Fast evaluation of minimum zone form errors of freeform NURBS surfaces

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Abstract

Nowadays freeform components are more and more commonly employed in precision engineering and it is crucial to evaluate their form qualities. The widely adopted PV parameters for form errors need to be evaluated in the sense of minimum zone, which is a non-differentiable optimization problem, and very difficult to be solved. Currently NURBS has become a ubiquitous data format in computer aided design and manufacturing. However, little can be found concerning the minimum zone fitting between the NURBS surfaces and measured data, due to the mathematical complexity of this problem. A fast evaluation strategy is proposed in this paper. It is implemented in three stages. The NURBS model is decomposed into Bezier patches first and the iterative closest point matching is implemented. And then the relative position is refined by the orthogonal distance least squares fitting. Finally the minimum zone fitting is carried out by the primal-dual interior point method. The solutions are recursively updated by line search until the Karush-Kuhn-Tucker conditions are satisfied. Numerical experiments proved that the proposed method is capable of minimum zone fitting for freeform NURBS surfaces with very high accuracy and efficiency.

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Keywords: freeform surface; form error; NURBS; minimum zone; primal-dual interior point method

1. Introduction

With the rapid development of advanced design and manufacturing technologies, freeform components are more and more widely used in modern opto-mechanical systems, because of their compact sizes, small weights, flexibility in design/utilization and some attractive capabilities of system integration, biological compatibility, realizing various novel functions and remedying the drawbacks of traditional components. The form qualities of precision freeform components are critical for their functionalities, e.g. the transmission accuracy of gears, imaging aberrations of complex lenses, the vibration and efficiency of fan blades etc. The form error is evaluated by the relative deviation between the measured data and the nominal template. In the field of surface metrology the Peak-to-Valley (PV) value is one of the most widely adopted parameters for the assessment of form qualities, which is determined by the tolerance zone limited by two surfaces enveloping spheres of diameter \( t \) equal to the PV parameter. The centers of these spheres are situated on the nominal/fitted ideal surface [1]. Such a PV parameter is consistent with the qualities of mechanical contacts between components and the tolerancing specifications in computer-aided design.

![Fig. 1. Definition of PV form error of complex surfaces](image)

Based on the theory of numerical optimization, the expected optimal form error parameter should be consistent with the objective parameter. Hence the Chebyshev fitting, i.e. the \( l_\infty \) norm fitting should be carried out to obtain the PV form error. However, the Chebyshev fitting is not continuously differentiable, and it is very difficult to be solved. At present most researchers in mechanical engineering and commercial software in precision instruments implement least squares fitting.
fitting for complex surfaces due to its simplicity and ease of utilization, and the difference between the maximal and minimal fitted residuals are taken as the PV parameter. But the solutions calculated in this way conflicts with the definition of the form deviations in ISO 1101; as a consequence the PV values will be seriously over-estimated, which will lead to false rejection to qualified parts.

2. Related Work

Minimum zone fitting originated from the evaluation of straightness and flatness. It is a linear programming problem [2] and some classic Chebyshev fitting algorithms can be applied directly. Although the roundness and sphericity problems are nonlinear, they can be linearized by simple modification. For some other standard geometry, like cylinders and cones, the solution of their form errors is highly nonlinear and not so readily to be simplified. Various methods have been proposed to evaluate the minimum zone form errors of simple geometries, such as computational geometry methods [3], support vector machines [4], simplex methods [5] and so on. But all these techniques utilize some special properties of the assessed shapes, say, the convexity of the nominal shapes, but these conditions do not hold for complex shapes. Therefore these methods cannot be extended to freeform surfaces.

