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Gauge fields on tachyon matter

Akira Ishida, Shozo Uehara

Department of Physics, Nagoya University, Chikusa-ku, Nagoya 464-8602, Japan

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Abstract

We study the rolling tachyon including the gauge fields in boundary string field theory. We show that there are no plane-wave solutions for the gauge fields for large time. The disappearance of the plane-wave solutions indicates that there are no excitations of the gauge fields on the tachyon matter, which is consistent with the Sen's conjecture.

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Recently, Sen [1,2] considered the decay process of unstable D-branes in bosonic and superstring field theories and constructed the classical time dependent solutions which describe the rolling tachyon field toward the bottom of the tachyon potential. It was shown in [2] that the energy density remains constant and the pressure approaches zero as the tachyon field rolls toward the minimum. This pressure-less gas with nonzero energy density is called the tachyon matter. This phenomenon was analyzed by using the Born–Infeld type effective field theory. It was shown in [3] that there are no plane-wave solutions for the tachyon field around the minimum of the tachyon potential and the pressure falls off at late time. Cosmological considerations for the tachyon matter have been studied in many papers [4].

Another analysis was carried out by using the boundary string field theory (BSFT) [5–11], which gives some exact results such as the exact form of the tachyon potential and the exact solutions of the lower-dimensional brane [8,9]. So it is reasonable to consider the rolling tachyon in the BSFT framework. Time dependent solution describing the rolling tachyon was constructed in [12,13]. The solution asymptotically approaches $T = x^0$ and its behavior for large time describes a pressure-less gas with nonzero energy density. So, the solution for large time represents the tachyon matter. Since the tachyon matter is no longer a D-brane, it is important to examine whether the excitations of the gauge fields are absent or not at the bottom of the tachyon potential. The absence of the open string excitations corresponds to the absence of the plane-wave solutions in the effective field theory. In this Letter, we consider the equations of motion of the gauge fields at the bottom of the tachyon potential by using

E-mail addresses: ishida@eken.phys.nagoya-u.ac.jp (A. Ishida), uehara@eken.phys.nagoya-u.ac.jp (S. Uehara).

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BSFT and show the absence of the plane-wave solutions.

Let us consider the non-BPS Dp-brane in type II string theory. Here we restrict ourselves to almost spatially homogeneous time dependent tachyon field and small fluctuations of the gauge fields. The BSFT action with the tachyon and the gauge fields is given by[1] [9,14,15]

$$S = -T_p \int d^{p+1}x e^{-\frac{1}{4}T^2}\sqrt{-\det(\eta+F)} \times \mathcal{F}\big(G^{\mu\nu}\partial_\mu T \partial_\nu T\big), \tag{1}$$

where

$$\mathcal{F}(x) = \frac{x4^x\Gamma(x)^2}{2\Gamma(2x)} = 1 + (2\log 2)x + \cdots, \tag{2}$$

$$G^{\mu\nu} = \left(\frac{1}{\eta+F}\right)^{(\mu\nu)} = \eta^{\mu\nu} + \mathcal{O}\big(F^2\big) = G^{\nu\mu}, \tag{3}$$

$$\det(\eta+F) = \det(\eta_{\mu\nu}+F_{\mu\nu}), \tag{4}$$

and T_p is the tension of the unstable Dp-brane. Since we consider almost spatially homogeneous tachyon field and small fluctuation of the gauge field, $\partial_i T$ and $F_{\mu\nu}$ (or A_μ) are small, which we symbolically represent as $\mathcal{O}(\varphi)$, so that we have

$$\begin{aligned} &G^{\mu\nu}\partial_\mu T\partial_\nu T \\ &\quad = G^{00}\dot{T}^2 + \eta^{ij}\partial_i T\partial_j T + \mathcal{O}\big(\varphi^3\big) \\ &\quad = -\dot{T}^2 + F^{0i}F_{0i}\dot{T}^2 + \eta^{ij}\partial_i T\partial_j T + \mathcal{O}\big(\varphi^3\big), \end{aligned} \tag{5}$$

where $\dot{*} \equiv \partial_0 *$ and $i, j = 1, \ldots, p$.

