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Quantitative techniques for financial risk assessment: a comparative approach using different risk measures and estimation methods

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Abstract

The aim of this paper is to highlight and illustrate the use of some quantitative techniques for risk estimation in finance and insurance. The first component involved in risk assessment concerns the risk measure used and the second one is based on the estimation technique. We will study the theoretical properties, the accuracy of modeling the economic phenomena and the computational performances of the risk measures Value-at-Risk, Conditional Tail Expectation, Conditional Value-at-Risk and Limited Value-at-Risk in the case of logistic distribution. We also investigate the most important statistical estimation methods for risk measure evaluation and we will compare their theoretical and empirical behavior. The quality of the risk estimation process corresponding to the quantitative techniques discussed will be tested for both real and simulated data. Numerical results will be provided.

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Keywords: Risk estimation; Value-at-Risk, Conditional Tail Expectation, Conditional Value-at-Risk; Limited Value-at-Risk; Logistic distribution;

1. Introduction

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The increasing complexity of the problems arising in various fields determined a strong demand for efficient methods of decision making. At the same time, the progress of information technology determined the development of advanced computational techniques to implement such methods. Risk assessment provides the theoretical basis for decision making processes in finance and insurance. Risk management has received a considerable interest among researchers in the last years. An important problem for portfolio managers, investors and financial regulators, refers to risk modeling and estimation. In his paper, Markowitz, 1959 underlined that investors should take into account not only the expected return, but also the variance of the return and to choose the portfolio with the highest expected return for a given level of the variance. Generally, the mean–variance analysis is applied when the returns are assumed to be normally distributed or when the investor's preferences can be accurately described using the mean and the variance. Recently, (Fulga and Dedu, 2010, Dedu and Fulga, 2011, Tudor, 2012, Toma, 2012, Toma and Leoni-Aubin, 2013), proposed new risk measures and optimization techniques to addressing various limitations of the mean–variance approach. In this paper we study some quantitative techniques for risk modeling and estimation in finance and insurance. In Section 2, we study theoretical properties, the accuracy of modeling the economic phenomena and the computational performance of different risk measures, such as: Value-at-Risk, Conditional Tail Expectation, Conditional Value-at-Risk and Limited Value-at-Risk. In Section 3, the most important Value-at-Risk estimation techniques are presented. In Section 4, we derive analytical formulas for Value-at-Risk and Conditional Value-at-Risk in the case when the loss random variable follows a Logistic distribution. Section 5 provides computational results of the case study. Section 6 presents the conclusions.

2. Risk measures used in the finance and insurance

2.1. Value-at-Risk

In the class of quantile-based risk measures, the most used is Value-at-Risk, which evaluates the maximal loss that can occur in a time horizon with a given probability level. Let $X : \Omega \to \mathbf{R}$ a random variable defined on the probability space (Ω, K, P), with cumulative distribution function $\mathrm{F}_X(x) = \mathrm{P}(X \le x)$, $\forall\, x \in \mathbf{R}$.

Let $\alpha \in (0, 1)$. The Value-at-Risk corresponding to a random variable X at the probability level α is given by:

$$\mathrm{VaR}_{\alpha}(X) = \inf\{x \in \mathbf{R} \mid \mathrm{P}(X \le x) \ge \alpha\}.$$

If the random variable X has a continuous one-to-one cumulative distribution function, then $\mathrm{VaR}_\alpha(X)$ can be computed as the unique solution of the equation:

$$\mathrm{VaR}_{\alpha}(X) = \mathrm{F}_X^{-1}(\alpha). \qquad (1)$$

Value-at-Risk is used for setting the capital adequacy limits for banks and other financial institutions and plays an important role in investment, risk management and regulatory control of financial institutions.

2.2. Conditional Tail Expectation

The limitations of the most common used risk measures (like variance, which is a symmetric measure, or Value-at-Risk, which does not take into account the extreme values) and the criticism addressed to some of these measures (Artzner, 1999), led to the development of new risk measures, with good analytical properties.

