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Didactics management of dichotomy "meaning / form"

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Abstract

In this research, we have shown that the meaning and form of a mathematical concept form a fundamental dichotomy of mathematical activity. In particular, we have been able to adapt the Calculation of Assimilations into a tool for structuring the contents of our teaching that would enable us to preserve at the same time the meaning and the form as two properties of mathematical objects, concomitant, disjoint and exhaustive. Thus, we have established -for example- that it is impossible to preserve the meaning and form of a numerical function such as the logarithm function, without developing a "geometry" of this function.

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1. Introduction

The teaching of mathematics is facing a lot of difficulties and challenges. Mathematicians and Didacticians consider that these difficulties are related to the problems of teaching the meaning. Each concept in mathematics has a meaning and a form, so how should the education system handle this dichotomy "meaning/form"? And how important is this dichotomy in teaching mathematics? And what is the role of ICT (Information and Communication Technologies) in the realization of this dichotomy? To find the answers to these questions, we necessary believe and appropriate to find a proper description of the dichotomy "meaning/ form" and the dialectic that is associated with it, knowing that we have two clarifying ideas concerning this dichotomy. This is the definition of "meaning" proposed by René Thom, and the dialectic "meaning/ form" proposed by Emile Benveniste.

2. Literature review

In his article "Mathematics and mathematics of always", Thom (1974) stated that the real problem of teaching mathematics is being the construction of meaning. Lemoyne (1993) agrees that the question of meaning in didactic research of mathematics is always addressed when it comes to interpreting student's errors. In the founding text of the theory of conceptual fields, Vergnaud (1996) defines meaning as a relationship between subject and both

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situation and meaning.

Based on a series of research concerning the teaching of division in compulsory education, Brousseau (1998) gives analytical definition of meaning:

"The meaning of a knowledge recognized as such by the student (institutionalized) is formed:

-The "tissue" of reasoning and evidence.

-The "fabric" of reformulations and formalizations.

-Implicit models associated with it.

-And more or less assumed relations between these different components, essentially dialectical relationships [...]" (p.50).

The meaning and form are present in the pedagogical discourse but we often oppose the meaning to the form and forget that they form a fundamental dichotomy, it is important to know the reasons of this omission.

3. Problem

3.1. Dichotomy «meaning/ form" according to René Thom

According to Thom (1980) "The meaning is always related to the allocation of a spatial place to a formal and coded expression" (p.62). In fact for Thom “this is because the mathematics result in the space is beyond the semantic takeoff created by axiomatisme of algebraic operations"(p.62).

The problem for us is how to translate it as teaching tools for teaching of mathematics. More precisely, we recognize the quotes René Thom dichotomy "meaning/ form", according to him, the meaning is related to space, to the spatial translation of ideas, as to form; it is related to the formal expressions of mathematical language. Therefore, a mathematical statement is always related to a dichotomy "meaning/ form". The problem is to know the dialectic that governs this dichotomy: is probably the dialectic "meaning/ form" as a back and forth between the formal expression and the real property even spatial of the object. What is the nature of this dialectic? The answer is in our view, in the distinction between meaning and form made by Emile Benveniste in linguistics.

3.2. Dialectic " meaning/ form" of Benveniste

According to Benveniste (1980) "the form of a linguistic unit is defined as its ability to dissociate into lower level. The meaning of a linguistic is defined as its ability to integrate higher level unit. Thus, the form and the meaning appear as joint properties, given necessarily and simultaneously and inseparable in the functioning of the language" (p.127).

In our opinion, the mathematical language, or rather its function possesses the same properties. Let’s emphasize that these properties which are the meaning and the form are, in the functioning of the mathematical language, concomitant, exhaustive and disjoint.

Thus, the teaching of mathematics, when it has to operate the mathematical language, needs two feet. One of the two feet is the meaning, the other is the form. Walking with both feet, the dialectic "meaning/form" must be a learning object: we study the ability of any mathematical object to fit into a high level structure (it is the meaning), but it also implies that one is interested in the ability of object of this level to dissociate into lower level components. The problem of mathematics is the implantation of this dialectic. Hence, the fundamental question of our research.