Let us compute the Hamiltonian density. The conjugate momenta P and Π^μ of the tachyon T and the gauge field A_μ, respectively, are given by

$$\begin{aligned} P &= \frac{\delta S}{\delta \dot{T}} \\ &= -2T_p e^{-\frac{1}{4}T^2}\sqrt{-\det(\eta+F)}\,G^{0\mu}\partial_\mu T\mathcal{F}', \end{aligned} \tag{6}$$

$$\begin{aligned} \Pi^\mu &= \frac{\delta S}{\delta \dot{A}_\mu} \\ &= -T_p e^{-\frac{1}{4}T^2}\sqrt{-\det(\eta+F)} \\ &\quad \times \big[f^{\mu 0}\mathcal{F} - 2\big\{G^{\mu\lambda}f^{\rho 0} + G^{\rho 0}f^{\mu\lambda}\big\} \\ &\qquad \times \partial_\lambda T\partial_\rho T\mathcal{F}'\big], \end{aligned} \tag{7}$$

where

$$\begin{aligned} f^{\mu\nu} &= \left(\frac{1}{\eta+F}\right)^{[\mu\nu]} \\ &= -F^{\mu\nu} + \mathcal{O}\big(F^3\big) = -f^{\nu\mu}. \end{aligned} \tag{8}$$

Then the Hamiltonian density is

$$\begin{aligned} \mathcal{H} &= P\dot{T} + \Pi^\mu\dot{A}_\mu - \mathcal{L} \\ &= -T_p e^{-\frac{1}{4}T^2}\sqrt{-\det(\eta+F)} \\ &\quad \times \Big[\big(f^{\mu 0}\dot{A}_\mu - 1\big)\mathcal{F} + 2\big\{G^{0\nu}\partial_\nu T\dot{T} - \dot{A}_\mu\partial_\nu T\partial_\lambda T \\ &\qquad \times \big(G^{\mu\lambda}f^{\nu 0} + G^{\nu 0}f^{\mu\lambda}\big)\big\}\mathcal{F}'\Big]. \end{aligned} \tag{9}$$

We consider the behavior of the rolling tachyon T at $x^0 \to \infty$ as was discussed in [12,13]. Here, we set the initial conditions of $T = 0$ and $\dot{T} = +0$ for simplicity. It was shown that the tachyon, which is rolling down to the bottom of the tachyon potential, never stops [12]. Thus, T becomes infinity and hence $e^{-T^2/4} \to 0$ as $x^0 \to \infty$. Since $\mathcal{H}$ should be finite, the rolling tachyon must hit a singularity and $\mathcal{F}$ should become infinity as $x^0 \to \infty$.

$\mathcal{F}(z)$ and $\mathcal{F}'(z)$ have singularities at $z = -n$ ($n = 1, 2, \ldots$) and the nearest singular point from $z = 0$[2] is $z = -1$, and hence we require the following condition,

$$G^{\mu\nu}\partial_\mu T\partial_\nu T \to -1. \tag{10}$$

The asymptotic behavior of $\mathcal{F}(z)$ and $\mathcal{F}'(z)$ near $z = -1$ are as follows,

$$\mathcal{F}(z) \sim \frac{-1}{2(z+1)}, \qquad \mathcal{F}'(z) \sim \frac{1}{2(z+1)^2}. \tag{11}$$

Since $\mathcal{F}'(z)$ is more singular than $\mathcal{F}(z)$ at $z = -1$, $e^{-T^2/4}\mathcal{F}'$ should be finite as $x^0 \to \infty$ because of energy conservation. From Eq. (10), we write the asymptotic equation for $\dot{T}$ as (cf. Eq. (5))

$$\dot{T} \sim \frac{1}{\sqrt{-G^{00}}}\left(1 + \frac{1}{2}\eta^{ij}\partial_i T\partial_j T + \mathcal{O}\big(\varphi^3\big) + \epsilon(x)\right), \tag{12}$$

[1] We set $\alpha' = 2$ and rescale the gauge fields properly for simplicity. This action is exact as long as $\partial_\mu\partial_\nu T = 0$ and $\partial_\rho F^{\mu\nu} = 0$ and it is difficult to examine higher derivative corrections in the BSFT framework. Here we proceed to investigate with the action Eq. (1) and we will comment on the validity of our result later.

