Estimation of wind power density at a wind farm site located in Western Rajasthan region of India

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Abstract

Wind power density is an important parameter for the potential estimation of a wind farm and is used to compare different sites. This paper estimates wind power density of Soda site located at Western Rajasthan in India. Modified Maximum Likelihood, Empirical, and Graphical methods are used for estimating Weibull distribution parameters, which are used for calculating wind power density. The calculated wind power density is then compared with actual measured values for validation. The results obtained using Empirical method shows better agreement with the measured data of wind power density.

1. Introduction

Wind power density (WPD) of a site helps in the comparison and selection of best-suited sites for the wind turbines. Turbines that are installed at sites having higher WPD usually generate higher electric energy. WPD of Soda site located in the Western Rajasthan region of India is estimated in this paper. Weibull distribution parameters of Soda site are estimated using Modified Maximum Likelihood, Empirical, and Graphical methods, which are used for calculating the WPD.

This paper is organized as follows: Section 2 explains the statistical analysis of wind data, section 3 discusses the three different methods of estimating Weibull distribution parameters, section 4 explains the method of estimating WPD.
WPD from Weibull parameters of the site, section 5 shows the results of WPD estimations at Soda site in Jaisalmer district of Rajasthan, and section 6 draws the conclusions of analysis.

2. Wind data analysis using statistics

Wind speed probability distribution function gives the probability of occurrence for a particular wind speed at a site, depending on the location parameters [1]. Weibull probability distribution function \( f(v) \) is expressed as [2]:

\[
f(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} e^{-\left(\frac{v}{c}\right)^k}
\]

where \( v \) is the wind speed, \( k \) is the shape parameter, and \( c \) is the scale parameter.

3. Estimation of Weibull distribution parameters

Rocha et al. [3] explained seven numerical methods for estimating the scale and shape parameters of Weibull probability distribution function. This paper uses three methods for estimating the Weibull parameters.

3.1. Modified Maximum Likelihood (MML) method

It is an iterative method and uses wind speed frequency distribution of the site. Rocha et al. [3] explained that in this method, the Weibull distribution shape and scale parameters are calculated by using the expressions

\[
k = \left[ \frac{\sum_{i=1}^{n} v_i^k \ln(v_i) f(v_i)}{\sum_{i=1}^{n} v_i^k f(v_i)} - \frac{\sum_{i=1}^{n} \ln(v_i) f(v_i)}{f(v \geq 0)} \right]^{-1}
\]

\[
c = \left( \frac{1}{f(v \geq 0)} \sum_{i=1}^{n} v_i^k f(v_i) \right)^{1/k}
\]

where the number of bins is \( n \), frequency of wind speed occurrence in bin \( i \) is \( f(v_i) \), wind speed at \( i \)th bin midpoint is \( v_i \), and the probability of wind speed \( \geq 0 \) is \( f(v \geq 0) \).

3.2. Empirical method (EM)

Manwell et al. [1] and Rocha et al. [3] estimated the Weibull parameters from the expressions

\[
k = \left( \frac{\sigma}{\bar{v}} \right)^{-1.086}
\]

\[
c = \left( \frac{\bar{v}}{\Gamma(1+1/k)} \right)
\]

where \( \bar{v} \) is mean wind speed, \( \sigma \) is standard deviation of wind speed, and \( \Gamma \) is gamma function defined by integral
3.3. Graphical method (GM)

Johnson [4] explained that this method uses Weibull cumulative distribution function \( F(v) \) as

\[
F(v) = 1 - e^{-\left(\frac{v}{c}\right)^k}.
\]  

(7)

By rearranging and taking the logarithm of both sides of (7) two times, the expression transforms to

\[
\ln \ln 1 \ln \ln = k \ln (v) - k \ln (c).
\]  

(8)

and is of the form of straight line expressed as \( y = ax + b \) where

\[
y = \ln \ln 1 \ln \ln , \quad a = k, \quad x = \ln (v), \quad \text{and} \quad b = -k \ln (c).
\]  

