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Several problems about strain calculation and analysis and correction of related deviation*

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Abstract: There exists many kinds of calculation models of plane and spherical strain fields, but the results of these models are different. The representative models was analyzed, and got some useful conclusions, in which some models are unbiased, some have deviations that can be corrected, some can only be used to compute strain in a uniform medium and can not be extended, and some can be used in the calculation and analysis of continuous strain field as well. Meanwhile, the correction relationship for spherical difference movement (displacement) computed from strain results was given, and the meaning of the non - differential term in spherical strain model was demonstrated.

Key words: GNSS; horizontal strain calculation; model applicability; deviation correction; conversion relationship; strain filtering

1 Introduction

In the early research on the strain field, as the scale of research was small, the regional spherical movement was usually projected onto Gauss plane, and in the plane the strain field was solved by using the relationship between movement and strain. With the introduction of GNSS, the scale of research expanded, the method mentioned above became inappropriate, so many calculation models of spherical strain have been created[1-11]. However, these models are not exactly the same in theory and application field, Some are even biased, and therefore the results are influenced to some extent. In addition, the fact that its characteristic of broad spectrum for strain has not been confirmed and fully understood, which makes the analysis on strain to be in confusion. In this paper, we intend to clarify the question and make the calculation and analysis of strain more accurate and effective.

2 Calculation model or formula of horizontal strain field

2.1 Calculation model or formula of plane horizontal strain field

In a plane rectangular coordinate system, there are four types of calculation models or formulas of strain in the analysis of surface deformation. The first one is to use differential equation to solve strain when the movement (displacement) is known; the second one is to establish the analytic expression of strain and movement according to the displacement of discrete points; the third one is to solve the normal strains and their shear strain in two orthogonal directions based on the relative variation in length of discrete baselines; the last one is to base on the Delaunay triangular element method.

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If the analytic expression in east component ($f_x(x,y)$) and north ($f_y(x,y)$) are known, strain can be expressed as

$$\begin{cases} \varepsilon_x(x,y) = \dfrac{\partial f_x(x,y)}{\partial x} \\ \varepsilon_y(x,y) = \dfrac{\partial f_y(x,y)}{\partial y} \\ \varepsilon_{xy}(x,y) = \dfrac{1}{2}\left[\dfrac{\partial f_y(x,y)}{\partial x} + \dfrac{\partial f_x(x,y)}{\partial y}\right] \end{cases} \tag{1}$$

If the displacement in east component ($v_x(x_j,y_j)$) and north ($v_y(x_j,y_j)$) of discrete point are known, strain can be expressed as

$$\begin{cases} V_x(x,y) = V_{x0} + x\varepsilon_x + y\varepsilon_{xy} - y\omega_0 \\ V_y(x,y) = V_{y0} + x\varepsilon_{xy} + y\varepsilon_y - x\omega_0 \end{cases} \tag{2}$$

If the relative variation in length (ε_j) and azimuth (ϕ_j) of discrete baselines are known, according to relational equation between movement and strain, strain can be expressed as

$$\varepsilon_j = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\phi_j + \frac{1}{2}\varepsilon_{xy}\sin 2\phi_j \tag{3}$$

Assuming the relationship between coordinates and movements is linear and coordinates (x_i, y_i) of three nodes in Delaunay triangular element and their corresponding horizontal movements($v_x(x_i,y_i), v_y(x_i,y_i)$) are known, the equation of movement and position can be defined as[9]

$$\begin{cases} v_x(x,y) = a_1 x + a_2 y + a_3 \\ v_y(x,y) = b_1 x + b_2 y + b_3 \end{cases} \tag{4}$$

With equation (1), the average strain in triangular element can be expressed as

$$\begin{cases} \varepsilon_x(x,y) = a_1 \\ \varepsilon_y(x,y) = b \\ \varepsilon_{xy}(x,y) = \dfrac{1}{2}(a_2 + b_1) \end{cases} \tag{5}$$