[2] Note that $z = 0$ corresponds to $\dot{T} = 0$.

where $\epsilon(x)$ is a small perturbation.[3] Note that we can see from Eq. (12) that the tachyon field becomes infinity as $x^0 \to \infty$. Furthermore, Eqs. (11) and (12) lead to the asymptotic form of $\mathcal{F}'$,

$$\mathcal{F}'\big(G^{\mu\nu}\partial_\mu T\partial_\nu T\big) \sim \frac{1}{8\epsilon(x)^2}\big(1+\mathcal{O}(\varphi^2)\big). \tag{13}$$

Due to the requirement that $e^{-T^2/4}\mathcal{F}'$ should be finite, we can determine $\epsilon(x)$ as

$$\epsilon(x) \sim -C(x)\exp\left(-\frac{1}{8}T^2\right), \quad C(x) = C + \mathcal{O}\big(\varphi^2\big), \tag{14}$$

where C is a constant to be determined by energy conservation. Thus, in the $x^0 \to \infty$ limit, we have

$$e^{-\frac{1}{4}T^2}\mathcal{F}\big(G^{\mu\nu}\partial_\mu T\partial_\nu T\big) \to 0, \quad e^{-\frac{1}{4}T^2}\mathcal{F}'\big(G^{\mu\nu}\partial_\mu T\partial_\nu T\big) \to \frac{1}{8C^2} + \mathcal{O}\big(\varphi^2\big). \tag{15}$$

Now we discuss the equations of motion of the gauge fields both at $x^0 = 0$ and $x^0 \to \infty$. They are derived from Eq. (1),

$$\partial_\nu\Big[e^{-\frac{1}{4}T^2}\sqrt{-\det(\eta+F)} \times \big\{f^{\mu\nu}\mathcal{F} - 2\big(G^{\mu\rho}f^{\lambda\nu} + G^{\nu\lambda}f^{\mu\rho}\big)\partial_\lambda T\partial_\rho T\mathcal{F}'\big\}\Big] = 0. \tag{16}$$

First, we consider the case where the tachyon is on the top of the potential. The equations of motion of the gauge fields at $x^0 = 0$ can be obtained by plugging the initial conditions, $T = 0$, $\dot{T} = +0$, into Eq. (16). Since we consider small fluctuations of the gauge fields, we ignore $\mathcal{O}(\varphi^2)$ terms and hence the second term in the brace does not contribute in this case. Using Eq. (8), we have the ordinary Maxwell equation,

$$\partial_\mu F^{\mu\nu} = 0. \tag{17}$$

Then it becomes, in the Coulomb gauge,[4]

$$\partial_\mu\partial^\mu A^i = 0. \tag{18}$$

Plugging a plane-wave solution,

$$A^i = a^i e^{ik_\mu x^\mu}, \tag{19}$$

into this equation, we get

$$k_\mu k^\mu = 0. \tag{20}$$

Therefore, there exist the plane-wave solutions at $x^0 = 0$.

Next, we consider the gauge fields at $x^0 \to \infty$. Similarly, we ignore $\mathcal{O}(\varphi^2)$ terms and hence, for example, only $\dot{T}^2$ in $\partial_\lambda T\partial_\rho T$ contributes. From Eqs. (8), (12) and (15), we obtain the equations of motion at $x^0 \to \infty$,

$$\partial_0 F^{0k} = 0, \qquad \partial_i F^{i0} = 0, \tag{21}$$

where $k = 1, \ldots, p$. Then, at $x^0 \to \infty$ we have

$$\partial_0\partial^0 A^k = 0. \tag{22}$$

Plugging (19) into Eqs. (22), we get

$$k^0 = 0. \tag{23}$$

Thus, contrary to the $x^0 = 0$ case, the plane-wave solution is absent for large time. From the above analysis, we conclude that the excitations of the gauge fields on a brane disappear as the tachyon field evolves toward the minimum of the potential, even though the energy density is conserved. This is consistent with the Sen's conjecture in which the brane will disappear at the minimum of the potential. One comment is in order: one may think that the result will be altered if the action (1) has higher derivative terms. However, such terms as can change Eq. (22) for a plane-wave equation will take the form of $f(\partial^2 T, \partial^3 T, \ldots)\cdot F^2$ where f is a function satisfying $f(0) = 0$, and since $\dot{T} \to 1 + \mathcal{O}(\varphi^2)$ and hence $f(\partial^2 T, \partial^3 T, \ldots) \to 0$ at $x_0 \to \infty$, we can expect that existence of those terms does not alter our result.[5]

Although we have focused on the decay process of an unstable D-brane, it will be interesting to extend our analysis to brane–antibrane systems to see pair annihilation processes of D-branes. It will be also interesting to consider the space dependent rolling tachyon in the decay process of a non-BPS brane into a lower-dimensional brane.

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[3] We have assumed that the BSFT action is valid for this form of T as is mentioned in the previous footnote. We note that a reliable result has been obtained in the similar situation [12].

[4] We can put $A^0 = 0$, as usual.

[5] $\partial F, \partial^2 F, \ldots$ terms in the action will not make the transverse modes of the gauge fields.

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