As for the general fitting of complex shapes, Goch and Lübke [6] converted the Chebyshev norm fitting into the \(l_p\) norm fitting with \(50 \leq p \leq 100\). The new merit function is equivalent to the Chebyshev norm fitting when \(p \rightarrow \infty\). But when \(p\) becomes larger, the observation matrix will become severely ill conditioning, and the solution will become very unstable. Al-Subaihi and Watson [7] approximated the original minimax optimization problem by recursive linearization, and the new optimization problem is solved using the conventional Gauss–Newton algorithm. Zhang et al [8] converts the minimum zone fitting problem by the exponential penalty function, and the new surrogate objection function can be guaranteed sufficiently close the original one under a given threshold as long as the exponential parameter \(p\) is large enough. All these approximation techniques suffer from drawback of slow convergence, especially when the solutions achieve the regions of the optima.

Some evolution-based heuristic optimizers have also been adopted, for example, genetic algorithms [9], immune evolutionary algorithm [10], particle swarm optimization [11], differential evolution [12] and so on. There methods initially generate some candidate solutions uniformly or randomly in the solution space. In each generation, these solutions are updated. The value of the merit function corresponding to each solution is used as the fitness to measure the degree of optimization. They are very powerful and can be applied for any complex problems. Convergence at the global optima can be guaranteed. But their computational complexity is unacceptably high and the optimization solutions are not deterministic.

In this paper a powerful minimum zone fitting method is presented for freeform surfaces with NURBS templates. The results possess fast convergence rate and great numerical stability even for very large data sets.

3. Minimum zone fitting of NURBS surfaces

Currently NURBS (Non-Uniform Rational B-Spline) is a standard format for computer aided design and manufacturing [13],

\[
S(u, v) = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,k}(u)N_{j,l}(v)w_{ij}p_{ij}
\]

In the equation, \(\{N_{i,k}(u)\kappa=1,2,\cdots,S\}\) and \(\{N_{j,l}(v)\|i=1,2,\cdots,T\}\) are the basis functions with respect to the footprint parameters \(u\) and \(v\), respectively, \(m\) and \(n\) are the degrees of the splines in the \(u\) and \(v\) directions, and \(\{w_{ij}\}\) are the control points specifying the position of the surface. While \(\{w_{ij}\}\) are weighting parameters used to measure the relative influence of each control point onto the NURBS surface. The foot-point parameters \(u\) and \(v\) are usually normalized into an interval \([0, 1]\).

NURBS is a very powerful representation and able to represent any complex shapes to a desired level of accuracy. For standard geometries, just several control points will be sufficient to describe them exactly. But a lot of control points will be needed to represent a complex freeform shape. In these cases, the densities and locations of the control points will be arranged carefully, instead of tuning the weighting factors. As a result all the weighting values are set to be equal; hereby the complex NURBS function becomes a simple B-spline surface.

In this paper the minimum zone fitting of NURBS surfaces is conducted in a nested manner,

\[
\min_{\mathbf{u}} \max_{\mathbf{v}} \min_{\mathbf{p}} \| \mathbf{p} - \mathbf{q}(\mathbf{u}) \|^2
\]

First solve the foot-point parameters \(\mathbf{u} = (u, v)\) associated with the orthogonal projection point \(\mathbf{q}\) on the template for each measured point \(\mathbf{p}\), with \(i=1, 2, \ldots, N\) at the inner iteration, and then update the six motion parameters \(\mathbf{m} = [\theta_{x}, \theta_{y}, \theta_{z}, t_{x}, t_{y}, t_{z}]^{T}\) at the outer iteration to minimize the largest distance between all the corresponding point pairs. This procedure is implemented recursively until the solution \(\mathbf{m}\) converges. It is proved that moving the measurement data is equivalent to moving the reference surface, and their evaluation results are the same for surface fitting. As a result transformations (rotation and translation) are always performed to the data for the sake of simplicity in programming.
3.1. Reliable point projection on NURBS surfaces

Due to the shape complexity, CAD models are usually modeled using multiple NURBS patches with continuity conditions enforced at the boundaries between adjacent patches. While within each NURBS patch we need to find the correct span associated with the foot-point parameters, because NURBS is a kind of 'spline', i.e. the surface representation differs at different parameter spans. As a result if the correct parameter scan is not supplied, the nearest projection point corresponding to each measured data point cannot be obtained directly by iteration, since the parameter increment and step length are very difficult to control and the solution will 'jump' between different patches.

