Modified Frequency Scaling Algorithm for FMCW SAR Data Processing

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Abstract

This paper presents a modified frequency scaling algorithm for frequency modulated continuous wave synthetic aperture radar (FMCW SAR) data processing. The relative motion between radar and target in FMCW SAR during reception and between transmission and reception will introduce serious dilation in the received signal. The dilation can cause serious distortions in the reconstructed images using conventional signal processing methods. The received signal is derived and the received signal in range-Doppler domain is given. The relation between the phase resulting from antenna motion and the azimuth frequency is analyzed. The modified frequency scaling algorithm is proposed to process the received signal with serious dilation. The algorithm can effectively eliminate the impact of the dilation. The algorithm performances are shown by the simulation results.

Keywords: FMCW radar; SAR; radar imaging; frequency scaling algorithm

1 Introduction

Frequency modulated continuous wave synthetic aperture radar (FMCW SAR) is a small, lightweight, cost-effective, and high-resolution imaging radar[1-5]. It is suited for small scale application such as observation of enemy lines with unmanned aerial vehicles (UAV). FMCW SAR transmits large time-bandwidth product signal, then it has the low probability of intercept (LPI) characteristic and can be used widely in military applications.

Compared with the real aperture radar, the most distinguished feature of SAR is the motion of the antenna. This motion can be further characterized into three categories[6]: motion occurring between successive transmitted pulses; motion occurring during transmission and reception of a pulse; motion occurring in the interval between transmission and reception of a pulse. In current pulsed SAR systems, since the pulse duration is very short, the last two categories of motion are ignored. However, in FMCW SAR systems, the motion during reception and between transmission and reception will induce serious dilation in the received signal. The dilation can cause serious distortions in the reconstructed images.

Dechirp on receive is often used in FMCW SAR to obtain very good range resolution. Some processing algorithms suitable for dechirp signal in SAR processor are polar format algorithm (PFA), range migration algorithm (RMA) and frequency scaling algorithm (FSA). The PFA[6] is attractive since it requires only two FFT’s, but two interpolations are needed in range and azimuth. Three FFT’s and the Stolt interpolation are required for the RMA[7]. The major advantage is the complete correction of the range cell migration (RCM). The disadvantage is the need of two-dimensional interpolation in the wavenumber domain. The FSA[8] corrects
the RCM by using a frequency scaling approach without interpolation. The problem is the azimuth data increasing extensively in large squint mode. Especially, all the above three algorithms are demonstrated on pulsed SAR signal model.

Basic signal model and signal processing flow were discussed in Ref.[9], but the RCM which is the key of imaging algorithms had not been considered. FMCW SAR image formation using RMA was discussed in Ref.[10]. Modified range-Doppler algorithm was used in FMCW SAR data processing[11]. This method is efficient, and, in principle, solves the problems of azimuth focusing and range cell migration correction (RCMC). One disadvantage is the need of interpolation in performing RCMC. Another disadvantage is the secondary range compression (SRC) cannot easily incorporate azimuth frequency dependence.

This paper derives the FMCW SAR signal model rigorously considering all kinds of antenna motions. The signal in range-Doppler domain is given. In range-Doppler domain, the extra phase introduced by the continuous antenna motion is the linear term of azimuth frequency. The modified frequency scaling algorithm compensates this phase by multiplying an appropriate operation factor in range-Doppler domain.

2 FMCW SAR Raw Signal

Amplitude has little effect in SAR signal processing. For simplicity, antenna gain, amplitude effects of propagation on the signal and any additional time delays due to atmospheric effects are ignored.

Let the transmitted signal of FMCW SAR be written in complex form

$$s_t(n,t) = C \exp \left( j2\pi \left( f_c t + \frac{1}{2} k e ( t - nT) \right) \right)$$  \hspace{1cm} (1)

where \( f_c \) is the center frequency, \( k_e \) is the chirp modulation rate, \( T \) is the sweep period, \( n \) is the sweep number, and \( C \) is a complex constant.