2.2 Calculation model or formula of spherical horizontal strain field

If the analytic expression in east component ($f_x(x,y)$) and north ($f_y(x,y)$) are known, strain can be expressed as

$$\begin{cases} \varepsilon_e(\lambda,\varphi) = \dfrac{1}{R\cos\varphi}\dfrac{\partial f_e(\lambda,\varphi)}{\partial\lambda} - \dfrac{f_n(\lambda,\varphi)}{R}\tan\varphi \\ \varepsilon_n(\lambda,\varphi) = \dfrac{1}{R}\dfrac{\partial f_n(\lambda,\varphi)}{\partial\varphi} \\ \varepsilon_{en}(\lambda,\varphi) = \dfrac{1}{2}\left[\dfrac{1}{R\cos\varphi}\dfrac{\partial f_n(\lambda,\varphi)}{\partial\lambda} + \dfrac{1}{R}\dfrac{\partial f_e(\lambda,\varphi)}{\partial\varphi} + \dfrac{f_e(\lambda,\varphi)}{R}\tan\varphi\right] \\ \omega(\lambda,\varphi) = \dfrac{1}{2}\left[\dfrac{1}{R\cos\varphi}\dfrac{\partial f_n(\lambda,\varphi)}{\partial\lambda} - \dfrac{1}{R}\dfrac{\partial f_e(\lambda,\varphi)}{\partial\varphi} - \dfrac{f_e(\lambda,\varphi)}{R}\tan\varphi\right] \end{cases} \tag{6}$$

where R denotes the average radius of the earth.

If the displacement in east component ($v_e(\lambda_j,\varphi_j)$) and north ($v_n(\lambda_j,\varphi_j)$) of discrete point are known, strain can be expressed as

$$\begin{bmatrix} v_e \\ v_n \end{bmatrix} = \begin{bmatrix} -R\sin\varphi\cos\lambda & -R\sin\varphi\cos\lambda & R\cos\varphi \\ \sin\lambda & -R\cos\lambda & 0 \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \begin{bmatrix} \varepsilon_e & \varepsilon_{en} \\ \varepsilon_{ne} & \varepsilon_n \end{bmatrix}\begin{bmatrix} \Delta\lambda R\mathrm{coa}\varphi \\ \Delta\varphi R \end{bmatrix} \tag{7}$$

If the relative variation in length (ε_j) and azimuth (φ_j) of discrete baselines are known, according to Jaeger method, strain can be expressed as

$$\varepsilon_j = \frac{\varepsilon_e + \varepsilon_n}{2} + \frac{\varepsilon_e - \varepsilon_n}{2}\cos 2\varphi_j + \frac{1}{2}\varepsilon_{en}\sin 2\varphi_j \tag{8}$$

If coordinates of three nodes in Delaunay triangular element and their corresponding horizontal movements ($v_e(\lambda_j,\varphi_j), v_n(\lambda_j,\varphi_j)$) are known, the method to obtain average strain in triangular element is similar to the one in a plane.

2.3 Difference and applicability of calculation models in different conditions

2.3.1 Analytical method

Theoretically speaking, as the value of the high term in strain calculation is so small that it can be ignored in application. Although the plane model (1) ignores the high term, it still can be considered as rigorous, and be applied for computing the continuous strain in a plane with non - uniform medium. In order to compare with the plane model, the equivalent transformation of spherical model (6) can be expressed as