(9)

The values of \( a \) and \( b \) that are expressed for \( n \) pairs of values \((x, y)\) where all summations are from 1 to \( n \), are

\[
a = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}
\]  

(10)

\[
b = \bar{y} - a \bar{x} = \frac{1}{n} \sum y - \frac{a}{n} \sum x.
\]  

(11)

Weibull distribution shape and scale parameters are then obtained from the expressions

\[
k = a
\]  

(12)

\[
c = e^{-\left(\frac{b}{k}\right)}.
\]  

(13)

4. Estimation of wind power density from Weibull distribution parameters

Jamil et al. [5], Chang et al. [6], and Jowder [7] showed that WPD of a site could be estimated from

\[
\text{WPD} = \frac{1}{2} \rho c^3 \Gamma(1 + 3/k)
\]  

(14)

where \( \rho \) is air density, \( c \) is scale parameter, \( k \) is shape parameter, and \( \Gamma \) is the gamma function defined by (6). The error between estimated and measured WPD is obtained by using the expression

\[
\text{Error} = \frac{\text{measured value} - \text{estimated value}}{\text{measured value}}.
\]  

(15)
5. Results and discussion

Wind frequency distribution data used for calculations are measured at Soda site located in the Jaisalmer district of Rajasthan, from April 2011 to March 2012 at 65 m height [8]. Air density is based on the measured temperature and pressure data. Weibull parameters of Soda site are estimated using MML, EM, and GM and used for estimating WPD. And finally the estimated and measured values of WPD are compared for validation.

5.1. Weibull distribution parameters and estimated Weibull wind speed probability at Soda site

MML method estimated monthly shape parameter varies from 1.7922 to 3.2595 and monthly scale parameter varies from 4.0250 to 10.1471 in the year. EM estimated monthly shape parameter varies from 1.8902 to 3.3229 and monthly scale parameter varies from 4.0376 to 10.1866 in the year. GM estimated monthly shape parameter varies from 1.5196 to 2.3472 and monthly scale parameter varies from 3.4343 to 9.7378 in the year. It was observed in all the three methods used for Weibull parameter estimations that the lowest value of shape parameter is in October and the highest is in May; and the lowest value of scale parameter is in November and the highest is in June.

Wind speed probability are calculated using Weibull parameters estimated from MML, EM, and GM for January to June months and are shown in Figs. 1(a) to 1(c) respectively. Similarly Figs. 2(a) to 2(c) shows the wind speed probability for July to December months.

Fig. 1. Estimated Weibull wind speed probability from January to June (a) MML; (b) Empirical method; (c) Graphical method.
5.2. Monthly wind power density at Soda site

Measured values of monthly air density at Soda site are shown in Fig. 3 [8]. WPD is estimated by using air
density and Weibull parameters in (14). Measured monthly WPD and estimated monthly WPD using Weibull parameters from MML, EM, and GM for Soda site at 65 m height are shown in Table 1. The results obtained using Empirical method shows better agreement with the measured data of WPD.

Percentage error between the measured WPD and estimated WPD using MML, Empirical, and Graphical methods are shown in Table 2. It is observed that the graphical method has highest percentage error.

Table 1. Measured and estimated monthly wind power density at Soda site.