The whole CAD model can be subdivided into a set of Bézier patches by knot insertion [14], so that the representation of \( S(u,v) \) within each Bézier patch becomes unique. A Bézier surface is a special B-spline surface with no interior knots, and then the convex hull property of the control polygon can be applied directly. The whole model can be regarded as a graph, with a node representing a patch and edges between nodes recording the neighboring relationships between these patches, as depicted in Fig. 2.

(a) NURBS surface  
(b) Bézier patches

Fig. 2. Representation of NURBS model

However, another problem arises: given an arbitrary measured point outside the NURBS surface, to find the correct Bézier patch where the projection point is located. Fortunately, this can be solved based on the control polygon.

For a Bézier surface \( S(u,v) \) of degree \((m,n)\) defined by \((m+1)(n+1)\) control points \( P_{i,j} \), \( k = 1,2,\ldots,m+1, l = 1,2,\ldots,n+1 \), if the query point \( p \) is closer to the corner point \( P_{i,j} \) than to all the other points on the surface, the following conditions should be satisfied [15],

\[
\langle P_{i,j} - p, p_{i,j} - p \rangle < 0, k = 1,\ldots,m+1, l = 1,\ldots,n+1 \tag{3}
\]

For the sake of clarity, only a Bézier curve with control points \( P_k \), \( k = 1,2,\ldots,m+1 \) is shown in Fig. 3 to demonstrate the inequality conditions. If the conditions in Equation (3) are met, the current patch is not the correct one, and the projection point will be located on the lower left direction. Otherwise the same testing procedure can be performed for the other three corner points \( P_{i+1,j} \) and \( P_{i,j+1} \).

If \( p \) projects onto the boundary \( S(u,0) \), the following conditions will be checked [16],

\[
\langle P_{i,j} - p, P_{j+1,j} - p \rangle < 0, j = 1,\ldots,m, i = 1,\ldots,n \tag{4}
\]

It suggests the projection point lies on the \(-v\) direction of the current patch. If it is not satisfied, the other three boundaries \( S(u,1) \), \( S(0,v) \), \( S(1,v) \) are all examined successively.

If none of the conditions in Equations (3)-(4) are satisfied, the projection point \( q \) can be thought located at this Bézier patch. The above searching process is repeated recursively until the correct Bézier patch of a projection point is determined. Then the foot-point parameters can be refined by the Newton algorithm [13],

\[
\begin{aligned}
S^T S S^T e & = 0, \\
S^T S S^T e & = S^T S S^T e + S^T S S^T e + S^T S S^T e \Delta u \approx -(e^T S S^T e)
\end{aligned}
\tag{5}
\]

with \( e = S - p \). \( S_{uv} = \partial S / \partial u \) , and \( S_{uv}, S_{uu}, S_{uv}, S_{vv} \) are defined similarly.

The convergence region of the Newton's method is not very large, and it requires the design matrix to be positive definite, and this needs particular attention when the query point \( p \) is far away from the surface. One effective technique is to add an appropriate damping term \( \lambda I_{2 \times 2} \) onto the design matrix to guarantee it is always positive definite.

3.2. Primal-dual interior point method

The outer iteration of Equation (2) is a non-differentiable minimax problem. It can be converted into a differentiable optimization problem with nonlinear constraints by using an auxiliary variable,

\[
\min_{x} D, \text{s.t.} \quad d_i \geq 0, i = 1,\ldots,N \tag{6}
\]

with \( d_i = D - \| p - q \| \) and \( x = [D,\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6] \).