The received signal is a delayed version of the transmitted one, that is

$$s_r(n,t) = s_t(n,t - \tau(t))$$  \hspace{1cm} (2)

where \( \tau(t) \) is the roundtrip travel time. If \( R(t) \) is the time-dependent range between the receiver and the object at time \( t \), then the signal received at time \( t \) is reflected from the object at time \( t - \tau(t)/2 \). At that time, the object is at range \( R(t - \tau(t)/2) \). Thus, the roundtrip time delay is

$$\tau(t) = \frac{2}{c_0} R(t - \tau(t)/2)$$  \hspace{1cm} (3)

where \( c_0 \) is the velocity of light. From the geometry of FMCW SAR, the instant slant range is

$$R(t, r_0) = \sqrt{r_0^2 + V^2 (nT)^2}$$  \hspace{1cm} (4)

where \( V \) is the velocity of the platform, and \( r_0 \) is the closest distance between the antenna and the scatter. Substitute Eq.(4) into Eq.(3), the roundtrip time delay can be rewritten as

$$\tau(t) \approx \tau_0 + \frac{2V^2 (nT)}{c_0 r_0} (t - nT - \tau_0)$$  \hspace{1cm} (5)

where

$$\tau_0 = \frac{2}{c_0} \sqrt{r_0^2 + V^2 (nT)^2}$$  \hspace{1cm} (6)

then Eq.(2) can be rewritten as

$$s_r(n,t; r_0) = C \exp \left( j2\pi f_c \left( \mu(t - \tau_0) + \frac{2V^2 (nT)^2}{c_0 r_0} \right) \right) \cdot \exp \left( jk e \left( \mu(t - \tau_0) + \frac{2V^2 (nT)^2}{c_0 r_0} - nT \right) \right)$$  \hspace{1cm} (7)

where

$$\mu = 1 - \frac{2V^2 (nT)}{c_0 r_0}$$  \hspace{1cm} (8)

is the dilation introduced by the continuous antenna motion.

Dechirp on receive is often used in FMCW SAR. In order to decrease the sampling frequency, and then to decrease the data in range direction, the radar demodulates the receive signal by mixing it with a replica of the transmitted waveform delayed by a time \( \tau_{ref} = 2r_c/c_0 \), where \( r_c \) is the slant range distance of the swath center. The reference signal is

$$s_{ref}(n,t; r_0) = \exp \left( j2\pi f_c (t - \tau_{ref}) \right) \cdot \exp \left( jk e (t - \tau_{ref} - nT)^2 \right)$$  \hspace{1cm} (9)

The intermediate frequency signal resulting from mixing Eq.(9) with Eq.(7) is
\[ ss(n, t; r_0) \approx C \exp\left[-j2\pi f_c (r_0 - r_{\text{det}})\right] \cdot \exp\left[-j2\pi f_c 2\sqrt{\frac{\nu}{c_0}} (t - nT)\right] \cdot \exp\left[-j2\pi k_\nu (r_0 - r_{\text{det}}) (t - nT - r_{\text{det}})\right] \cdot \exp\left[j\pi k_\nu (r_0 - r_{\text{det}})^2\right] \exp(j2\pi k_\nu r_{\text{det}}^2) \]  

(10)

After neglecting the constant phase term and letting \( t_a = nT \), \( t_t = t - nT \) represent azimuth time and the range time respectively, Eq.(10) becomes

\[ ss(t_a, t_t; r_0) = C \exp\left(-\frac{4\pi}{\lambda} r_t \right) \exp\left(-2\frac{\nu}{\nu_0} t_a r_t \right) \cdot \exp\left[-\frac{4\pi k_\nu}{c_0} (r_t - r_\nu) \right] \exp\left[\frac{4\pi k_\nu}{c_0} (r_t - r_\nu)^2\right] \]  

(11)

where \( r_t(t_a; r_0) = \sqrt{r_0^2 + \nu^2 t_a^2} \) is the azimuth dependent distance to the scatter, and \( \lambda \) is the wavelength. In Eq.(11), the first exponential term is the azimuth phase history and the second term represents the RCM due to the Doppler frequency shift, which can be ignored in pulsed SAR but must be considered in FMCW SAR. The third term represents the range signal, which is a sinusoidal signal with a constant frequency value corresponding to the azimuth dependent distance to the point target. The last exponential term represents the residual video phase (RVP) term.

