ell(s_i)$ represent the experimental phase shifts and their errors, respectively. The χ^2 of the fit to Roy's equations is defined as

$$\chi^2_{\text{Roy}}(I) = \sum_{j=1}^{12} \left[\frac{\text{Re}\, f^I_{\text{in}}(s_j) - \text{Re}\, f^I_{\text{out}}(s_j)}{\Delta f}\right]^2, \tag{11}$$

where $s_j = [4j + 0.001]\mu^2$ for $j = 1, \ldots, 11$ and $s_{12} = 46.001\mu^2$. We take a Δf value of 0.5×10^{-2} to obtain reasonable values of χ^2_{Roy} corresponding to an accuracy of one-half percent. Simultaneously we require that the $\chi^2_{\exp}$ should not be larger than about 18 for the fit to the 18 data points between 600 and 970 MeV. We use the CERN MINUIT program which provides errors of the fitted parameters.

For the isospin 0 wave the number of data points N_0 is 24 in (10). It consists of the 18 "down-flat" or 18 "up-flat" data points [2] between 600 and 970 MeV and of the 6 points below 400 MeV from the K_{e4} decay experiment [17]. In the isotensor wave we use the 12 data points [18] from 350 to 1450 MeV. For the isospin 1 we generate 8 pseudo-data points using the K-matrix parameterization between 600 and 970 MeV [19] and we choose $\Delta\varphi^1_1 = 2°$.

4. Results

The Padé approximants for the isoscalar S-wave amplitude supplemented by the model amplitudes [15] for $m_{\pi\pi} > 970$ MeV together with the P-wave parameterization and that of the isotensor S-wave as described above, constitute our input to Roy's equations.

A good global fit to data and Roy's equations can be achieved only for the "down-flat" data. The resulting $\ell = 0, I = 0$ phase shifts, plotted as solid lines in Fig. 1 are compared to the data. The χ^2 values are summarized in Table 1. For the 18 points between 600 and 960 MeV the χ^2 is 16.6 and it is 5.7 for the 6 near threshold points. The χ^2 per degree of freedom on the isoscalar phase shifts is 1.2. Roy's equations are very well fulfilled, the differences between the real parts "in" and "out" being smaller than 0.8×10^{-3} for the isoscalar wave and smaller than 2×10^{-3} for the isotensor amplitude. For the P-wave this difference does not exceed 6×10^{-3}. The corresponding parameters are given in Table 2. The low-energy parameters a^0_0, b^0_0, a^2_0, b^2_0 and a^1_1, compare well within errors with those of [12] but b^1_1 is smaller by about 50%. Relative errors of those parameters vary from about 5% for a^0_0, b^0_0 and a^1_1 to 25% for b^1_1. For the other parameters, they are less than 20% with the exception of α_2. This parameter is negative, so its influence on the values of δ^0_0 is small as can be seen from (4). It furthermore does not appear in the two first coefficients of the low energy expansion of $\tan\delta^0_0$. One then expects the error of α_2 to be large.

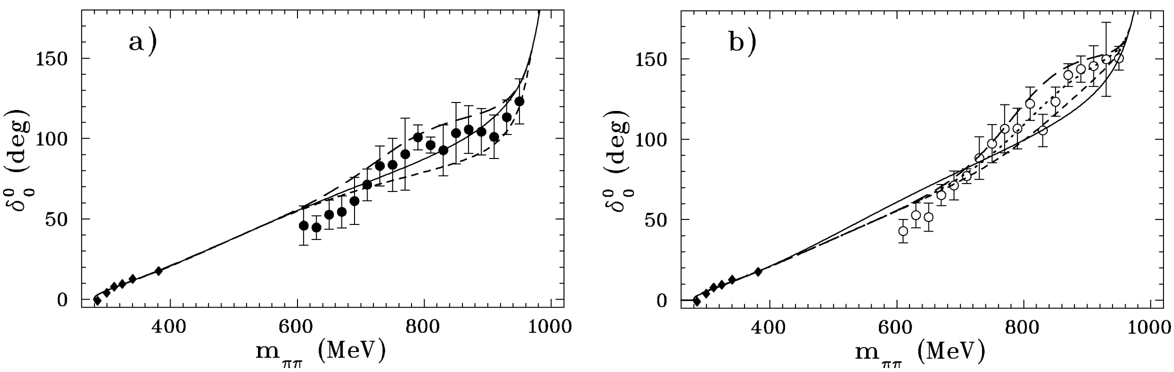

Fig. 1. Scalar–isoscalar phase shifts for (a) “down-flat” data (full circles) and (b) “up-flat” data (open circles) [2]. Diamonds denote the K_{e4} data [17]. Solid lines represent fits to Roy’s equations and to data. Dashed lines represent bands of the Padé fits to the data of [2]. The dotted line in (b) shows the special Padé fit.