The new constrained optimization problem can be solved using the primal-dual interior point method [17] via a sequence of barrier or interior point sub-problems,

\[
\min_{x} D - \mu \sum_{i=1}^{N} \ln s_i \tag{7}
\]

for decreasing values of the barrier parameter \( \mu \rightarrow 0 \). This method gets an approximate solution to the barrier problem in Equation (7) for a fixed value of \( \mu \), and continues for the next barrier problem with a decreased value of \( \mu \) from the solution of the last one.

For a Bézier surface \( S(u,v) \) of degree \((m,n)\) defined by \((m+1)(n+1)\) control points \( P_{i,j} \), \( k = 1,2,\ldots,m+1, l = 1,2,\ldots,n+1 \), if the query point \( p \) is closer to the corner point \( P_{i,j} \) than to all the other points on the surface, the following conditions should be satisfied [15],

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Its Lagrangian function with multipliers (also called dual variables) \( \{ \lambda_i \} \) is given as

\[
L(x, s, \lambda) = D - \mu \sum \phi (s_i) + \sum \lambda_i (d_i - s_i)
\]

A local minimum will be obtained when the first-order Karush-Kuhn-Tucker (KKT) conditions are satisfied,

\[
\begin{align*}
W & \frac{\partial L}{\partial x} + \sum \lambda_i \frac{\partial L}{\partial x_i} = 0 \\
B & \frac{\partial L}{\partial s} + \lambda = 0 \\
d - s & = 0 \\
s & > 0
\end{align*}
\]

Here \( g = \partial W / \partial x \) is the gradient, \( \partial d / \partial x \) the Jacobian matrix, \( s \in \mathbb{R}^n \) the slack parameters and \( t = [1/s_1, 1/s_2, \ldots, 1/s_n]^T \).

Equation (8) is solved by the Newton algorithm,

\[
\begin{bmatrix}
W & \gamma
\\
B & 0
\end{bmatrix}
\begin{bmatrix}
\delta x \\
\delta s
\end{bmatrix} = \begin{bmatrix}
\gamma + B^T \lambda \\
d - s
\end{bmatrix}
\]

with

\[
\begin{bmatrix}
W = \frac{\partial^2 L}{\partial x^2} + \sum \lambda_i \frac{\partial^2 L}{\partial x_i^2} \\
B = \left[ \partial d / \partial x \right] - \text{diag} \{ r \}
\end{bmatrix}
\]

\[
\gamma = \left[ \frac{\partial d}{\partial x} \right] - \mu r
\]

and \( \delta x = \frac{\partial x}{\partial x} \). Here \( r \) is a vector of ones.

As is common for Newton-like methods, the top-left block \( W \) in the design matrix has to be definite positive to guarantee the descending properties of the solution. Here the design matrix is modified as

\[
\begin{bmatrix}
W + \delta_x I & B^T \\
B & -\delta_s I
\end{bmatrix} \frac{\partial z}{\partial x} = \begin{bmatrix}
\gamma + B^T \lambda \\
d - s
\end{bmatrix}
\]

The damping coefficients \( \delta_x, \delta_s > 0 \) need to be selected carefully.

A line search filter strategy is applied to determine the optimal step length \( \alpha \in (0, 1] \) during the iteration.

\[
\begin{align*}
x^{(k+1)} &= x^{(k)} + \alpha \delta x \\
s^{(k+1)} &= s^{(k)} + \alpha \delta s \\
\lambda^{(k+1)} &= \lambda^{(k)} + \alpha \delta \lambda
\end{align*}
\]

The value of \( \alpha \) is decreased exponentially from 1 until the merit function value \( D \) improves to enforce progress toward the correct solution. As a result the program can obtain optimal results even when the starting point is poor. The solution is terminated when the conditions in Equation (8) are satisfied or the solution stagnates under a given threshold.

