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Procedia Engineering 15 (2011) 3371 – 3375

**Procedia  
Engineering**[www.elsevier.com/locate/procedia](http://www.elsevier.com/locate/procedia)

Advanced in Control Engineering and Information Science

## Multiple Swarms Multi-objective Particle Swarm Optimization Based on Decomposition

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### Abstract

Particle swarm optimization is a very competitive swarm intelligence algorithm for multi-objective optimization problems, but because of it is easy to fall into local optimum solution, and the convergence and accuracy of Pareto solution set is not satisfactory. So we proposed a multi-swarm multi-objective particle swarm optimization based on decomposition (MOPSO\_MS), in the algorithm each sub-swarm corresponding to a sub-problem which decomposed by multi-objective decomposition method, and we constructed a new updates strategy for the velocity. Finally, through simulation experiments and compare with the state-of-the-art multi-objective particle swarm algorithm on ZDT test function proved the convergence and the accuracy of the algorithm.

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Selection and/or peer-review under responsibility of [CEIS 2011]

*Keywords:* Multi-objective particle swarm optimization; Multi-swarm particle swarm optimization; Multi-objective decomposition; Performance indicator

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### 1. Introduction

Real-world optimization problems are in fact multi-objective optimization problem with conflicting goals, and widely exist in the field of science and engineering, but hardly to be solved, so the researching on that has very important theoretical and practical significance. Multi-objective optimization problem

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(MOP) is defined as follows:

$$\begin{aligned} & \text{minimize } F(x) = (f_1(x), \dots, f_m(x))^T \\ & \text{subject to } x \in \Omega \end{aligned} \quad (1)$$

where,  $\Omega$  is the solution space,  $F: \Omega \rightarrow \mathbb{R}^m$  constituted by the  $m$  objective functions.

Particle swarm optimization (PSO)<sup>[1]</sup> is a adaptive evolution algorithm based on population search, the thought comes to the prey behavior of birds, and its advantages is simple and fast convergence. In PSO algorithm, each particle represents a potential solution. Algorithm initially generates a random population and gives each particle a random velocity, which on particle flight process, the particle track their self optimum  $P_{id}$  and entire swarm global optimum  $P_{gd}$ . Each iteration, the particles velocity and position updates according to the following formula:

$$v_{id}^{k+1} = wv_{id}^k + c_1r_1(p_{id} - z_{id}^k) + c_2r_2(p_{gd} - z_{id}^k) \quad (2)$$

$$z_{id}^{k+1} = z_{id}^k + v_{id}^{k+1} \quad (3)$$

In 1999 Moore<sup>[2]</sup> first introduced the PSO to solve multi-objective problem, in the following many scholars have proposed a lot of new method to promote the performance , which Coello<sup>[3]</sup> proposed algorithm is now classical algorithms used PSO for MOP; Wei<sup>[4]</sup> introduced strategy of multi-objective decomposition to the PSO, then proposed multi-objective PSO algorithm based on decomposition. Although the varieties of multi-objective PSO algorithm have improve the performance on certain extent, however, such as convergence and accuracy of Pareto optimal set is still the difficult problem of research, this paper proposes a new multiple swarm multi-objective particle swarm optimization based on decomposition (MOPSO\_MS).

## 2. Multi-objective decomposition evolutionary strategy

Zhang<sup>[5]</sup> in 2007 had proposed a new framework for multi-objective optimization based on decomposition, and this method used the traditional mathematic method decomposes the MOP into a number of single objective problems that is also known as sub-problems to solve. Each sub-problem corresponds to a multi-objective weight vector, and each sub-problem optimal solution is a Pareto solution of the MOP. With each sub-problem that the neighborhood sub-problems with closest weight vector gave the greatest help for optimization, because they are the closest Pareto solution on the Pareto front, by calculating the Euclidean distance of the weight vectors can determine the neighborhood problem of sub-problem, this is the main motivate of the multi-objective decomposition evolution strategy.

In the field of multi-objective optimization, there are several ways to decompose the MOP into several sub-problems, we implement with a simple linear weighting method. Linear weighting method for solving MOP is most widely used, set  $\lambda = (\lambda_1 \dots \lambda_m)^T$ , and  $\lambda$  is non-negative weight coefficient vector, and  $\sum \lambda_i = 1$ . Construct single-objective evaluation function as follows:

$$\begin{aligned} & \text{minimize } g^{ws}(x | \lambda) = \sum_{i=1}^m \lambda_i f_i(x) \\ & \text{subject to } x \in \Omega \end{aligned} \quad (4)$$

According to the formula different non-negative weight vector can obtain different Pareto solution.

