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Stochastic Contaminant Transport Equation in Porous Media

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Abstract—Stochastic equations for the prediction of contaminant migration in porous media are considered by the use of the decomposition method. The results are easily generalized to the nonlinear case as well. Important applications of significance in the environmental sciences and engineering are beginning to appear in the literature, such as the forecasting of contaminant plume evolution in natural soils and aquifers after chemical spills, aquifer restoration, and groundwater pollution management.

Keywords—Decomposition, Groundwater pollution, Mathematical models.

DISCUSSION

A general analytic approach is necessary for the prediction of time and space evolution of contaminant migration in porous media without the usual restrictive assumptions (linearization, perturbation, and limitations on the stochastic processes).

The paper will consider equations of the form:

$$\frac{\partial u}{\partial t}(x,t,\omega) + A(x,t,\omega)u = g(x,t,\omega), \qquad (x,t,\omega) \in G \times [0,T] \times \Omega,$$

where g is a second-order stochastic process, G is subset of $\Re^3$ with boundary ∂G on $(0 < T < \infty)$, x represents three-dimensional space, A is an m^{th} order random partial differential operator.

The general three-dimensional stochastic advective-dispersive equation in porous media, given by Serrano [1], is

$$\frac{\partial u}{\partial t} + \Lambda u = g, \tag{1}$$

where $u = u(x,t,\omega)$, $g = g(x,t,\omega)$, and Λ is an m^{th} order random partial differential operator. We write (1) as

$$Lu = g - Au.$$

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Using decomposition [2], we define L^{-1} as the definite integral from zero to t, with decomposition of u into $\sum_{n=0}^{\infty} u_n$ and identification of u_0 as $u(0)$; we have

$$\begin{aligned} L^{-1}Lu &= L^{-1}g - L^{-1}Au, \\ u - u(t=0) &= L^{-1}g - L^{-1}Au, \\ u &= u(t=0) + L^{-1}g - L^{-1}Au. \end{aligned}$$

Define $u_0 = u(t=0) + L^{-1}g$ and $u = \sum_{n=0}^{\infty} u_n$,

$$\begin{aligned} u &= u_0 - L^{-1}\Lambda \sum_{n=0}^{\infty} u_n, \\ u_1 &= -L^{-1}\Lambda u_0, \\ u_2 &= -L^{-1}\Lambda u_1 = (-L^{-1}\Lambda)^2 u_0, \\ &\vdots \\ u_n &= -L^{-1}\Lambda u_{n-1}. \end{aligned}$$

We can then write the approximant

$$\phi_m[u] = u_0 - \sum_{n=0}^{m-2} L^{-1}\Lambda u_n = \sum_{n=0}^{m-1} (-L^{-1}\Lambda)^n u_0.$$

EXAMPLE

$$\frac{\partial u}{\partial t} = g - \Lambda u,$$

where

$$\begin{aligned} \Lambda &= -D\frac{\partial^2}{\partial x^2} + v\frac{\partial}{\partial x}, \\ u &= u(t=0) + L^{-1}g, \\ u_1 &= -L^{-1}\left[-D\frac{\partial^2}{\partial x^2} + v\frac{\partial}{\partial x}\right] u_0 = L^{-1}D\frac{\partial^2}{\partial x^2}u_0 - L^{-1}v\frac{\partial}{\partial x}u_0, \\ &\vdots \end{aligned}$$

$\langle u \rangle$ is found from $\langle \phi_m[u] \rangle$:

$$\langle u_0 \rangle = \langle u(t=0) \rangle + L^{-1}\langle g \rangle \quad \text{or} \quad u(t=0) + L^{-1}\langle g \rangle,$$

depending on the given initial conditions

$$\langle u_1 \rangle = L^{-1}D\frac{\partial^2}{\partial x^2}\langle u_0 \rangle - L^{-1}v\frac{\partial}{\partial x}\langle u_0 \rangle,$$

or even

$$\begin{aligned} \langle u_1 \rangle &= L^{-1}\langle D \rangle \frac{\partial^2}{\partial x^2}\langle u_0 \rangle - L^{-1}\langle v \rangle \frac{\partial}{\partial x}\langle u_0 \rangle \\ &\vdots \end{aligned}$$

if D, v are stochastic. g could be a general stochastic process. First- and second-order statistics can be found easily. Because of the fast convergence of the series, it is usually accurate to obtain

$\phi_4[t]$ and $\phi_4[t']$ and average to derive the correlation function (the errors are discussed in [3]). The results are easily generalized to three dimensions and, by assuming Gaussian behavior, to higher moments. Finally by using the Adomian polynomials [2], the results can be extended to nonlinear equations as well.

With decomposition the usual restrictions, on which today's small perturbation transport theory is based, are not needed [4]. New foundations for a physically based theory of dispersion (i.e., one where the natural hydrology may be considered) are beginning to appear in the literature, along with promising results that reproduce the phenomenon of scale dependency in the transport parameters [5–9].

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