### 3.3. Remarks on implementation

It is worth noting that during the primal-dual interior point optimization, the correspondence pairs of the measured data points \( p_i \) and the projection points \( q_i \) should always keep closest to each other. That is to say, every time the variables \( x \) are changed, the foot-point parameters \( u_i \) associated with the projection points \( q_i \) need to be updated simultaneously. Most researchers omit the dependency between the foot-point parameters and the motion parameters due to the complexity of calculation. But this will seriously slow down the convergence rate of the optimization process, especially when the number of data points is large and the design matrix is numerically unstable. The derivation of the dependence \( \partial u / \partial m \) is presented below. For the sake of concision and clarity, the subscript \( i \) for \( p_i \) and \( q_i \) is omitted.

The point \( q \) is always the closest point to \( q \) on the NURBS surface, thus the derivatives with respect to the footpoint parameters are zero,

\[
\frac{\partial q}{\partial m} (q - p ) = 0
\]

So that

\[
\frac{\partial q}{\partial m} (q - p ) = 0
\]

In equation (2) with \( q \) replaced by \( q \),

\[
\frac{\partial q}{\partial m} (q - p ) = 0
\]

It is obtained straightforwardly

\[
\begin{align*}
\frac{\partial q}{\partial m} &= \frac{\partial q}{\partial u} \frac{\partial u}{\partial m} \\
\frac{\partial q}{\partial m} &= \frac{\partial q}{\partial u} \frac{\partial u}{\partial m}
\end{align*}
\]

Then the elements of the Jacobian matrix can be written explicitly as,

\[
\begin{align*}
\frac{\partial q}{\partial m} &= \frac{\partial q}{\partial u} \frac{\partial u}{\partial m} \\
\frac{\partial q}{\partial m} &= \frac{\partial q}{\partial u} \frac{\partial u}{\partial m}
\end{align*}
\]

For the sake of concision, denote

\[
A = \frac{\partial q}{\partial u} \frac{\partial u}{\partial m}
\]

The terms in the Hessian matrix,

\[
\frac{\partial^2 q}{\partial m^2} = \frac{\partial^2 q}{\partial u^2} \frac{\partial u}{\partial m} + \frac{\partial q}{\partial u} \frac{\partial^2 u}{\partial m^2}
\]

Ignoring the term \( \frac{\partial A}{\partial m} \), it is obtained as,
\[
\frac{\partial^2 d}{\partial \omega^2} = -2p^T_A p + 2e^T A \frac{\partial^2 \omega}{\partial \omega^2} p
\]

Then all the terms in Equation (9) can be calculated straightforwardly and the convergence process of the optimization program can be accelerated greatly.

The primal-dual interior point method employed here is a local optimization problem and a good initial guess needs to be provided. In practice, the whole NURBS model is subdivided into Bézier patches first. The iterative closest point (ICP) matching is conducted to find a rough relative position between the measured data set and the set of corner points of the Bézier patches [18], followed by the orthogonal distance least squares (ODLS) fitting using the Levenberg-Marquardt algorithm [19].

The implementing procedure is shown below,

**STEP 0**: INPUT data set \( P \), and NURBS surface \( S \).

**STEP 1**: Subdivide the NURBS surface into Bézier patches \( B \).

**STEP 2**: Find the rough relative position between \( P \) and \( B \).

**STEP 3**: Find the correct Bézier patches for the projection points.

**STEP 4**: Orthogonal distance least squares fitting to supply good initial solution.

**STEP 5**: Update projection points \( Q \) by the Newton’s method.

**STEP 6**: Minimum zone fitting by primal interior point method; Note \( P \) and \( Q \) are moved simultaneously.

**STEP 7**: IF the termination conditions are satisfied, STOP; ELSE go to **STEP 5**.

4. Numerical example

In the present paper all the programs were coded with MATLAB 2009b and run on a PC, with Intel(R) Core(TM) i5-2400 CPU@3.10GHz, 4 GB of RAM and 32 bit Windows 7 operation system. The open source IPOPT package [16] is adopted for the primal dual interior point optimization.

