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Supply function auction for linear asymmetric oligopoly: equilibrium and convergence

Marina Dolmatova[a,] *, Alexander Vasin[a], Hongwei Gao[b]

[a]Lomonosov Moscow State University, Leninskiye Gory 1-52, Moscow 119991, Russia
[b]*College of Mathematics, Qingdao University, 308 Ningxia Road, Qingdao, 266071, P.R.China*

Abstract

We study the supply function auction for an asymmetric oligopoly with uncertain linear demand function and linear marginal cost functions of producers. We examine existence of a supply function equilibrium (SFE) in the model and convergence of the best response dynamics to this equilibrium. We show that the dynamics converges to the SFE for a duopoly, but in general the SFE and the strong best response do not exist in the model.

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1. Introduction

The present paper considers a model of the supply function auction under a linear and uncertain demand function and linear marginal cost functions of producers. The literature on supply function auctions usually studies symmetric oligopolies. A corresponding model was introduced by Klemperer and Meyer [9] and developed in later papers [1-5], [8], [10-12], [15]. In this model, a bid of a producer is a monotone smooth function of the price and a demand function depends on an uncertain parameter besides the price. For each parameter value, the market clearing price is determined from the balance of the aggregate supply function and the current demand function. A bid profile is called *a supply function equilibrium (SFE)* if, for any demand realization, the bid of each firm maximizes its profit under fixed bids of other producers. Klemperer and Meyer [9] derive a necessary condition for an equilibrium bid as a differential equation and describe the set of SFE. Green and Newbery [5] obtain an explicit formula of the SFE for a symmetric oligopoly with linear marginal cost and demand functions. Holmberg and Newbery [7] show that, under an inelastic demand and binding

*Corresponding author. Tel.: +7-495-9392491; fax: +7-495-9392596.
E-mail address: ms.marina.dolmatova@gmail.com

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capacity constraints, there exists a unique SFE in the model. Rudkevich [13] considers the supply function auction for an asymmetric oligopoly with linear marginal cost functions and zero fixed marginal costs. The paper establishes existence of the unique SFE with linear supply functions and convergence of the best response dynamics to this equilibrium. Anderson and Hu [2] show that an SFE with a jump of the supply function at price p may exist only if all the other producers with positive outputs reach their capacity constraints at this price and for one producer his fixed marginal cost coincides with it. Holmberg [6] studies the social welfare at the SFE depending on the number of producers and their asymmetry characteristic. Our paper studies the supply function auction with linear marginal cost functions under different fixed marginal costs of producers. We focus on the strong best response dynamics (SBRD) examination. We determine the price intervals where the SBRD exists and show its convergence to the SFE for each interval. We also establish existence of the SBRD and its convergence to the SFE without price constraints for a duopoly. However, if the permitted price interval covers the fixed marginal cost of producer $i > 2$ (ordered by the fixed marginal cost increase), the SBR of the first producer does not exist in the second period.

2. Oligopoly with linear marginal cost functions of producers

Consider supply function auction model introduced by [9], for an asymmetric oligopoly with n producers. Each producer i is characterized by cost function $C_i(q)$ of production volume q with standard properties $C'_i(q) > 0$, $C''_i(q) \geq 0$ for all $q \geq 0$, $i \in N = [1, \ldots, n]$. Demand function $D(p,t)$ depends on price p and random parameter t. The demand function meets the following conditions: $D_p < 0$, $D_{pp} \leq 0$, $D_{pt} = 0$, $D_t > 0$. Each participant $i \in N$ submits his strategy which is a smooth non-decreasing supply function $S_i(p)$. It determines the volume of electricity producer i proposes to supply depending on the market price p. The producers don't know the demand random parameter t when setting their strategies. Once parameter t becomes a common knowledge, a strategy profile $\vec{S} = (S_1(p), S_2(p),.., S_n(p))$ determines the price $p(\vec{S},t)$ balancing the demand and the aggregate supply function: $D(p(t)) = \sum_{i=1}^{n} S_i(p(t))$. If the market clearing price does not exist or is not unique, then no production takes place and the participants have zero profits. Producers aim to maximize their profit functions: $\pi_i(p,t) = p(D(p,t) - S_{-i}(p)) - C_i(D(p,t) - S_{-i}(p))$, where $S_{-i}(p) = \sum_{j \neq i} S_j(p)$ is a total supply function of all players except i. A strategy profile $\vec{S}^* = (S_i^*, i \in N)$ is a *supply function equilibrium (SFE)*, if for all $t \geq 0$ and $i \in N$, $S_i^* \in Argmax_{S_i}(\pi_i(S_i, S_{-i}^*, t))$. First order condition for profit maximization is

$$D(p,t) - S_{-i}(p) + (D'_p(p,t) - S_{-i}'(p))(p - C_i'(D(p,t) - S_{-i}(p))) = 0.$$