3.3. Fundamental question

The meaning and the form of a mathematical concept are two sides of a coin. If you tear one of them, the other will be destroyed, so, by what means necessary to preserve the correlation between the form and the meaning in teaching mathematical concepts? And what would be the place of ICT in these ways?

3.4. Research hypothesis

Our research hypothesis is that our system of education gives importance to the meaning to the primary level and to the form to the higher level (high school).

4. Approach, method

Very often, learning difficulties about calculation are rooted in the fact that this calculation is taken by most students without its reference to any of his realization in space. For these students, this calculation is then struck by insignificance. To restore the meaning of calculation, the allocation of spatial place for students to a formal expression is needed. According to Thom, the meaning is the allocation of a spatial place to formal coded expression, so we must seek a management tool of the dichotomy "meaning/form".

Once, this tool is developed, the organization of its confrontation with contingency should allow among other things to validate our research hypothesis. We selected for this confrontation two questionnaires:

1) A questionnaire to "grade 12" students, option: science.
2) A questionnaire for mathematics teachers in high school.

5. Proposal of a tool for managing the dichotomy «meaning/form"

5.1. Tool's Presentation

5.1.1. Regime of Assimilations according to Rene Guitart

Guitart (2003) considers two sets $\boldsymbol{E}$ and $\boldsymbol{F}$, a binary relation $\boldsymbol{\varepsilon} \subseteq \boldsymbol{EXF}$, and notes, $\boldsymbol{(x,y) \in \varepsilon, y \rightarrow , x}$ – what he proposes to read:

" $\boldsymbol{y}$ is assimilable to $\boldsymbol{x}$, from the point of view $\boldsymbol{\varepsilon}$"

5.1.2. Proposal of a regime of Assimilations for the numerical functions study

Let $\boldsymbol{S}$ be the set of surfaces and $\boldsymbol{C}$ the set of plane curves.

We consider the function of assimilation a defined from $\boldsymbol{S}$ to $\boldsymbol{C^2}$

$$\begin{aligned} \mathbf{a} : \boldsymbol{S} &\rightarrow \boldsymbol{C^2} \\ \boldsymbol{s} &\rightarrow \mathbf{a}\boldsymbol{(s)} \end{aligned}$$

By all of the following pairs:

$$\boldsymbol{c} \rightarrow_{\mathbf{a}\,(s)} \boldsymbol{c'} \Leftrightarrow \boldsymbol{c} \text{ and } \boldsymbol{c'} \text{ two parallel plane sections of } \boldsymbol{s}$$

Then, we say that: "In the regime **(a)** and from the point of view $\boldsymbol{s}$, $\boldsymbol{c}$ is assimilable to $\boldsymbol{c'}$ ".

The regime **(a)** is to integrate a curve of cartesian equation of the form $\boldsymbol{y= f(x)}$ in a unit of higher level.

For example, the curve $\boldsymbol{y= artg(x)}$ appears in the regime **(a)** as a component of the surface $\boldsymbol{z=artg(\frac{y}{x})}$, the surface $\mathbf{z=artg(\frac{y}{x})+(x-1)y}$ or the surface $\boldsymbol{z=xy+artg(y)}$. Thus, we can see the curve of a function as a plane section of the surface $\boldsymbol{z= xy + f(y)}$ (Nachit, Namir, Bahra, Kasour & Talbi, 2012).

5.2. The Calculation of Assimilations and ICT

The calculation of assimilation joint to possibilities offered by ICT let us find a tool with which we can, when desired, engage the student activity in the dialectic " meaning/ form" through a good marriage of the speech with the image.

6. Tool's confrontation to the contingency

6.1. Questionnaire 1 addressed to students

Questionnaire objective: The aim of the questionnaire 1 is to know whether students can be spontaneously engaged in a dialectic "meaning/form" about the "logarithm function". This questionnaire was addressed to 200 to "grade 12" students, option: science.

