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Stochastic poromechanical modeling of anthropogenic land subsidence

Massimiliano Ferronato [a,*], Giuseppe Gambolati [a], Pietro Teatini [a], Domenico Baù [b]

[a] *Department of Mathematical Methods and Models for Scientific Applications, University of Padova, via Belzoni 7, 35131 Padova, Italy*

[b] *Department of Geological Engineering and Sciences, Michigan Technological University, 1400 Townsend Drive, Houghton, MI, USA*

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Abstract

A key issue in poromechanical modeling, e.g. for predicting anthropogenic land subsidence due to fluid withdrawal, is the evaluation and use of representative mechanical properties for the deforming porous medium at a regional scale. One such property is the vertical uniaxial rock compressibility c_M which can be obtained through either laboratory oedometer tests or in situ measurements, and typically exhibits quite a marked scattering. This paper addresses the influence of the c_M uncertainty on the predicted land settlement using a stochastic simulation approach where c_M is regarded as a random variable and a large number of equally likely c_M realizations are generated and implemented into a poroelastic finite element model. A compressibility law, characterized by a log-normal distribution with depth-dependent mean, constant variance and exponential covariance, is assumed. The Monte Carlo simulation provides a set of responses which can be analyzed statistically. The results from a number of numerical experiments show how the c_M variance and covariance affect the reliability of the simulated land subsidence and provide a quantitative evaluation of the intrinsic uncertainty of the model prediction.

Keywords: Stochastic compressibility; Land subsidence; Finite elements; Monte Carlo simulation

* Corresponding author. Tel.: +39 049 8275929; fax: +39 049 8275995.
E-mail address: ferronat@dmsa.unipd.it (M. Ferronato).
URL: http://www.dmsa.unipd.it/~ferronat (M. Ferronato).

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1. Introduction

Anthropogenic land subsidence due to the production of subsurface fluids, such as oil, gas, or water, has been observed worldwide over the last few decades. Well-known examples of subsidence above compacting oil fields may be found for instance in Long Beach, California (Colazas and Strehle, 1995), in Venezuela (Finol and Sancevic, 1995), or in the North Sea Ekofisk field (Hermansen et al., 2000). In Italy the Northern Adriatic coastland has experienced a pronounced settlement due to both groundwater and gas removal (e.g. Gambolati et al., 1974, 1991; Baù et al., 2001).

The need for a more reliable prediction of the impact that the development of subsurface reservoirs may have on the ground surface has led to a continuous improvement of the numerical tools employed in poromechanics. At present the use of advanced models for the most accurate prediction of land subsidence can be considered quite a common effort (Ferronato et al., 2005; Gambolati et al., in press). However, although sophisticated poro-visco-plastic constitutive models have been developed for a realistic description of the actual soil behavior (e.g. Alonso et al., 1990; Coussy et al., 1998; Barthelemy and Dormieux, 2002), the geomechanical analysis over producing fields is usually performed at the macroscale, with the reservoir being regarded as a homogeneous structure from a mechanical point of view and the solution to the poro-elasto-plastic equations addressed deterministically. By distinction, the importance of describing the heterogeneous nature of the subsurface has been somewhat definitely recognized in the simulation of several geohydrodynamical processes. To overcome the limits of a deterministic approach, which would require an extensive medium characterization neither supported by the available data nor allowed by the available resources, the rock hydraulic heterogeneity at the field and regional scale has been incorporated stochastically into geostatistical models (Dagan, 1989; Gelhar, 1993). While the geostatistical approach has been extensively used over the last few decades for modeling flow and transport in random porous media, a limited number of works have addressed the influence of stochastic rock heterogeneity on stress and displacements (e.g. Darrag and Tawil, 1993; Griffith and Fenton, 2001; Frias et al., 2004). In particular, to our knowledge no study has been performed on the stochastic analysis of geomechanically heterogeneous porous media using actual field observations.