A NURBS model and a measured data set of 3136 points are shown in Fig. 4(a). The NURBS model is subdivided into Bézier patches. Rough matching by the iterative closest point method is conducted and the motion parameters are solved by the singular value decomposition method [20]. The matching residual is depicted in Fig. 5(a). Then the orthogonal least squares fitting is implemented, with the fitting residual shown in Fig. 5(b).

Then the fitting result of the ODLS is supplied as the initial guess of the primal dual interior point based minimum zone fitting. The fitted results are shown in Fig. 6. From Fig. 6(b) it can be seen the fitted deviation distributes symmetrically around the model surface, and clearly differs from the least squares fitting results in Fig. 5(b).

Concerning the distance between the transformed data points and the NURBS model, there are seven data points with their distances reaching the maximal value, 0.12082175 mm, i.e. these are the critical points determining the tolerance zone. It has to be stressed that this number seven is one more than the number of the real variables in the minimum zone fitting, i.e. three rotation angles and three translation components. Recalling that three extreme points are required to determine flatness, four points for flatness, four points for roundness etc, the numbers of extreme points are always one more than the number of the variables in the optimization programs. This implies the correct global optimum is obtained and the calculated form error depicts the real tolerance zone.

To quantitatively compare the two fitted results of ODLS and primal-dual interior point based minimum zone (PDIP MZ) method, the arithmetic average (AA), root-mean square (RMS) and peak-to-valley (PV) parameters of the two deviations are calculated. It needs to be clarified that the deviations are evaluated along the normal vectors of the nominal surface, and their signs are assigned according to the \( z \) components. In Table 1 it can be clearly seen that the PV parameter of the minimum zone fitting is much smaller than the least squares fitting, while the corresponding AA and RMS parameters are greater a little.
Table 1. Comparison of fitting results of ODLS and MZ (in mm)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>ODLS</th>
<th>DE MZ</th>
<th>PDIP MZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>0.0285</td>
<td>0.0425</td>
<td>0.0425</td>
</tr>
<tr>
<td>RMS</td>
<td>0.0367</td>
<td>0.0463</td>
<td>0.0463</td>
</tr>
<tr>
<td>PV</td>
<td>0.3207</td>
<td>0.2416</td>
<td>0.2416</td>
</tr>
</tbody>
</table>

This is straightforward to understand. The minimum zone fitting intends to minimize the maximal deviation between the data and nominal template, thus the resulting tolerance zone is optimized. As a result it is consistent with the situations when the functionalities are related to the width of deviations between two contacting surfaces, e.g. gears, cams, and so on. On the contrary, the least squares fitting intends to minimize the RMS deviation, thus it is more appropriate for the cases when the functionalities are more relevant to the average amplitude of form deviations, like optical aberrations, heat diffusion and so forth. The minimum zone fitting program of PDIP converges very quickly and only 1.6 seconds is taken.

For the purpose of comparison, a heuristic searching method, differential evolution (DE) is also employed to conduct the minimum zone fitting [21]. Before that, 66 ‘significant points’ with the greatest least squares fitted deviations are selected by the alpha hull method. The point number should be determined appropriately large to make sure that the actual vertex points defining the tolerance zone of the NURBS surface are among them. Only these significant points are utilized in the minimum zone fitting to accelerate the program. As proved in Table 1, both of the two minimum zone fitting methods obtain the same results, but the running time of differential evolution turns out to be 298 seconds, almost two hundred times slower than the PDIP method.

5. Conclusions

A minimum zone fitting method is presented in this paper for the form error evaluation of freeform components with respect to the NURBS templates. The non-differentiable minimax optimization problems are converted into differentiable optimization problems with constraints, and they can be solved using the primal-dual interior point algorithm. Numerical experiments proved that the fitted results can obtain the global optimum with a symmetric deviation map, and the tolerance zone is greatly reduced compared to the ordinary least squares fitting. As a result the fault rejection rate can be avoided by to a large extent when the peak-to-valley parameter is used as a measure of the form qualities of freeform components.

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References