For smooth supply functions, Anderson and Hu [2] obtain a necessary condition for a strategy profile to be a supply function equilibrium as a system of differential equations:

$$S'_{-i}(p) = \frac{S_i(p)}{p - C'_i(S_i(p))} + D'_p(p,t), \ i = 1,..,n. \tag{1}$$

Consider a model with linear marginal cost functions of producers $C'_i(q) = c_i^0 + c_i^1 q, \ i \in N = [1, \ldots, n]$, and linear demand function $D(p,t) = \overline{D}(t) - dp$. For this case, the system (1) is:

$$S'_{-i}(p) = \frac{S_i(p)}{p - c_i^0 - c_i^1 S_i(p)} - d, \; i = 1,..,n \tag{2}$$

Since there are no capacity constraints, system (2) provides the necessary condition for SFE in the area where functions $S_i(p)$, $i = 1,..,n$, are smooth.

3. Strong best response dynamics

Strategy $S_a(p)$ is a *strong best response* to competitors' strategies $S_{-a}(p)$, if for each $t \in [\underline{t}, \bar{t}]$ the price $p(t)$, balancing the demand $D(p,t)$ and aggregate supply $S_a(p) + S_{-a}(p)$, provides maximum of the producer's profit:

$$p(S_a, S_{-a}, t) \to \max_p \{(D(p,t) - S_{-a}(p))p - C(D(p(t) - S_{-a}(p)))\}.$$

We study the best response dynamics for the repeated auction with $n \geq 2$ producers. In each period $\tau = 1,2,...$ producer a sets his bid $S_a(p,\tau)$ as a strong best response to the competitors' bids $\vec{S}_{-a}(p,\tau-1)$ in the previous period.

For the case where $c_i^0 = c^0 = 0$ for all $i \in N$, Rudkevich [13] shows, that the best response dynamics converges to the unique SFE of the static model. Consider the case where the fixed components of producers' marginal cost functions differ. Without loss of generality, assume that producers $i \in N$ are ordered by the increase of their fixed marginal costs: $c_i^0 : c_1^0 < c_2^0 < \ldots < c_n^0$. We study the best response dynamics in the separate price intervals (c_i^0, c_{i+1}^0), since, as it is shown below, the SBRD for the whole domain does not exist already in the second period.

Let the starting bid be $S_a(p,0) = 0$, for all $a \in N$. The SBR of a producer in the first period is

$$S_a(p,1) = \begin{cases} \dfrac{d(p - c_a^0)}{1 + dc_a^1}, & \text{for } p \geq c_a^0, \\ 0, & \text{for } p \leq c_a^0. \end{cases}$$

The BRD is special for each of the price intervals $p \in (c_i^0, c_{i+1}^0), i \in N$, since passing to the next price interval changes the set of players for whom bidding in the auction is profitable (possible market clearing prices are greater or equal to their fixed marginal costs). Let producer's a bid in the interval i for the period τ be of the following form: $S_a(p,\tau) = k_a(\tau,i)(p - c_a^0), \; a \in N_i, \; p \in (c_i, c_{i+1})$, where $N_i = \{a \mid a \leq i\}$ is a set of active players in this interval. For $a \in N \setminus N_i$, $S_a(p,\tau) = 0$. Producer's a strong best response to the rivals' strategies $S_j(p,\tau), \; j \in N_i \setminus a$ in the interval i maximizes his profit: $p_i^a(S_a(\tau), S_{N_i \setminus a}(\tau-1), t) = \underset{c_i \leq p \leq c_{i+1}}{\arg max}\{p(D(p,t) - S_{N_i \setminus a}(p,\tau-1)) - C_a(D(p,t) - S_{N_i \setminus a}(p,\tau-1))\}$.

The bid $S_a(p,\tau)$ provides an optimal price for any value of parameter t such that the price value stays within the interval i.