3 Modified Frequency Scaling Algorithm

3.1 Signal in range-Doppler domain

After a range Fourier transformation, the signal formulation in Eq.(11) can be expressed as follows

\[ sS(t_a, f_r; r_0) = \left\{ C \exp\left(-\frac{4\pi}{\lambda} r_t \right) \exp\left(-2\frac{\nu}{\nu_0} t_a r_t \right) \right\} \cdot \exp\left[-\frac{4\pi k_\nu}{c_0} (r_t - r_\nu) \right] \exp\left[\frac{4\pi k_\nu}{c_0} (r_t - r_\nu)^2\right] \cdot \]  

\[ T \cdot \exp\left[\pi T \left(f_r + \frac{2k_\nu}{\nu_0} (r_t - r_\nu) + \frac{2\nu^2 t_a}{\nu_0}\right)\right] \]  

(12)

where \( f_r \) is the range frequency, \( \sin(x) = \sin (x)/x \).

After introducing two exponential terms to aid in the following analysis, Eq.(12) becomes

\[ sS(t_a, f_r; r_0) = \left\{ C \exp\left(-\frac{4\pi}{\lambda} r_t \right) \exp\left(-2\frac{\nu}{\nu_0} t_a r_t \right) \right\} \cdot \exp\left[-\frac{4\pi k_\nu}{c_0} (r_t - r_\nu) \right] \exp\left[\frac{4\pi k_\nu}{c_0} (r_t - r_\nu)^2\right] \cdot \exp\left[-j\pi k_\nu r_t^2\right] \exp\left[j\pi f_r^2\right] \cdot \]  

(13)

After an inverse range Fourier transformation, the two-dimensional time domain signal of FMCW SAR can be approximated to be

\[ ss(t_a, t_t; r_0) = \left\{ C \exp\left(-\frac{4\pi}{\lambda} r_t \right) \exp\left(-2\frac{\nu}{\nu_0} t_a r_t \right) \right\} \cdot \exp\left[-\frac{4\pi k_\nu}{c_0} (r_t - r_\nu) \right] \exp\left[\frac{4\pi k_\nu}{c_0} (r_t - r_\nu)^2\right] \cdot \exp\left[j\pi k_\nu r_t^2\right] \exp\left[-j\pi f_r^2\right] \]  

(14)

where \( \otimes \) denotes convolution. The complete derivation is given in the Appendix. After applying the principle of stationary phase for the azimuth Fourier transformation, the signal in range-Doppler domain is obtained,

\[ Ss(f_a, t_t; r_0) = \left\{ C \exp\left(-\frac{4\pi r_t \beta}{\lambda} \right) \exp(j2\pi f_a t_t) \right\} \cdot \exp\left[-\frac{4\pi k_\nu}{c_0} (r_t - r_\nu) \right] \exp\left[\frac{4\pi k_\nu}{c_0} (r_t - r_\nu)^2\right] \cdot \exp\left[-j\pi k_\nu r_t^2\right] \]  

(15)

where \( f_a \) is the azimuth frequency, which varies within the following range

\[ \frac{f_a}{2} \leq f_a \leq f_a \leq \frac{f_a}{2} + f_{\text{DC}} \]  

(16)

where \( f_a \) is the sweep frequency of FMCW SAR, and \( f_{\text{DC}} \) is the Doppler frequency centroid. The factor \( \beta \) is dependent on the azimuth frequency, and is defined by

\[ \beta(f_a) = \sqrt{1 - \frac{f_a^2 \lambda^2}{4 V^2}} \]  

(17)