## 3. Multi-swarms multi-objective PSO base on decomposition

### 3.1. multi-swarm evolutionary strategy

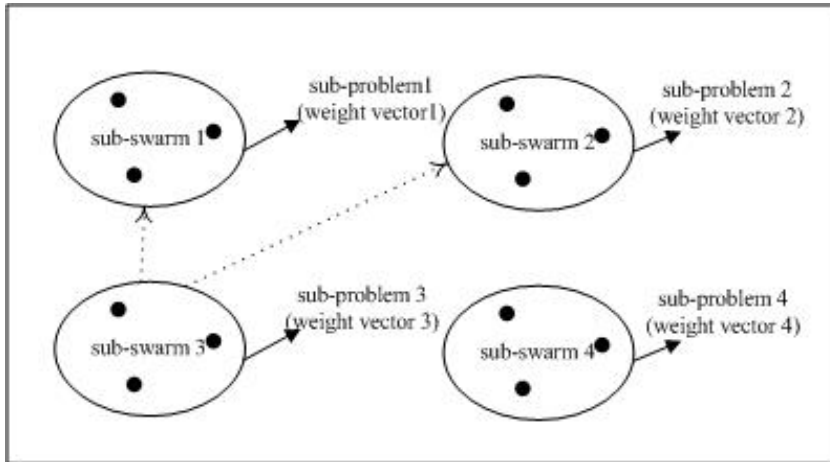


Fig.1 The illustration of multiple swarm co-evolution

Multi-swarm PSO is divided entire particle swarm into several sub-swarm, parallel optimization of these sub-swarms, the particle updates based on itself history optimal value and the sub-swarm optimal value, which can enhance the local search capacity and delay the evolution speed to avoid premature to local minimum. The introduction of multi-objective decomposition evolutionary strategy to sub-swarm PSO for MOP with a natural advantage, so that each sub-swarm can be related to a sub-problem, by sub-swarm optimization can quickly converge to the Pareto optimal solution. Shown in Fig.1, the whole swarm is divided into four sub-swarms, each sub-swarm corresponds to a sub-problem that decomposition of MOP, and each sub-problem corresponds to a weight vector.

Information exchange mechanism is a key issue of multiple swarms PSO taking into account the sub-problems using multi-objective decomposition method decomposed can determine a group of problem set with closely evolutionary related according to Euclidean distance of weight vectors, and then corresponding sub-problem to a sub-swarm of the PSO and each sub-swarm can determine its neighborhood sub-swarm. As shown in Fig.1, the neighborhood of sub-swarm 3 is the sub-swarm 1 and 2, for which we propose a multiple swarm co-evolution update strategy, update the particle according to itself history optimal value  $P_{id}$ , sub-swarm optimal value  $P_{ld}$  and neighborhood optimal value  $P_{nd}$ , and uses a comprehensive learning strategy from the three extreme value, the formula is as follows:

$$v_i^{t+1} = \chi \left[ v_i^t + \sum_{p_k \in N_i} \varphi_k U_k^t (p_k^t - x_i^t) \right] \tag{5}$$

Where  $\varphi$  is the sum of acceleration factor and  $\varphi_k = \varphi/3$ ,  $N_i \in \{P_{id}, P_{ld}, P_{nd}\}$ .

### 3.2. Algorithm description

MOPSO\_MS algorithm described in the following:

**Input:**

- (1) MOP;
- (2) N: number of sub-problems which multi-objective decomposed;
- (3) Uniform distribution weight vector:  $\lambda_1, \lambda_2 \dots \lambda_N$ ;
- (4) T: each sub-swarm's neighborhood size;

**Output:** Archives EP

**Step 1 Initialization**

- Step 1.1 Set archive  $EP = \Phi$ ;  
 Step 1.2 calculated the Euclidean distance between Weight vectors to determine the neighbourhood of each weight vector,  $B(i) = \{i_1, \dots, i_T\}$ , which  $\lambda^{i_1}, \lambda^{i_2} \dots \lambda^{i_T}$  is the closest T weight vector of  $\lambda^i$ ;  
 Step 1.3 Randomly initialize the position and velocity of particles swarm, divided sub-swarm, then each sub-swarm corresponds to a weight vector and the corresponding neighbourhood;  
 Step 1.4 Initialize the particles optimal value  $P_i$  and the sub-swarm optimal value  $P_i$  and sub-swarm neighbourhood optimal value  $P_n$ .

**Step 2 Main loop**

- Step 2.1 According to formulas (3) and (4) Update the particle velocity and position;  
 Step 2.2 Delete the individual in the EP which dominated by sub-swarm optimal value;  
 Step 2.3 Add the individual to EP which non-dominated by any individuals in EP;  
 Step 2.4 If the number of EP is greater than the maximum size, according the crowded degree of spaced grid to remove the exceed member;  
 Step 2.5 Update  $P_i$ ,  $P_i$  and  $P_n$ ;  
 Step 2.6 If satisfaction the terminating condition, then go to Step 3, otherwise go to Step 2.

