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Preface

This special issue is devoted to applications and numerical analysis of *Nonlinear Parabolic Problems with Blow-up Solutions*. It is a subject that has experienced a steady growth since S. Kaplan's paper on blow-up solutions for nonlinear partial differential equations (PDEs) [1] appeared in 1963. The increase in interest in this type of problem is related to the concomitant developments in mathematical analysis of nonlinear parabolic systems. Significant developments in appropriate numerical techniques for this type of problem are confined to the more recent past, however, and we anticipate that improvements in this area will soon lead to a deeper understanding of some of the phenomena that occur.

Blow-up is of great importance when considering nonlinear evolution processes. It usually means an explosive growth in the magnitude of some quantity (for example, temperature, pressure or density) in a finite time. Explosive growth occurs in a wide range of practical situations that includes chemical reactions, gaseous ignition, flow in porous media, ohmic heating and chemotaxis in biological systems. It is extremely important that we gain a deeper understanding of blow-up phenomena. For example, blow-up might be identified with gaseous self-ignition or explosion, and, for reasons of safety, it is of great practical interest to identify conditions under which blow-up occurs. If these conditions are satisfied, it is also of considerable interest to be able to estimate when blow-up will occur.

In this issue of the Journal of Computational and Applied Mathematics we have collected nine papers which, in our view, give a good impression of the 'state of the art' and of current work and open problems in the theory, applications, and the numerical analysis of blow-up problems. We hope these papers (and their extensive bibliographies) will also serve as a reasonably self-contained basis and a convenient reference for future work in these areas.

The opening paper by *Bandle and Brunner* contains a survey of the theory and the numerical analysis of blow-up solutions for quasilinear reaction–diffusion equations; in many ways, it complements the comprehensive survey paper by Levine [2]. Its first part gives a description of the most important criteria for finite-time blow-up and the methods for analysing blow-up solutions and their asymptotic behaviour near their singularities. A good understanding of these methods and results is crucial for the numerical analysis of, and the design of effective algorithms for blow-up problems. As the second part of this survey paper shows, much work remains to be done to bring the numerical analysis and computational solution of blow-up problems to a level of maturity. This is particularly true for higher-dimensional problems.

The next two papers, by *Bricher and Akdoğu* and by *Lacey*, describe a number of specific diffusion models with blow-up (arising in chemical reactions, electrical heating, fluid flow, and gaseous ignition) and the various quantitative and qualitative properties of their solutions, as well as their physical significance. Here, as in the paper by *Budd, Collins and Galaktionov*, techniques from the theory of centre manifolds and dynamical systems prove to be important tools in the analyses of such solutions. In the latter paper the blow-up behaviour of two reaction–diffusion problems with a quasilinear degenerate diffusion and a superlinear reaction is investigated, and the results are contrasted with those that hold for the linear diffusion limit.

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The important notion of self-similarity in blow-up solutions (which plays a central role in the contribution by *Budd, Collins and Galaktionov*) is also studied in detail in the paper by *Herrero, Medina and Velázquez*: their analysis is motivated by reaction–diffusion systems arising as models for biological and physical problems; for example, in chemotaxis and the evolution of self-attracting clusters.

The paper by *Roberts* presents analytical tools for a very different class of blow-up problems, namely for nonlinear Volterra integral equations of the second kind that model the explosive behaviour in diffusive media (for example, in the shear band formation, where material failure resulting from high-strain rates is predicated by the formation of localized shear bands). However, this seemingly different paper shows that there exist many close links between these nonlinear integral equations and nonlinear PDEs.

The remaining three papers focus on the numerical analysis and the computational treatment of blow-up problems. *Dimova, Kaschiev, Koleva and Vasileva* consider the combustion process for a nonlinear heat conducting medium with a nonlinear volume source: they describe and analyse blow-up self-similar solutions which describe the evolution of radially nonsymmetric waves. The numerical solution of such problems is based on semidiscrete Galerkin finite element methods and certain explicit difference schemes, using special adaptive meshes that are consistent with the structure of the self-similar solutions.

In her paper, *Meyer-Spasche* studies the question of how to construct implicit difference schemes for the time integration of blow-up problems. Here, the 'optimum degree of implictness' (which can be used in an adaptive way) is the central ingredient, and schemes with this property are compared with a number of standard difference schemes. Some of the ideas in this paper are related to those underlying the time integration methods analysed by *Le Roux*. Her work focuses on problems with fast or slow diffusion where again the solutions may blow-up (or become extinct) in finite time. Such problems arise typically in the modelling of the evolution of temperature and density in a fusion plasma. The nonstandard schemes are constructed so that they preserve the (extinction or blow-up) properties of the exact solutions.

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References

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