The secondary range compression term \( \text{src}(f_a, t_t; r_0) \) is defined by

\[ \text{src}(f_a, t_t; r_0) = \exp\left[-\frac{2\pi r_\nu k_\nu^2 \lambda^2 (\beta^2 - 1) t_t^2}{\beta^3 \nu_0^2} \right] \cdot \exp\left[\frac{2\pi r_\nu k_\nu^2 \lambda^3 (\beta^2 - 1)^3 t_t^3}{\beta^5 \nu_0^3} \right] \]  

(18)
3.2 Principle of RCMC

In Eq.(15), RCM can be expressed by

\[ \Delta RCM = \frac{r_0}{\beta(f_a)} - \frac{f_a c_0}{2k_r} - r_0 \]  

(19)

The RCM is dependent on not only target range \(r_0\), but also azimuth frequency \(f_a\).

RCMC is performed in three steps by using modified frequency scaling algorithm. First, the range time \(t_r\) is scaled by \(\beta\), which causes the RCM term in Eq.(15) becomes

\[ \exp \left[ -j \frac{4\pi k}{c_0} (r_0 - r_c) \beta t_r \right] \exp (j2\pi f_a \beta t_r) \]

In the second step, the RCM introduced by Doppler frequency shift is compensated. The RCM term in Eq.(15) after the step becomes

\[ \exp \left[ -j \frac{4\pi k}{c_0} (r_0 - r_c) \beta t_r \right] \]

Here, the RCM is no longer dependent on the point target range \(r_0\) but instead only on the reference range \(r_c\). This means that the RCM trajectories of all targets are equalized to the RCM trajectory of the target at the reference range. In the last step, the RCMC is completed by a bulk shift, which causes the following change in the RCM term in Eq.(15)

\[ \exp \left[ -j \frac{4\pi k}{c_0} (r_0 - r_c) t_r \right] \]

At the position, the frequency of the range signal is azimuth independent. Thus, the RCM is completely corrected.

3.3 Algorithm description

The block diagram of the modified frequency scaling algorithm for FMCW SAR data processing is shown in Fig.1.

After an azimuth Fourier transformation on FMCW SAR raw data, the RCM trajectories of all targets with the same range but different azimuths are equalized. But the targets with different ranges also have different RCM trajectories. After multiplying Eq.(15) with the following frequency scaling factor

\[ H_{FS} (f_a, t_r) = \exp \left[ j \pi k (1 - \beta) t_r^2 \right] \]  

(20)

all targets in swath have the same RCM trajectories, which are independent on the target ranges but dependent on the reference range. At this position, a small linear frequency modulation is introduced. The following factor is used to remove the RVP term in wavenumber domain

\[ H_{RVPC} (f_a, f_t) = \exp \left[ -j \frac{\pi f_a^2}{k \beta} \right] \]  

(21)

After multiplying the following inverse frequency scaling factor in range-Doppler domain

\[ H_{IFS} (f_a, t_r) = \exp \left[ -j \pi k \left( \beta^2 t_r^2 - \beta t_r^2 \right) \right] \]  

(22)

the small linear frequency modulation introduced during the frequency scaling is removed. After the frequency scaling processing described in Eqs.(20)-(22), the FMCW SAR signal in range-Doppler domain becomes

\[ S_{s} (f_a, t_r, r_0) = C \exp \left[ -j 4\pi f_0 \beta \right] \exp (j2\pi f_a \beta t_r) \cdot \exp \left[ -j \frac{4\pi k}{c_0} (r_0 - r_c) \beta t_r \right] \cdot \exp \left[ -j \frac{4\pi k}{c_0} (r_0 - r_c) \beta t_r \right] \]

(23)

Compared with the pulsed SAR, the RCM introduced by the continuous antenna motion in FMCW SAR cannot be ignored.
quency scaling algorithm introduces an extra phase factor in range-Doppler domain to correct the RCM accurately. The Doppler frequency correction factor \( H_{\text{DFC}}(f_s,t_t) \) is defined by

\[
H_{\text{DFC}}(f_s,t_t) = \exp(-j2\pi f_s \beta_t) \tag{24}
\]