$$\begin{cases} \varepsilon_e(\lambda,\varphi) = \dfrac{\partial f_e(\lambda,\varphi)}{\partial s_\lambda} - \dfrac{f_n(\lambda,\varphi)}{R}\tan\varphi \\ \varepsilon_n(\lambda,\varphi) = \dfrac{\partial f_n(\lambda,\varphi)}{\partial s_\varphi} \\ \varepsilon_{en}(\lambda,\varphi) = \dfrac{1}{2}\Big[\dfrac{\partial f_n(\lambda,\varphi)}{\partial s_\lambda} + \dfrac{\partial f_e(\lambda,\varphi)}{\partial s_\varphi} + \dfrac{f_e(\lambda,\varphi)}{R}\tan\varphi\Big] \\ \omega(\lambda,\varphi) = \dfrac{1}{2}\Big[\dfrac{\partial f_n(\lambda,\varphi)}{\partial s_\lambda} - \dfrac{\partial f_e(\lambda,\varphi)}{\partial s_\varphi} - \dfrac{f_e(\lambda,\varphi)}{R}\tan\varphi\Big] \end{cases} \tag{9}$$

Where s_λ denotes longitudinal length of geodesic and s_φ the latitudinal length. Comparing with the plane model (1), we can find that there is an additional item $-\dfrac{f_n(\lambda,\varphi)}{R}\tan\varphi$ in item $\varepsilon_e(\lambda,\varphi)$ *and* $\dfrac{f_e(\lambda,\varphi)}{R}\tan\varphi$ in item $\varepsilon_{en}(\lambda,\varphi)$. Since the two additional items are not differential terms, it would be misunderstood that the calculation of spherical strain is related to the reference of movement. As the reference of movement is relative, it is very difficult to understand very well. Furthermore, it contradicts with the understanding that the strain analysis is unrelated to the reference, for this reason, it is not widely used. Actually it is a common phenomenon due to the variance of coordinate system.

In order to describe this problem briefly, assuming the north direction of coordinates (longitude) is parallel, similar to the prime vertical, hence the element can move towards north with no deformation, but it cannot be described directly in current spherical system. In other words, the present movement towards north is just the projection of the above north movement with no deformation, so when the element moves towards north, the projection of the movement shows that the longitudinal distance is shortening, namely, a bias occurs. If the north movement of element is assumed as $f_N(\lambda,\varphi)$, when $\Delta\lambda\to0$, then $f_n(\lambda,\varphi)\to f_N(\lambda,\varphi)$, $\Delta\lambda\to\sin\Delta\varphi$, the variation of longitudinal movement due to north projection can be expressed as

$$\Delta f_e = \lim_{\Delta\lambda\to0}(f_N(\lambda,\varphi)\sin\varphi\sin\Delta\varphi) = f_n(\lambda,\varphi)\sin\varphi\Delta\varphi \tag{10}$$

The relative bias of strain in EW-trending can be expressed as

$$\Delta\varepsilon_e = \lim_{\Delta\lambda\to0}\frac{f_n(\lambda,\varphi)\sin\varphi\Delta\lambda}{R\cos\varphi\Delta\lambda} = \frac{f_n(\lambda,\varphi)}{R}\tan\varphi \tag{11}$$

It is equal to non - differential item of east strain in equation (9) numerically, and opposite in sign. For SN - trending strain, no matter the direction of the movement, the value of strain should be unbiased, but not for shear strain, because the angular velocity of the element's movement with no - deformation in sphere is constant, but linear velocity between parallel (e. g. latitude) is different. However, the calculation of shear strain is based on linear velocity, so bias occurs inevitably and it only exists in the EW - trending movement, and its value can be expressed as

$$\Delta f_E = \lim_{\Delta\varphi\to0} f_e(\lambda,\varphi)(\cos(\varphi+\Delta\varphi) - \cos\varphi) = -f_e(\lambda,\varphi)\sin\varphi\sin\Delta\varphi \tag{12}$$

Accordingly, the bias of shear strain can be expressed as

$$\Delta\varepsilon_{en} = \lim_{\Delta\varphi\to0}\frac{-f_e(\lambda,\varphi)\sin\varphi\sin\Delta\varphi}{2R\cos\varphi\Delta\varphi} = \frac{-f_e(\lambda,\varphi)}{2R}\tan\varphi \tag{13}$$

It is also equal to non-differential item of shear strain in equation (9) numerically, and opposite in sign.