The Conditional Tail Expectation of the random variable X at the probability level α is defined by:

$$\mathrm{CTE}_{\alpha}(X) = \mathrm{E}[X \mid X \ge \mathrm{VaR}_{\alpha}(X)].$$

2.3. Conditional Value-at-Risk

The lack of some important properties of Value-at-Risk, like subaditivity, led to the development of some new risk measures. The Conditional Value-at-Risk measure corresponding to the random variable X and to the probability level $\alpha \in (0,1)$ is defined by:

$$\mathrm{CVaR}_{\alpha}(X) = \int_{-\infty}^{\infty} x \, \mathrm{d}F_X^{\alpha}(x), \text{ where } \mathrm{F}_X^{\alpha}(x) = \begin{cases} 0, & x < \mathrm{VaR}_{\alpha}(X) \\ \dfrac{\mathrm{F_X}(x) - \alpha}{1-\alpha}, & x \geq \mathrm{VaR}_{\alpha}(X) \end{cases}.$$

If X is a continuous random variable, then

$$\mathrm{CVaR}_{\alpha}(X) = \mathrm{E}\left[X \mid X \geq \mathrm{VaR}_{\alpha}(X)\right].$$

In recent literature, we can find several representation formulas for CVaR. One of them was proposed by Acerbi (2002) and is given by:

$$\mathrm{CVaR}_{\alpha}(X) = -\frac{1}{\alpha}\int_{0}^{\alpha} \mathrm{VaR}_{\beta}(X)\,\mathrm{d}\beta \tag{2}$$

Conditional Value-at-Risk estimates the mean value of the losses greater than Value-at-Risk which can occur in a given time horizon. If we refer to a certain probability level α, $\mathrm{VaR}\alpha(X)$ represents the inferior bound of the $\mathrm{CVaR}\alpha(X)$ measure.

2.4. Limited Value-at-Risk

Let X be a random variable with cumulative distribution function $\mathrm{F}_X(x) = \mathrm{P}(X \leq x)$. Let $\alpha \in (0,1)$ and $l_0 \in \mathbf{R}$, such that the condition (C) $\mathrm{P}(X \geq l_0) > 0$ holds. The (α, l_0) - Limited Value-at-Risk of the random variable X corresponding to the threshold l_0 and to the probability level α is defined by:

$$\mathrm{LVaR}_{\alpha,l_0}(X) = \inf\left\{x \in \mathbf{R} \mid \mathrm{P}(\mathrm{X} \leq x \mid X \geq l_0) \geq \alpha\right\}.$$

We investigate some properties of LVaR risk measure and study the relationship between VaR and LVaR.

Proposition 1. For any loss random variable X, for any $\alpha \in (0,1)$ and any l_0 satisfying (C) we have:

$$\mathrm{LVaR}_{\alpha,l_0}(X) = \mathrm{VaR}_{\beta(\alpha,l_0)}(X), \text{ unde } \beta(\alpha, l_0) = \alpha + (1-\alpha)F_X(l_0). \tag{3}$$

The Limited Value-at-Risk of a random variable evaluates the worst expected loss over a time period, given that the loss random variable exceeds an upper threshold l_0 at a given probability level. Since the Value-at-Risk corresponding to the same probability level α underestimates the risk level, it follows that the LVaR approach is more realistic in risk modeling, because there always exists a certain range of losses which is of real concern for the investor. By increasing the threshold l_0 corresponding to the Limited Value-at-Risk, we obtain further information relative to the right tail of the distribution of the loss random variable.

3. Value-at-Risk estimation methods

Since in most cases the distribution of the loss random variable is not known, the methods for computing or approximating VaR represent very important topics. There are three main approaches for estimating VaR (Dedu and Fulga, 2011): the parametric or analytic method, the historical simulation or empirical method and the Monte-Carlo simulation method.

3.1. The parametric method

The parametric or analytic method requires an assumption about the statistical distribution from which data are drawn. The advantage of parametric VaR is that relatively little information is needed to compute it. But its main weakness is that the distribution chosen may not accurately reflect all the possible states of the market and may under or overestimate the risk. This problem is particularly acute when using value at risk to assess the risk of asymmetric distributions, such as portfolios containing options and hedge funds.

3.2. The historical simulation method

The historical simulation or empirical method is useful when empirical evidence indicates that we cannot make distributional assumptions. The historical simulation method computes the hypothetical value of a change in the current portfolio depending on historical variations of the risk factors. The great advantage of this method is that it makes no assumption regarding the probability distribution, it only using the empirical distribution obtained from the analysis of past data, the calculations being relatively simple. The disadvantage of the historical simulation method lies in the fact that it predicts the future development on the basis of past data, which could lead to inaccurate forecasts if the trend of the past no longer complies, or if the portfolio changes.

Let L_j be the loss random variable corresponding to the asset j, $j=\overline{1,s}$. Let $L_j^1, L_j^2, ..., L_j^n$ be n independent and identically distributed random samples of L_j and let $\hat{F}_n^j$ be the empirical cumulative distribution function of L_j. Then we have: $\hat{F}_n^j(z)=\frac{1}{n}\sum_{i=1}^{n} I_{\{L_j \le z\}}$, where I_A represents the indicator function of the set A. The historical estimation of VaR involves generating n independent and identically distributed random samples of L_j, denoted as $L_j^1, L_j^2, ..., L_j^n$ and estimating $\text{VaR}_\alpha(L_j)$ by: $\hat{v}_n^j(L_j)=(\hat{F}_n^j)^{-1}(\alpha)=\min\{z \in \mathbf{R} \mid \hat{F}_n^j(z) \ge \alpha\}$.