Using the approximation \( r_0 \approx r_c \), the phase multiplication for secondary range compression \( H_{\text{SRC}}(f_s,t_t;r_c) \) is given by

\[
H_{\text{SRC}}(f_s,t_t;r_c) = \exp\left[ j \frac{2\pi k_c^2 \lambda}{c_0^2} \left( \frac{\beta t - 2r_c}{c_0} \right)^2 \right]. \tag{25}
\]

while the bulk range shift phase is defined by

\[
H_{\text{BS}}(f_s,t_t) = \exp\left[ j \frac{4\pi k_c}{c_0} \left( \frac{1}{\beta} - 1 \right) \left( \beta t - 2r_c \right) \right]. \tag{26}
\]

In fact, the RCM trajectories of all targets are equalized in Eq.(23). And Eqs.(24)-(26) can be performed at the same time. That is to say, the extra phase factor \( H_{\text{DFC}}(f_s,t_t) \) of the modified algorithm can not introduce any other extra computational load. After a range Fourier transformation, the following phase correction factor is multiplied to the signal for achieving phase preservation in the processing,

\[
H_{\text{PPC}}(f_s,t;f_t;r_c) = \exp\left( j \frac{4\pi f_t}{c_0} \frac{f_s}{\beta} \right). \tag{27}
\]

At this time, the range processing is completed. In strip mode FMCW SAR, the matched filter method is more efficient than the spectral analysis (SPECAN) method in azimuth compression\[^{12}\]. This is another difference between the modified algorithm and the basic algorithm.

After the range processing, the following factor is used to achieve the azimuth compression:

\[
H_{\text{AMF}}(f_s,t;f_t) = \exp\left( j \frac{4\pi f_t}{\lambda} \right). \tag{28}
\]

The target image is achieved by performing an inverse azimuth Fourier transformation and it can be expressed by

\[
sS(t_s,f_t;f_0) = C_{\text{si}} \left[ \pi \frac{T}{\beta} \left( f_t + \frac{2k_c}{c}(r_0 - r_c) \right) \right] \sin \left( \pi \frac{2V^2}{\lambda r_0^3} t_s \right) \tag{29}
\]

where \( T_s \) is the synthetic aperture time.

4 Simulation Results

The FMCW SAR system parameters used in the simulations are given in Table 1. The parameters are similar to the FMCW SAR demonstrator system developed by DELFT University of Technology of Netherlands described in Ref.[11].

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center frequency</td>
<td>35 GHz</td>
</tr>
<tr>
<td>Azimuth beamwidth</td>
<td>2°</td>
</tr>
<tr>
<td>Sweep period</td>
<td>1 ms</td>
</tr>
<tr>
<td>Reference range</td>
<td>600 m</td>
</tr>
<tr>
<td>Maximum slant range</td>
<td>800 m</td>
</tr>
<tr>
<td>Platform velocity</td>
<td>60 m/s</td>
</tr>
<tr>
<td>Frequency sweep</td>
<td>500 MHz</td>
</tr>
<tr>
<td>Sampling frequency</td>
<td>4 MHz</td>
</tr>
</tbody>
</table>

The target raw data are simulated using the system parameters listed in Table 1. The raw data are processed using the basic frequency scaling algorithm described in Ref.[8] and the modified algorithm proposed in the paper respectively. The two-dimensional contour plots of the impulse response function are shown in Fig.2.

Fig.3 shows the magnitude of range and the azimuth slice of processed image of a point target respectively using the basic frequency scaling algorithm. The corresponding results using the modified algorithm are shown in Fig.4. The modified algo-
The modified algorithm provides better focusing performance in both range and azimuth than the basic algorithm. The slice plots of impulse response function of a point target using the basic algorithm are shown in Fig. 3. The slice plots of impulse response function of a point target using the modified algorithm are shown in Fig. 4.