Proposition 1 *Bid* $S_a(p) = (p - c_a^0)\dfrac{d + k_{N_i \backslash a}(i)}{1 + c_a^1(d + k_{N_i \backslash a}(i))}$, *where* $k_{N_i \backslash a}(i) = \sum_1^{N_i \backslash a} k_j(i)$, *of producer* a *is a unique strong best response to the competitors' bids* $S_j(p) = k_j(i)(p - c_j^0),\ j \in N_i \backslash a$ *in the interval* $p \in (c_i^0, c_{i+1}^0), i \in N$.

Proof: According to [13], the first order condition for profit maximization gives: $S_a(p,\tau) = (p - c_a^0)\dfrac{d + k_{N_i \backslash a}(\tau - 1, i)}{1 + c_a^1(d + k_{N_i \backslash a}(\tau - 1, i))}$. Thus, the SBRD in the interval i looks like $S_a(p,\tau) = k_a(\tau, i)(p - c_a^0),\ a \in N_i$, where

$$k_a(\tau, i) = \left(\frac{1}{d + k_{N_i \backslash a}(\tau - 1, i)} + c_a^1\right)^{-1},\ a \in N_i \tag{3}$$

The vector form of Equ. (3) is $\vec{k}(\tau + 1, i) = \vec{\Phi}(\vec{k}(\tau, i))$. Proceeding from [13], Lemma 2, we obtain the following result on convergence of the SBRD in each interval i, $i = 1,\ldots,n$ (assuming $c_{n+1}^0 = \infty$). Consider system (4) for a fixed point of the dynamics (3):

$$\vec{k}(i) = \vec{\Phi}(\vec{k}(i)) \tag{4}$$

Proposition 2 *For every* i, *the system (4) has a unique solution* $\overrightarrow{k^*}(i) > 0$. *For any initial point (3)* $\vec{k}(0, i) > 0$, *the sequence* $\vec{k}(\tau, i)$ *converges to* $\overrightarrow{k^*}(i)$.

This result implies that in each period there exists a strong best response in each interval i, and the sequence of BR profiles converges to the SFE for $\tau \to \infty$. However, the combination of all the BR functions for all the intervals in the single bid is not the best response to the competitors' bids. Examine $n \geq 3$ and the second period of the dynamics. Fig.1 shows the total supply of producers $2,\ldots,n$ and the residual demand for the producer 1.

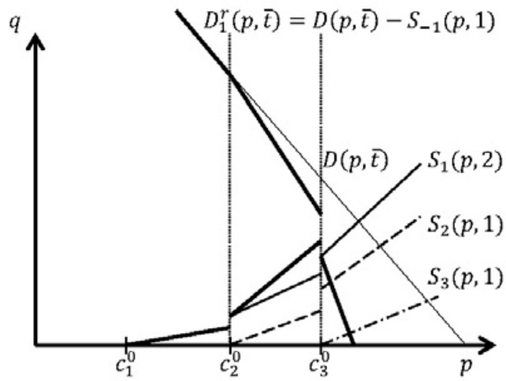

Fig. 1. The best response supply functions and the residual demand function for the first player at $\tau = 2$

Assume that c_a^1, $a = 1,\ldots,n$ are small. Denote $\bar{t}$ the value corresponding to the price $c_3^0 + \varepsilon$. Consider the residual demand faced by the first producer for $\tau = 2$, $t = \bar{t}$ (Fig. 1). Now we show that $S_1(p,2)$ is not the BR for $S_{-1}(p,1)$ under $t = \bar{t}$. The increase of $S_1(p,2)$ up to the value $D(c_3^0) - S_{2-}(c_3^0,1)$ in the neighbourhood of the point c_3^0 shifts the market clearing price to the left semi-neighbourhood of the point c_3^0. Under almost the same market price equal to c_3^0, the jump of the sales volume provides the profit growth.

Therefore, already in the second period the SBR does not exist under general assumptions. This result conforms to the proposition in [2] implying absence of SFE in the considered model. The only exception is the case of duopoly. Then the residual demand function stays continuous and concave for $p > c_i^0$ for all t and τ for the both agents.