Table 1
χ^2 values for the different fits; N_I being the number of data and n_I that of free parameters, $N_0 = 24$, $n_0 = 6$, $N_1 = 8$, $n_1 = 3$, $N_2 = 12$, $n_2 = 4$

	“Down-flat” case			“Up-flat” case					
	Global fit			Global fit			Special fit		
I	0	1	2	0	1	2	0	1	2
$\chi^2_{\text{exp}}(I)$	22.3	7.0	8.1	53.0	8.4	6.9	19.0	7.0	8.1
$\chi^2_{\text{Roy}}(I)$	0.1	6.0	0.4	0.3	7.4	0.5	1.2×10^4	5.5	5.8
χ^2_{total}		43.9			76.5			1.2×10^4	

Table 2
Parameters from the fit to Roy’s equations and to the “down-flat” data of [2]; α_3, α_7, α_9, B, C, D, Λ_1 and Λ_2 are dependent parameters

Isoscalar S-wave			Isovector P-wave			Isotensor S-wave		
a_0^0	0.224 ± 0.013		a_1^1	$(3.96 \pm 0.24) \times 10^{-2}$	μ^{-2}	a_0^2	$(-3.43 \pm 0.36) \times 10^{-2}$	
b_0^0	$0.252^{+0.012}_{-0.010}$	μ^{-2}	b_1^1	$(2.63^{+0.67}_{-0.66}) \times 10^{-3}$	μ^{-4}	b_0^2	$(-7.49^{+1.01}_{-1.65}) \times 10^{-2}$	μ^{-2}
α_2	$-4.41^{+2.06}_{-5.05}$	μ^2	s_ρ	30.87 ± 0.14	μ^2	β_1	4.88 ± 0.25	μ
α_4	7.53 ± 0.32	μ^2				β_2	1.23 ± 0.21	μ
α_5	$(2.29^{+0.35}_{-0.45}) \times 10^{-2}$	μ^{-5}						
α_6	$12.0^{+0.5}_{-0.3}$	μ^2						
α_3	-0.153	μ^{-3}	B	-3.27×10^{-3}	μ^{-4}	Λ_1	0.268	
α_7	-0.320×10^{-3}	μ^{-7}	C	5.24×10^{-4}	μ^{-6}	Λ_2	-0.0257	
α_9	-0.312×10^{-4}	μ^{-9}	D	-2.66×10^{-5}	μ^{-8}			

In the “up-flat” case a good global fit cannot be obtained, as the χ^2_{exp} on the 18 data points of [2] between 600 and 960 MeV is as large as 46.4. Including the χ^2 of 6.6 from the 6 points of the K_{e4} experiment, one obtains the total value equal to 53.0 (see Table 1). The χ^2 per degree of freedom on the isoscalar data is as large as 2.9 which shows that this fit for the “up-flat” case, plotted as the solid line in Fig. 1(b), is not acceptable. The agreement with Roy’s equations is, however, almost as good as in the “down-flat” case. In Fig. 2 we plot, together with the data, the solid curves of Fig. 1(a) and (b) representing Roy’s fits to the “down-flat” and “up-flat” data. These two curves define a band of isoscalar phase shifts fitting

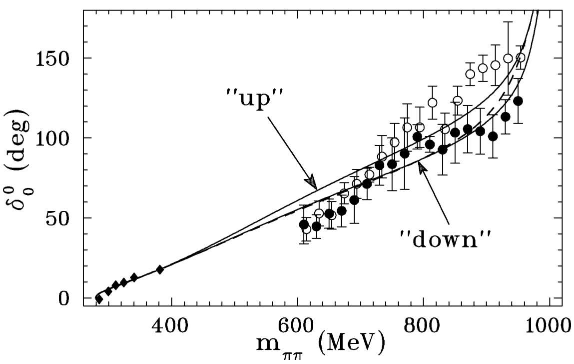

Fig. 2. Solid lines—fits to Roy's equations and to the "down-flat" data (full circles) and to the "up-flat" data (open circles) of [2] as well as to the K_{e4} data of [17] represented by diamonds. The dashed curve is Roy's fit to the K_{e4} data and to the "up-flat" data in the restricted range of $m_{\pi\pi}$ up to 740 MeV.

well threshold data and Roy's equations for two sets of data above 600 MeV. The "down"curve reproduces well the "down-flat" data while the "up" curve *does not reproduce* the "up-flat" data.