**Step 3 Output global optimal values, stop the algorithm.****4. Experiments and results analysis***4.1. Test function*

Multi-objective evolutionary algorithm test functions are often relatively simple, because the test function too complex is difficult to visualize, and the exact shape of Pareto front and location is hard to describe. The ZDT test function set is the widely used and consists of six different types of test functions, and the six test functions test the performance of optimization algorithms from different points. Therefore, we focus on using this test functions set of the five test functions (ZDT5 are discrete optimization problems) to verify the proposed algorithm. In this paper, we use the literature [3] mentioned convergence indicators GD and validity indicators ER to assess the convergence and correctness of Pareto solution set.

*4.2. Results analysis*

Simulation experiment completed in the Matlab, because the result of MOP is a Pareto solution set, in our algorithm, the optimal value of each sub-swarm is a candidate Pareto solution, and the number of sub-swarm is not too small, so set to 100 sub-groups, each sub-swarm contains three particles, sub-swarm neighborhood is set to 10, each PSO algorithm for each test function independently run 30 times under the same conditions. The results shown in Table 1, the results MOPSO\_MC compared with MOPSO<sup>[3]</sup> and MOPSO/D<sup>[4]</sup> of the performance indicator averages shown in Table 2 and the best results in bold.

Table 1 The experiment result of MOPSO\_MC

Metric	Status	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6
ER	Best	1.0000E-02	0.0000E+00	1.0000E-01	1.0000E+00	1.0000E-02
	Worst	3.6000E-01	6.6667E-01	4.7000E-01	1.0000E+00	1.0000E-01
	Average	1.2800E-01	3.6745E-01	2.5962E-01	1.0000E+00	4.9667E-02
	Std.Dev.	8.9381E-02	1.7622E-01	1.1750E-01	0.0000E+00	2.3706E-02
GD	Best	1.1441E-04	1.8519E-04	9.8582E-04	3.7203E-02	1.5013E-04
	Worst	3.0319E-02	6.3997E-02	7.5451E-03	3.5207E-01	8.0670E-02

Average	3.3439E-03	1.4006E-02	4.1547E-03	1.8608E-01	2.9211E-03
Std.Dev.	6.2088E-03	1.2389E-02	1.4082E-03	7.7974E-02	1.4684E-02

In the experimental results shows in Table 1, validity indicator ER of MOPSO\_MS on test functions except ZDT4 are very small, indicating the validity is relatively high, and the convergence indicators GD in all functions tend to zero, indicating the convergence better. Although ZDT4 not converge to the actual Pareto front, but in Table 2 the ZDT4’s GD of MOPSO\_MS is the smallest, MOPSO\_MS is the closest species to the actual Pareto front in the three algorithms.

It can be seen from Table 2 in the test function ZDT1, ZDT4 and ZDT6, MOPSO\_MS comparison with MOPSO/D and MOPSO have obvious advantages in the all performance indicators, in ZDT2 and ZDT3 MOPSO\_MS have a small gap than the MOPSO, But better than MOPSO/D.

Table 2 The compared result of algorithms performance

Metric	Algorithms	ZDT1	ZDT2	ZDT3	ZDT4	ZDT6
GD	MOPSO_MS	<b>3.3439E-04</b>	1.4006E-02	4.1547E-03	<b>1.8608E-01</b>	<b>2.9211E-03</b>
	MOPSO	5.8687E-04	<b>1.9715E-04</b>	<b>2.5495E-04</b>	5.7144E-01	8.4904E-02
	MOPSO/D	6.1620E-03	2.3548E-02	1.3612E-02	1.6187E+01	8.3080E-02
ER	MOPSO_MS	<b>1.2800E-01</b>	3.6745E-01	2.5962E-01	<b>1.0000E+00</b>	<b>4.9667E-02</b>
	MOPSO	2.2400E-01	<b>1.0200E-01</b>	<b>1.5200E-01</b>	1.0000E+00	2.5800E-01
	MOPSO/D	9.9718E-01	1.0000E+00	1.0000E+00	1.0000E+00	1.4200E-01

Experimental analysis shows the convergence and accuracy of MOPSO\_MS algorithm is better, so it is a kind of competitive multi-objective evolution algorithm.

### 5. Conclusions

Introduce the decomposition of the multi-objective evolutionary strategy to PSO is a new attempt, this paper analyzes the multi-objective PSO and multi-swarm PSO, then proposed multi-swarm multi-objective PSO based on multi-objective evolutionary strategy and the algorithm is proved by experiments.

### Acknowledgements

This work was supported by the science and technology foundation of Jiangxi Provincial under grant No. GJJ10616 and No. GJJ11616.

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