The above analysis indicates that in the calculation of spherical strain, if the formula of strain calculation in rectangular coordinate system is used, the results are biased. Model (6) is the unbiased formula, can be applied for computing and analyzing the continuous strain in sphere with non-uniform medium.

As mentioned above, the non - differential term derives from the correction for bias, whereas the movement is related to the reference inevitably. The derivation above shows that this movement has an absolute property in the sphere, thus a rigorous result can not be given at present. Supposing the sum of variation on earth's surface due to the horizontal movement of the earth is equal to zero, besides that, the global movements of the earth satisfy the following condition

$$\int_D r \times V\mathrm{d}m = 0 \tag{14}$$

The movement can be concerned with an absolute property on the surface of the earth, where D denotes spatial dimension of whole surface of the earth, dm denotes an element; r and V denote displacement vector and velocity vector of dm. The early solutions of ITRF are ensured by using the above criterion, but ITRF2000 and ITRF2005 are not sufficient concordant with this criterion, and the difference is very slight. This means that the movement in ITRF can be used to calculate the strain correction, and the continuous unbiased spherical strain field can be obtained when equation (6) is applied appropriately.

2.3.2 Discrete movement method

When the analytic expression of movement field could not be established and there are some discrete points with movement in the research field, usually the strain is solved according to relationship between movement and strain. The equation (2) can be used in a plane and equation (7) can be used in a sphere. The equation (2) is rigorous and the results are unbiased, but it can only be applied in a uniform medium. In the solving of three strain parameters, it can be transformed to solve the analytic expression of the parameters by some means, but a perfect result can not be obtained currently, because the information obtained is in low frequency or under intermediate frequency. As far as equation (7) is concerned, it is exactly the same as equation (2) in essence, three parameters of entire movement and three strain parameters can be used in calculation of strain parameters from relative movement, but they are applied in different coordinate system. The above analysis indicates that as the coordinate system is not rectangular, equation (7) is biased, and when the correction is added, its application is the same as equation (2).

2.3.3 Linear strain method

In equation (3) and (8), the strains in different directions are used to solve the normal strains and their shear strains in two orthogonal directions. Whether in a plane or in a sphere, the results are unbiased. But if the strains on sides of the triangle are used to solve the normal strains and shear strains, simulation analysis of continuous strain field from the above results is inappropriate, and the result can not be corrected. To explain the problem, an example is given as illustrated in Figure 1.

Supposing there are five monitoring sites in a continuous deformation block A and their different movements illustrated in figure 1, evidently, there is deformation in block A. If the movement agrees with the result of the whirl in which the center of each circle is the one of gravity of each triangle and the two circles have the same radius, the strain calculated of equation (3) and (6) is equal to zero, and it is obviously wrong. Thus this method can only be applied in the calculation of entire strain in deformation block with uniform medium and can not be applied in the calculation of continuous strain in block with no-uniform medium.

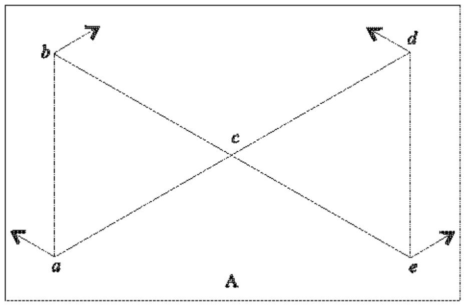

Figure 1 Schematic diagram of the relative motion of block A

2.3.4 Delaunay triangular element method

The method usually used to solve strain today is Delaunay triangular element method. In this method, the strain can be continuous in space according to the principle that fanned out from point to area. In a plane, above all, the result is related to the component of the triangle (namely multiplicity); furthermore, there is no connection between two adjacent elements; finally, the most important thing that we should pay attention to is that the results are not in the same frequency domain due to the asymmetrical distribution of the triangle's vertexes (namely un-equilateral). As the difference of strain between different frequency domains can reach some orders of magnitude, there will come to wrong conclusions in the spatial comparative analysis. In addition, when equation (2) and (5) are used in calculation, there will be distortion besides the above problem as shown in figure 1.