The main differences between the "down-flat" and "up-flat" solutions are for energies between 800 and 970 MeV. Below 740 MeV phase shifts of both solutions are compatible within their error bars and the "down" curve reproduces well the "up-flat" data (see Fig. 2). We have checked that there is a possibility to find a good fit to Roy's equations and to the "up-flat" data in this limited range of effective mass. This is shown as the dashed line in Fig. 2. Differences of phase shifts between this line and the "down" one are smaller than one degree up to 840 MeV. Therefore, based on the "down-flat" fit, we build up a special Padé fit to the "up-flat" data in order to reproduce as well as possible their $m_{\pi\pi}$ dependence. We proceed in the following way. Below 600 MeV this fit (dotted line in Fig. 1(b)) is constrained to approximate the previously obtained "down-flat" isoscalar amplitude and, in particular, to reproduce its scattering length a_0^0 and slope parameter b_0^0 as well as its two values at 500 and 550 MeV. The corresponding $\chi^2_{\text{exp}}(0)$ of 19.0 is good but the $\chi^2_{\text{Roy}}(0)$ of 1.2×10^4 is very large. The differences between the "in" and "out" real parts are as large as 0.25 around 900 MeV. So, if one tries to improve the fit to the "up-flat" data then one spoils the fit to Roy's equations.

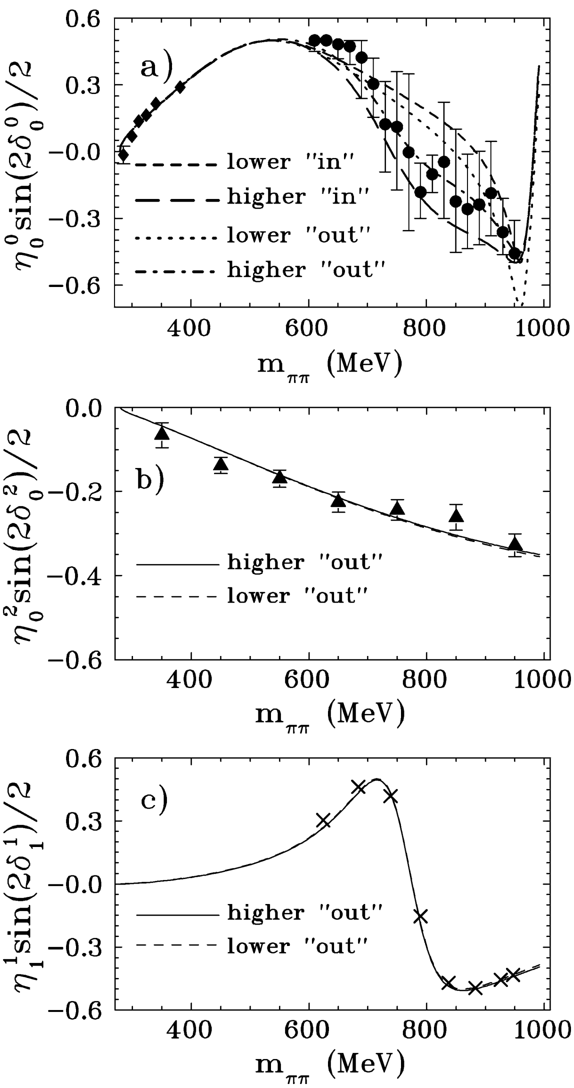

Fig. 3. Real parts of the $\pi\pi$ amplitudes (multiplied by $2ks^{-1/2}$) corresponding to the "down-flat" data [2] (full circles). In (a) input and output bands of the scalar–isoscalar real parts are shown. Diamonds denote the K_{e4} data [17]. Solid and dashed lines in (b) and (c) represent the output bands. In (b) triangles denote the isotensor data of [18]. Crosses in (c) are the isovector pseudo-data calculated from the K-matrix fit of [19].

We have studied the influence of the experimental errors on the $\pi\pi$ input amplitudes by calculating Roy's equations for two extreme isoscalar amplitudes fitted to the data points shifted upwards ("higher-in") or downwards ("lower-in") by their errors. Below 600 MeV these fits were constrained in the same way as in the special Padé fit to the "up-flat" data just described above. The corresponding long- and short-dashed lines in Fig. 1(a) and (b) show typical bands delimiting the possible values of the experimental data within their errors. The resulting output curves, lower "out" and higher "out", of the numerical integrations of Roy's equations for the three partial-wave real parts, multiplied by $2ks^{-1/2}$, are displayed in Figs. 3 and 4. In Figs. 3(a) and 4(a) we also show the input band for the real part of the isoscalar wave, limited by two dashed lines, called lower "in" and higher "in". These dashed lines correspond to the dashed lines shown in Figs. 1(a) and (b). We do not show the "in" lines in Figs. 3(b), (c), and 4(b), (c) since they are almost indistinguishable from the "out" lines.