The point target analysis results for this impulse response function are shown in Table 2. The results of the simulation show that the range resolution of the modified algorithm is 4.94 percentage points better than that of the basic algorithm, and the azimuth resolution is 4.97 percentage points better. The performances of the peak sidelobe ratio (PSLR) and the integrated sidelobe ratio (ISLR) in azimuth are comparable in both algorithms. But the modified algorithm has much better performances of PSLR and ISLR in range.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Results of point target analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Basic algorithm</td>
</tr>
<tr>
<td></td>
<td>Azimuth (m)</td>
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<tr>
<td>Res. Calculated</td>
<td>0.122 8</td>
</tr>
<tr>
<td>Res. Measured</td>
<td>0.128 9</td>
</tr>
<tr>
<td>PSLR/dB</td>
<td>-12.654 8</td>
</tr>
</tbody>
</table>

5 Conclusions

In this paper, the raw data of FMCW SAR are modeled in detail. All kinds of antenna motion are considered. The continuous antenna motion introduces an extra exponential term in the raw data. The formulation for dechirped raw data in range-Doppler domain is deduced. The phase term introduced by the continuous antenna motion is linear in azimuth frequency. The modified frequency scaling algorithm adds an appropriate factor to compensate the phase term precisely. The extra compensation factor can not introduce extra computational consumption because of incorporating with other compensation factors in data processing. In the simulation, the accuracy and image quality of the modified algorithm are compared favorably with those of the basic algorithm. An improvement in range and azimuth resolution is measured.

References

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Appendix

By introducing the substitution

\[ F = f + \frac{2k_r}{c_0} (r_1 - r_c) + \frac{2V^2 t_k}{\lambda r_0} \]  

(A1)

into Eq.(13) and after an inverse Fourier transformation, Eq.(13) becomes

\[ ss(t_a, t_c, t_r; r_0) = C \exp \left( -j \frac{4\pi r_1}{\lambda} \right) \exp \left( -j2\pi \frac{2V^2 t_k}{\lambda r_0} t_r \right) \cdot \exp \left[ -j4\pi k_r \left( \frac{t_c - 2r_1}{c_0} \right) \right] \exp \left[ -j2\pi \frac{2(r_1 - r_c)}{c_0} \right] \cdot \exp \left[ -j\pi \frac{2V^2 t_k}{\lambda r_0} \right] \exp \left[ -j2\pi F \left( \frac{t_r - 2r_1}{c_0} + \frac{2V^2 t_k}{\lambda r_0 k_r} \right) \right] \exp \left( -jn k_r t_r^2 \right) \]  

(A2)

In Eq.(A2), \( 2V^2 t_k/(\lambda r_0) \) and \( 2(r_1 - r_c)/c_0 \) represent the azimuth instantaneous Doppler frequency and the delay time from a target to the scene center. For FMCW SAR, the Doppler frequency is often in the thousands Hertz and the delay time is often in the microsecond. The chirp rate is often in \( 10^{11} \) Hz/s.

At this instance, the fourth and fifth exponential terms in Eq.(A2) can be neglected and \( 2V^2 t_k/\lambda r_0 k_r \) in the third exponential term of the integral in Eq.(A2) is also approximate to zero. Then Eq.(A2) can be rewritten as

\[ F^2 \exp \left[ -j2\pi F \left( \frac{t_r - 2r_1}{c_0} + \frac{2V^2 t_k}{\lambda r_0 k_r} \right) \right] dF \otimes \exp \left( -jn k_r t_r^2 \right) \]  

(A3)

The remaining integral in Eq.(A3) can be solved by the following approximation\[^6\]. The integral has significant magnitude only for \( -1/T < F < 1/T \). The exponential function, quadratic in \( F \) can be neglected if

\[ \exp \left( -j \frac{\pi}{k_r T^2} \right) \approx 1 \]  

(A4)

where \( k_r T \) is the time-bandwidth product of the FMCW SAR range dimension, which is often in \( 10^5 \). By using the approximation and the Fourier pair between \( si \) function and gate function, the FMCW SAR two-dimensional time domain signal described by Eq.(14) is obtained.