We can also write:

$$\hat{v}_n^j(L_j)=\min\left\{ z \in \mathbf{R} \mid \frac{1}{n}\sum_{i=1}^{n} I_{\{L_j^i \le z\}} \ge \alpha \right\}. \tag{4}$$

3.3. Monte Carlo simulation method

Monte Carlo method is helpful when the analytical approach is not applicable. The Monte Carlo method for VaR estimation is based on the statistical simulation of the joint behaviour of all the relevant variables and uses this simulation in order to generate future possible scenarios. This method is used in the first step of the scenario generation technique, that means producing a large number of future price scenarios. The next step, the portfolio valuation, consists in computing a portfolio value for each scenario. In the final step, the summary, we report the results of the simulation, either as a portfolio distribution or as a particular risk measure. The VaR of the loss random variable corresponding to an asset is estimated by creating a hypothetical time series of returns on that asset, obtained by running the asset through actual historical data and computing the changes that would have occurred in each period.

4. Risk modeling and assessment using the Logistic distribution

4.1. The Logistic distribution

The probability density function of the Logistic distribution with location parameter μ and scale parameter s is given by:

$$f(x;\mu,s)=\frac{e^{-\frac{x-\mu}{s}}}{s\left(1+e^{-\frac{x-\mu}{s}}\right)},\ x\in\mathbf{R};\ \mu\in\mathbf{R},s>0.$$

Figure 1 provides the plot of the probability density function of the Logistic distribution, for different values of the location and scale parameters.

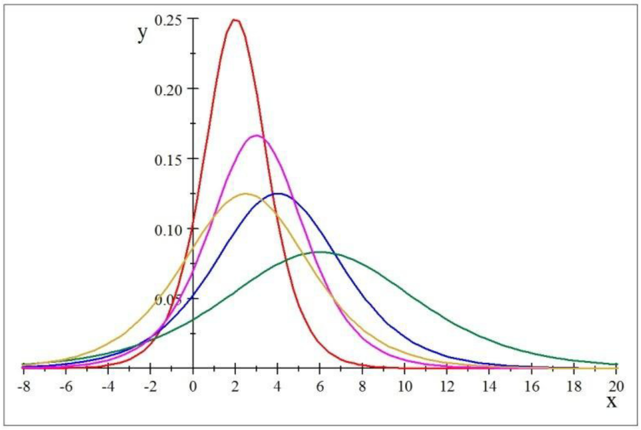

Fig.1. The probability density function of the Logistic distribution for different values of the location and scale parameters: μ = 2, s = 1 (red), μ = 2.5, s = 2 (orange), μ = 3, s = 1.5 (magenta), μ = 4, s = 2 (blue), μ = 6, s = 2 (green).

The cumulative distribution function of the Logistic distribution with location parameter μ and scale parameter s is given by:

$$F(x;\mu,s)=\frac{1}{1+e^{-\frac{x-\mu}{s}}},\ x\in\mathbf{R};\ \mu\in\mathbf{R},s>0. \qquad (4)$$

4.2. Risk assessment using the Logistic distribution

Our aim is to derive analytical formulas for risk computation using VaR and CVaR corresponding to the loss random variable and to the probability level α. We will use the analytic method in the case when the considered random variable can be well approximated by a Logistic distribution. Consider a set of S assets, with the asset j giving the return R_j at the end of the investment period. We model the return R_j using a random variable, since the future price of the asset is not known. Let L_j be the loss random variable corresponding to the asset j, $j=\overline{1,s}$. We will derive the analytical form of VaR risk measure of the loss random variable L_j in the case of Logistic distribution. We will also derive the analytical expression of CVaR risk measure of the portfolio corresponding to the probability level α, as stated in the next proposition.

Proposition 2. If the loss random variable X follows a Logistic distribution with location parameter μ and scale parameter s, then the Value-at-Risk measure of the loss random variable corresponding to the probability level α is given by:

$$\mathrm{VaR}_{\alpha}(X) = \mu - s \cdot \ln\left(\frac{1}{\alpha} - 1\right). \tag{5}$$

Proof. Let $\alpha \in (0,1)$. Since the cumulative distribution function of the random variable X is one-to-one continuous, it follows that $\mathrm{VaR}\alpha(X)$ is the solution of the equation (1) and using (4) it follows the conclusion.