Below 600 MeV Roy's equations are well satisfied for all waves. At higher energies a good agreement is found for the P-wave and $I = 2$ S-wave. As seen in Fig. 3, in the "down-flat" case both "out" curves lie inside the band limited by the "in" curves up to 937 MeV. Above this energy the lower "out" curve starts to lie below the higher "in" and at about 950 MeV the higher "out" starts to be above the lower "in" curve. The "in" band is then inside the "out" one. It means that there is still a possibility to find an "out" solution within the "in" band. We can then conclude that within the error bars the "down-flat" solution satisfies Roy's equations and consequently crossing symmetry. On the contrary, in Fig. 4, for the scalar–isoscalar "up-flat" solution, the output band lies outside the input one from 840 to 970 MeV. This eliminates the "up-flat" solution as it does not satisfy crossing symmetry in that energy range.

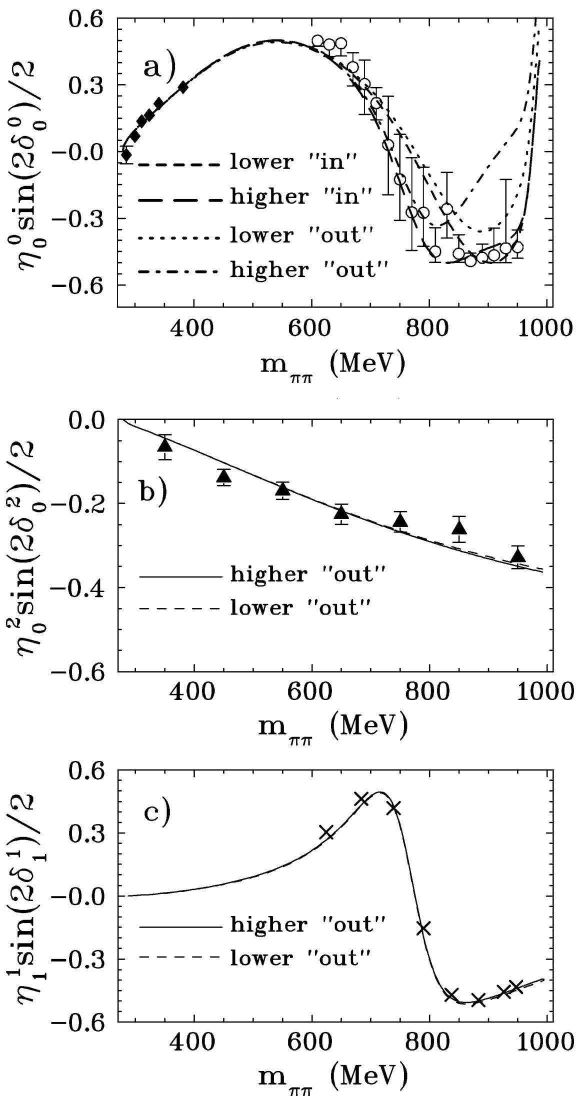

Fig. 4. Same as in Fig. 3, but for the "up-flat" data of [2].

5. Conclusions

We have studied the scalar–isoscalar $\pi\pi$ amplitudes constructed from two sets of phenomenological phase shifts called "up-flat" and "down-flat" [2]. It has been shown that only the "down-flat" solution satisfies well crossing symmetry. The conclusion about violation of Roy's equations for the "up-flat" data is insensitive to the variation of the different parameters in our input: reasonable changes of the $\ell = 1$, $I = 1$ and $\ell = 0$, $I = 2$ amplitudes, shift of the upper integration limit $s_{\max}$ from (2 GeV)^2 to (1.46 GeV)^2, modifications of the $f_2(1270)$ and $\rho_3(1690)$ resonance parameters and of the Regge amplitudes in the driving terms. All these changes do not lead to complete overlap of the input and output bands for the "up-flat" solution.

Recently a joint analysis of the S-wave $\pi^+\pi^-$ data [10,11] and of the new $\pi^0\pi^0$ data [23], obtained by the E852 Collaboration at 18.3 GeV/c, has been performed [24]. It has been shown that, using the one-pion and a_1-exchange model developed in [2], the calculated S-wave intensity of the $\pi^0\pi^0$ system agrees with the measured $\pi^0\pi^0$ intensity only for the $\pi\pi$ amplitudes obtained from the "down-flat" phase shifts. So, in [24] the "up-flat" solution has also been eliminated and a new "down-flat" set of phase shifts, compatible with the $\pi^+\pi^-$ and $\pi^0\pi^0$ data, has been found. We have verified that these new "down-flat" phase shifts, parameterized as described above, satisfy well Roy's equations.

To conclude, using the theoretical constraints of crossing symmetry, unitarity and analyticity, the fourfold ambiguities in the isoscalar S-wave $\pi\pi$ phase shifts, in the $\pi\pi$ invariant mass range from 800 to 1000 MeV, have been eliminated in favour of the "down-flat" solution.

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