3 The formula to calculate displacement from spherical strain

In some situation or for some research, continuous displacements are obtained by using continuous strain (rate). In a plane coordinate system, the calculation formula can be deduced from formula (2) as

$$\begin{cases} v_x(x,y) = \int_{x0}^{x} \varepsilon_x(x,y)\mathrm{d}x + \int_{y0}^{y} \varepsilon_{xy}(x,y)\mathrm{d}y \\ v_y(x,y) = \int_{x0}^{x} \varepsilon_{xy}(x,y)\mathrm{d}x + \int_{y0}^{y} \varepsilon_y(x,y)\mathrm{d}y \end{cases} \quad (15)$$

But in a sphere, the above method can not be used, and we need to use the following formula to get the relative movement

$$\begin{cases} \begin{bmatrix} v_e(\lambda,\varphi) \\ v_n(\lambda,\varphi) \end{bmatrix} = \\ \begin{bmatrix} \int_{\lambda 0}^{\lambda} \varepsilon_e'(\lambda,\varphi) R\cos\varphi d\lambda + \int_{\varphi 0}^{\varphi} \varepsilon_{en}'(\lambda,\varphi) R d\varphi \\ \int_{\lambda 0}^{\lambda} \varepsilon_{en}'(\lambda,\varphi) R\cos\varphi d\lambda + \int_{\varphi 0}^{\varphi} \varepsilon_n'(\lambda,\varphi) R d\varphi \end{bmatrix} \\ \varepsilon_e'(\lambda,\varphi) = \varepsilon_e(\lambda,\varphi) + \frac{f_n(\lambda,\varphi)}{R}\tan\varphi \\ \varepsilon_{en}'(\lambda,\varphi) = \varepsilon_{en}(\lambda,\varphi) - \frac{f_e(\lambda,\varphi)}{2R}\tan\varphi \end{cases} \quad (16)$$

4 Correction to horizontal strain field of biased model

At present, many researchers use equation (7) in strain calculation. The continuous strain obtained by using least square collocation is biased, but can be modified. For the strain in high frequency (order of 10^{-8} and higher), the correction can be considered to be ignored according to actual situation, but for low frequency (order of 10^{-9}) the correction can not be ignored, because the correction has the same order of magnitude as strain. The equation of correction can be expressed as

$$\begin{cases} \Delta\varepsilon_e(\lambda,\varphi) = \frac{f_n(\lambda,\varphi)}{R}\tan\varphi \\ \Delta\varepsilon_{en}(\lambda,\varphi) = -\frac{f_e(\lambda,\varphi)}{2R}\tan\varphi \end{cases} \quad (17)$$

The reference frame of movement used in equation above should be ITRF. The calculation based on GNSS average rate of movement (1999 – 2007) is given, and the results show that in Chinese Mainland the range of strain rate correction in EW direction is –2.5 – 3.0 and the process of spatial variation is large in the west and small in the east, but variation gradient is rather sharp in the west and flat in the east; the range of shear strain correction in EW and SN direction is 0.9 – 2.8, large in the north and small in the south. Furthermore, the other variable of horizontal strain field such as maximum principal strain, minimum principal strain, maximum shear strain and surface strain, etc. can be obtained based on above three variable, and they have corresponding variation. So it can be confirmed clearly that the strain correction is indispensable in strain analysis in the area with weak deformation or in which strain is in low frequency (eg. Eastern China).