A questionnaire description: The questionnaire consists of four successive activities:
Activity1: we ask the students to find a method of tracing curves C_k that $y= kx + ln(x)$; $k \in IR$ using two lines of equation ($y=-1$) and ($x=1$) and the curve plotted ($y = -1+ ln(x)$).
Activity 2: we give a method to plot curves C_k and ask the students to follow this method to plot other curves C_k .
Activity 3: we ask the students to demonstrate, using a suitable calculation, the method used to plot the curves C_k .
Activity4: we ask the students to describe the set of points $M(x,y,z)$ such as $z= xy + ln(y)$, using the previous activities around the curves C_k.

The following table summarizes the progress of this questionnaire:

Table 1. Activity progresses

Activity	Duration
Activity 1	15
Activity correction 1	5
Activity 2	15
Activity correction 2	5
Activity 3	15
Activity correction 3	5
activity 4	15
Activity correction 4	5

6.2. Questionnaire 2 addressed to Math teachers

Questionnaire objective: The aim of the questionnaire 2 is to know whether mathematics teachers in high school use ICT for teaching the mathematical concept "numerical function for real variables" and if they are for the integration of similar activities of questionnaire 1 in school textbooks. This questionnaire was addressed to 100 mathematics teachers in high school.

A questionnaire description: The questionnaire 2 consists of two parts:
Part 1: Use of ICT for teaching the concept “numerical function of real variables"
Part 2: View of teachers on the activities of questionnaire 1 addressed to students.

The following table gives a description of the contacted population:

Table 2. The contacted population description

Region	Number of teachers
Grande Casablanca	40
Doukkala-Abda	30
Tadla-Azilal	10
Souss-Massa-Draâ	20
Total	100

7. Analysis of results and discussion

7.1. Results analysis of the questionnaire 1 and discussions

Among 200 students who have participated in the test, only 2 students have succeeded in activity 4, and none has found the correct answer to the activity 1. The second remark is that 68 % of students have succeeded in activity 2 and 80 % have succeeded in activity 3.

As a conclusion, we notice that students do well in activities which turn around "form" and cannot do well in activities which turn around "meaning". They are able to do the study of functions, and cannot attribute a spatial place to these elements in this study, in particular, to see the family of functions as plane sections of a surface. This shows that the students, although they are given the form, they cannot find the real meaning of the concept.

7.2. Results analysis of the questionnaire 2 and discussions

7.2.1. Analyses and discussions of the first part

The results analysis of the first part shows us that 25% of teachers use the ICT for the teaching of numerical functions of real variables. This weak use of ICT is due, according to the teachers, to:

- Lack of computers in school (20%)
- The time allocated to teaching mathematics is not sufficient (45%)
- Lack of training in the use of ICT for pedagogical purposes (35%)

7.2.2. Analysis and discussion of second part

The results analysis of the second part shows us that:

100% of questioned teachers have never seen similar activities as found in questionnaire 1 in their school textbooks. This confirms the hypothesis that form is given more importance than meaning in the teaching of mathematics in high school.

67% of questioned teachers wish to use this kind of activity, while 33% of questioned teachers are against the use of this activity because they do not know that ICT is a means to find the lost meaning in a concept. ICT play an important role to attribute a spatial place to a formal coded expression, but this role is ignored in school practices. Mathematics teachers are conscious of the role that ICT can play in the management of the dichotomy "meaning / form".

For a better management of this dichotomy, we deduce that the school textbooks must contain similar activities as these in questionnaire 1.

8. Conclusion and perspectives

At the end of this research, we well understand the conditions under which our system of education is asked to function within a wrong hypothesis which gives priority to the meaning to primary levels, and to the form to the superior levels, in order that students spontaneously engage in a dialectic "meaning/form".

In particular, we have been able, to adapt the Calculation of Assimilation to make of it a tool of structuring teaching contents which are able to preserve the meaning and form as two properties of mathematical object, concomitant, disjoint and exhaustive. Thus, we have established that it is impossible to preserve the meaning and the form of a numerical function such as the "logarithm function" without developing a "geometry" of this function.

Can we declare that the proposed tool based on the Calculation of Assimilations is able to correct the actual functioning of teaching system? And to give to the meaning and the form the places they deserve? Some of the experiences with students and their teachers give encouraging signs in this respect, but we must multiply this kind of experiences before confirming definitely this tendency.

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