<table>
<thead>
<tr>
<th>Months</th>
<th>Measured WPD (W/m²)</th>
<th>Estimated WPD (GM) (W/m²)</th>
<th>Estimated WPD (EM) (W/m²)</th>
<th>Estimated WPD (MML) (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apr</td>
<td>180.11</td>
<td>159.64</td>
<td>180.34</td>
<td>181.28</td>
</tr>
<tr>
<td>May</td>
<td>429.96</td>
<td>469.73</td>
<td>430.06</td>
<td>421.07</td>
</tr>
<tr>
<td>Jun</td>
<td>568.45</td>
<td>659.07</td>
<td>578.02</td>
<td>569.70</td>
</tr>
<tr>
<td>Jul</td>
<td>318.93</td>
<td>322.62</td>
<td>319.00</td>
<td>318.30</td>
</tr>
<tr>
<td>Aug</td>
<td>165.86</td>
<td>153.36</td>
<td>169.99</td>
<td>167.94</td>
</tr>
<tr>
<td>Sep</td>
<td>171.12</td>
<td>174.95</td>
<td>173.10</td>
<td>169.91</td>
</tr>
<tr>
<td>Oct</td>
<td>111.39</td>
<td>101.06</td>
<td>113.33</td>
<td>118.18</td>
</tr>
<tr>
<td>Nov</td>
<td>49.03</td>
<td>44.05</td>
<td>50.46</td>
<td>52.41</td>
</tr>
<tr>
<td>Dec</td>
<td>92.51</td>
<td>91.84</td>
<td>94.82</td>
<td>97.33</td>
</tr>
<tr>
<td>Jan</td>
<td>95.08</td>
<td>89.08</td>
<td>96.57</td>
<td>98.08</td>
</tr>
<tr>
<td>Feb</td>
<td>124.12</td>
<td>113.79</td>
<td>124.84</td>
<td>125.58</td>
</tr>
<tr>
<td>Mar</td>
<td>175.84</td>
<td>163.95</td>
<td>173.58</td>
<td>176.74</td>
</tr>
</tbody>
</table>

Table 2. Percentage error between the measured and estimated monthly wind power density.

<table>
<thead>
<tr>
<th>Months</th>
<th>Error in WPD (GM) (%)</th>
<th>Error in WPD (EM) (%)</th>
<th>Error in WPD (MML) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apr</td>
<td>11.37</td>
<td>-0.13</td>
<td>-0.65</td>
</tr>
<tr>
<td>May</td>
<td>-9.25</td>
<td>-0.02</td>
<td>2.07</td>
</tr>
<tr>
<td>Jun</td>
<td>-15.94</td>
<td>-1.68</td>
<td>-0.22</td>
</tr>
<tr>
<td>Jul</td>
<td>-1.16</td>
<td>-0.02</td>
<td>0.20</td>
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<tr>
<td>Aug</td>
<td>7.54</td>
<td>-2.49</td>
<td>-1.25</td>
</tr>
<tr>
<td>Sep</td>
<td>-2.24</td>
<td>-1.16</td>
<td>0.71</td>
</tr>
<tr>
<td>Oct</td>
<td>9.27</td>
<td>-1.74</td>
<td>-6.10</td>
</tr>
<tr>
<td>Nov</td>
<td>10.16</td>
<td>-2.92</td>
<td>-6.89</td>
</tr>
<tr>
<td>Dec</td>
<td>0.72</td>
<td>-2.50</td>
<td>-5.21</td>
</tr>
<tr>
<td>Jan</td>
<td>6.31</td>
<td>-1.57</td>
<td>-3.16</td>
</tr>
<tr>
<td>Feb</td>
<td>8.32</td>
<td>-0.58</td>
<td>-1.18</td>
</tr>
<tr>
<td>Mar</td>
<td>6.76</td>
<td>1.29</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

6. Conclusions

Wind farm site situated in Rajasthan is analyzed in this paper for its wind power density. Measured WPD is varying from 49.03 W/m² in the month of November 2011, to 568.45 W/m² in month of June 2011. It is also noted that the wind power density is more than 300 W/m² during May, June, and July months. Thus it can be concluded that maximum energy production from wind turbines will occur during these three months.

Weibull parameters of the site are estimated using Modified Maximum Likelihood, Empirical, and Graphical methods. In all the three methods, the lowest value of shape parameter is in October and the highest is in May,
whereas the lowest value of scale parameter is in November and the highest is in June. Estimated Weibull parameters are then used for calculating the WPD. The results obtained using Empirical method shows better agreement with the measured WPD; whereas the results obtained using Graphical method yields maximum percentage error.

Acknowledgements

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References