Proposition 3 *Let* $N = \{1,2,3\}$. $S_i(p,0) = \begin{cases} 0 & for \quad p < c_i^0, \\ k_i(0)(p - c_i^0) & for \quad p \ge c_i^0, \end{cases}$ *, where* $k_i(0) > 0$ *,* $i = \{1,2\}$. *Then, for any* $\tau = 1,2,\ldots$ *, there exist SBR bids for the both producers: in the interval* $p \in (c_1^0, c_2^0)$ $S_1(p,\tau) = \dfrac{d}{1 + dc_1^1}(p - c_1^0)$, $S_2(p,\tau) = 0$, *for* $p > c_2^0$ $S_i(p,\tau) = k_i(\tau)(p - c_i^0)$, *where*

$k_i(\tau) = \left(\dfrac{1}{d + k_j(\tau - 1)} + c_i^1 \right)^{-1} := F_i(\vec{k}(\tau - 1))$, $i = \{1,2\}, i \ne j$. *The unique static point of the mapping* $\vec{k}(\tau) = \vec{F}(\vec{k}(\tau - 1)$ *is*

$$k_1^* = \frac{-2c_1^1 d - c_1^1 c_2^1 d^2 + \sqrt{d}\sqrt{2 + c_2^1 d}\sqrt{4c_1^1 + 4c_2^1 + 2{c_1^1}^2 d + 4c_1^1 c_2^1 d + {c_1^1}^2 c_2^1 d^2}}{2(c_1^1 + c_2^1 + c_1^1 c_2^1 d)} \tag{5}$$

$$k_2^* = (\frac{k_1^*}{1 - k_1^* c_1^1} - d) \tag{6}$$

and the SBRD $\{S_1(p,\tau), S_2(p,\tau)\}_{\tau=0}^{\infty}$ *converges to the pair of bids:*

$$S_1^*(p) = \begin{cases} k_1^*(p - c_1^0), & for \ p \ge c_2^0, \\ \dfrac{d}{1 + dc_1^1}(p - c_1^0), & for \ c_1^0 \le p \le c_2^0, \\ 0, & for \ p \le c_1^0. \end{cases} \quad S_2^*(p) = \begin{cases} k_2^*(p - c_2^0), & for \ p \ge c_2^0, \\ 0, & for \ p \le c_2^0, \end{cases}$$

corresponding to the SFE.

Proceeding from Lemma 5 in [2] on uniqueness of the SFE under an unbounded demand random component, we may conclude that this equilibrium is unique.

In order to complete our study of linear oligopoly, we should consider the case when $n \ge 3$ and fixed costs are the same for several producers. If there are at least 3 different values of c_i^0, then the situation is similar to the case on Fig. 1 for any price interval including some $c_i^0 > c_2^0$, the SBR for $\tau = 2$ and SFE do not exist.

Moreover, if there are several producers with the minimal fixed marginal cost $c_1^0 = c_2^0 = \ldots = c_m^0$, then the same proposition is true for any price interval including the next value c_{m+1}^0. The next proposition considers the only case where the SBRD exist.

Proposition 4 *Let* $c_1^0 < c_2^0 = c_3^0 = \ldots = c_n^0$, $S_i(p,0) = \begin{cases} 0 & for \quad p < c_i^0, \\ k_i(0)(p - c_i^0) & for \quad p \ge c_i^0, \end{cases}$ *, where* $k_i(0) > 0$ *,* $i \in N$*. Then, for any* $\tau = 1,2,\ldots$ *, there exist SBR bids determined as follows: in the interval* $p \in (c_1^0, c_2^0)$ $S_1(p,\tau) = \dfrac{d}{1 + dc_1^1}(p - c_1^0)$*,* $S_{-1}(p,\tau) = 0$*, for* $p > c_2^0$ $S_i(p,\tau) = \bar{k}_i(\tau)(p - c_i^0)$*,*

where $\bar{k}_i(\tau) = \left(\dfrac{1}{d + \bar{k}_{-i}(\tau - 1)} + c_i^1 \right)^{-1} := F_i(\vec{\bar{k}}(\tau - 1))$*,* $i \in N$.

The mapping $\vec{\bar{k}}(\tau) = \vec{F}(\vec{\bar{k}}(\tau - 1))$ *has a unique static point and the SBRD converges to this static point.*

4. Conclusion

Our study shows that properties of the supply function auction for the oligopoly with linear demand and marginal cost functions essentially depend on the fixed marginal costs of the producers. If these costs are the same for all producers (may be, except for one with the smallest cost), then the SBRD for such an auction converges to the SFE with linear supply functions. Proceeding from [13], the rate of convergence is rather high. However, in general the SBR exists in the price intervals limited by different marginal costs, but does not exist if there are no price constraints. This result confirms our conclusion in the previous paper [14] about rather limited possibilities for efficient implementation of the supply function auction under uncertain demand in practice.

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