Proposition 3. If the loss random variable X follows a Logistic distribution with location parameter μ and scale parameter s, then the Conditional Value-at-Risk measure of the loss random variable corresponding to the probability level α is given by:

$$\mathrm{CVaR}_{\alpha}(X) = s \cdot \ln\left(\frac{1}{\alpha} - 1\right) - \frac{s}{\alpha} \cdot \ln(1-\alpha) - \mu. \tag{6}$$

Proof. Let $\alpha \in (0,1)$. Since the cumulative distribution function of the random variable X is one-to-one continuous, $\mathrm{CVaR}\alpha(X)$ can be derived using formulas (1) and (2):

$$\mathrm{CVaR}_{\alpha}(X) = -\frac{1}{\alpha}\int_0^{\alpha} VaR_X(\beta)d\beta = -\frac{1}{\alpha}\int_0^{\alpha}\left[\mu + s \cdot \ln\left(\frac{1}{\beta} - 1\right)\right]d\beta = -\mu + s \cdot \ln\left(\frac{1}{\alpha} - 1\right) - \frac{s}{\alpha} \cdot \ln(1-\alpha).$$

5. Computational results

We consider the case of a portfolio composed by s assets and evaluate the outcome of the assets using the log-return function. Let $S_j(t)$ be the closing price of the asset j at the moment t. The log-return of the asset j corresponding to the time horizon $[t, t+1]$ is defined by: $R_j(t) = \ln S_j(t+1) - \ln S_j(t),\ j \in \overline{1,s},\ t > 0.$

The loss random variable of the asset j corresponding to the time horizon $[t, t+1]$ is given by:

$L_j(t) = \ln S_j(t) - \ln S_j(t+1),\ j \in \overline{1,s},\ t > 0.$

We used historical data available on Yahoo Finance website for a 5 years time horizon. We choose the loss random variable on the basis of 1260 daily closing prices of the Yahoo stock, from November 28, 2008 to November 29, 2013. The results of the Kolmogorov-Smirnov test, presented in Table 1, show that the Logistic distribution fits best the data for the goodness of fit test.

Table 1. The results of performing distribution fitting tests

Distribution	*p*-value	Decision
Normal	0.0000	Reject
Student	0.0001	Reject
Logistic	**0.1200**	**Accept**
Weibull	0.0001	Reject

The probability density function of the estimated Logistic distribution and the histogram of the real data are presented in Figure 2.

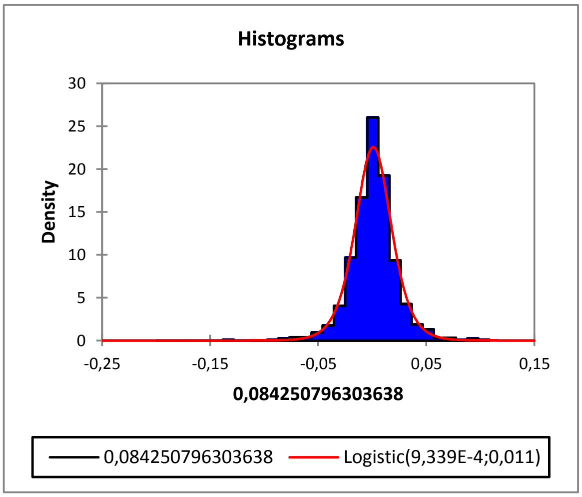

Fig. 2. The probability density function of the estimated Logistic distribution (red line) fitting the histogram corresponding to the real data

The risk corresponding to the loss random variable was estimated using different methods and measures. The results presented in Table 2 and Table 3 show that historical method underestimates risk. VaR, CVaR and LVaR measures were computed using the Propositions 1, 2 and 3.

Table 2. The results obtained for the VaR computation using three estimation methods for different values of the probability level

Method	$\alpha = 0.90$	$\alpha = 0.95$	$\alpha = 0.99$
Parametric	0.027	0.036	0.055
Historical	0.024	0.034	0.058
Monte Carlo simulation	0.028	0.036	0.060

Table 3. The values of different risk measures computed for different values of the probability level α, using the parametric method

Risk measure	$\alpha = 0.90$	$\alpha = 0.95$	$\alpha = 0.99$
VaR_{α}	0.027	0.036	0.055
$CVaR_{\alpha}$	0.037	0.046	0.065
CTE_{α}	0.037	0.046	0.065
$LVaR_{\alpha;0.01}$	0.031	0.038	0.058

The results obtained indicates that LVaR approach is more realistic in risk modelling because there always exists a certain range of losses which should be taken into account by the investor in order to evaluate risk as accurate as possible.

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