5 The problem about broad-spectrum in strain analysis

In current strain analysis, this problem is neglected or does not get duly attention, hence the results of strain obtained from the same movement field are different, and sometimes the differences are so significant that the analysis of strain is in confusion. The problem may be attribute to not only the incorrectly selection of calculation model or the defect of calculation model, but also the distribution and selection of sites, especially under the condition that there exists significant error in data. We can't deny the fact that based on the current distribution density of stations and observation error, the information of error has the same order of magnitude as interference, especially in Eastern China. Thus the problem should not be avoided, and the fact that the crustal medium is anisotropic and the spectrum of strain is broad should be also clearly understood. In spite of using unbiased calculation model, the subsequent processing is also important, and different methods of subsequent processing can still get different results. The shear strain rate in SN, EW direction in Sichuan-Yunnan Area (1999 – 2007) was calculated by using the equation (6), then different methods of subsequent processing[12] were used, consequently different results were obtained (shown in Fig. 2). The difference between the results may be caused by two reasons, one is the inappropriate application of calculation model that causes the result incorrectly, the other one is that the application of calculation model is appropriate but the collection of information are different. Consequently, the spectral ranges of strain are different, whereas, this can not be considered as a mistake. The spectral range of result in figure 2 (a) is relative

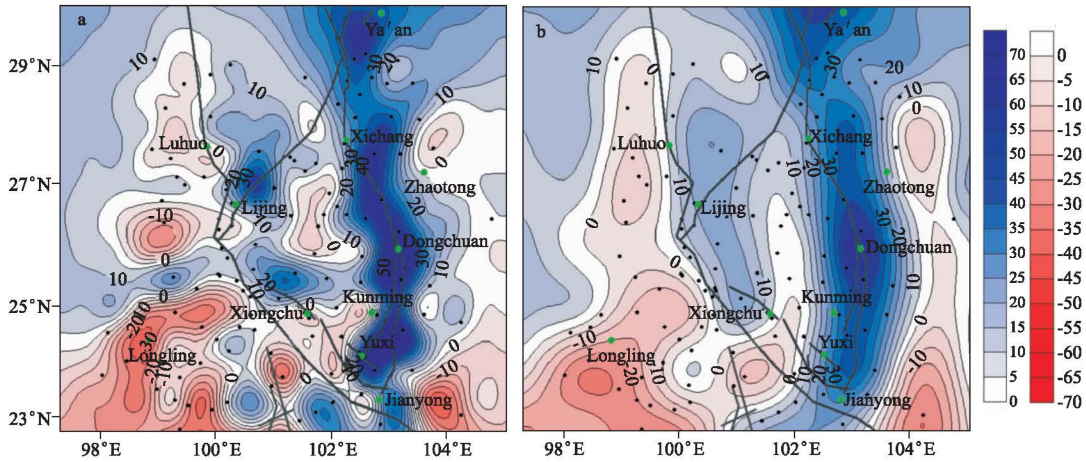

Figure 2 Results of EN shear strain rate by kernel point filtering of step-size 0.5°(a) and 1.0°(b) in Sichuan and Yunnan area

broad, component information of strain in high frequency is abundant; the spectral range in figure 2(b) is relative narrow, focusing on description of information in low frequency.

6 Conclusions

The problems of spherical continuous strain analysis are mainly in two aspects, the first one is the inappropriate use of calculation model and the second one is that some models are biased, accordingly the analysis of spherical strain is in confusion and incorrect. The spherical calculation model (equation (6)) is recommended, because it is not only unbiased, but also includes the information of strain in various frequency (depending on the density of spatial distribution of the sites), so the information of strain we need can be picked up through filtering[12]; Jaeger Method based on the relative variation of triangle in side length should be abandoned in continuous calculation and analysis; the continuous calculation of strain based on equation (7) is biased, and its value should not be ignored in analysis of strain in low frequency. However, it can be modified to be unbiased by correction, thus it should be used with utmost care. Although Delaunay triangular element method can be used for continuous analysis of strain after correction, the condition that the side lengths are equal can not be satisfied in practical applications, and therefore the information of strain is in a mess. In addition, if the unbiased result of strain is used to calculate the relative movement, the correction should be required.

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