A most fundamental geomechanical parameter controlling the compaction caused by pore pressure drawdown in a depleted formation is the vertical uniaxial rock compressibility c_M. The parameter c_M is often measured in the laboratory on samples with a few centimeter size taken from exploratory wells, but many difficulties may arise when upscaling such data to the field macroscale. An alternative and promising technique relies on the measurement of the in situ compaction during the field production life by the radioactive marker technique. Originally developed more than 30 years ago (De Loos, 1973), the marker records allow for a straightforward rock mechanical characterization by relating the measured compaction to the pore pressure drawdown experienced by the depleted formation. Since 1992, the radioactive marker technique has been implemented by Eni-E&P, the Italian national oil company, in several offshore boreholes of the Northern Adriatic Sea (see Fig. 1), in order to derive a most reliable assessment of c_M. Marker data processing has provided a constitutive law of c_M at the basin scale in both the first and the second loading cycle (Baù et al., 2002). The significant scattering of the original measurements, however, required a statistical analysis, based on the weighted moving average method and on a logarithmic regression, with fairly large confidence intervals.

The present paper addresses the impact of the c_M uncertainty on the predicted land subsidence using a stochastic simulation approach where c_M is regarded as a spatial random variable. Experience suggests that in normally pressurized and consolidated sedimentary basins the rock mechanical properties usually exhibit a low horizontal variability and primarily depend on depth. Since the data from the Northern Adriatic basin fulfil such a requirement (Baù et al., 2002), c_M is assumed to vary with depth only. A stochastic 1-D ergodic process for the c_M generation is implemented into a Finite Element (FE) 3-D axial-symmetric poro-elastic model solved by a Monte Carlo simulation to predict anthropogenic land subsidence due to fluid

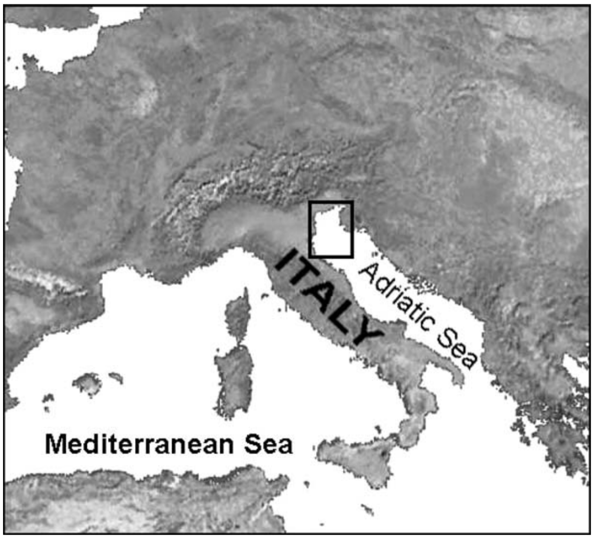

Fig. 1. The Mediterranean Sea. The geographic location of the Northern Adriatic basin is indicated by the panel.

withdrawal. It is worth noting that the 1-D stochastic generation of c_M, suggested by field observations, does not detract from the three-dimensionality of the analysis. Several numerical examples are discussed for a hydro-geological setting similar to that of the Northern Adriatic basin. A sensitivity analysis to the vertical c_M covariance, medium permeability and depth of the depleted formation is then performed. Finally, some conclusions are provided in order to describe the influence of the c_M stochastic characterization on the reliability of the predicted land subsidence and the potential of the proposed approach to address the geomechanical medium uncertainty of macroscale regional models.

2. Stochastic compressibility

The c_M value provided by the ith pair of markers located within a producing formation is estimated as

$$c_{M,i} = \frac{\Delta s_i}{s_{i,0}\Delta p_i} \tag{1}$$

where Δs_i is the shortening of the ith spacing due to the pore pressure drawdown Δp_i, and $s_{i,0}$ is the initial distance between the markers, i.e. about 10.5 m (Mobach and Gussinklo, 1994). Each $c_{M,i}$ value given by Eq. (1) can be associated to the vertical effective stress $\sigma_{z,i}$ at the average depth of the ith spacing, with the pairs ($c_{M,i}$; $\sigma_{z,i}$) thus obtained regressed to derive a constitutive relationship for the uniaxial vertical compressibility. Several Δs_i values are obtained during each monitoring survey in order to offset as much as possible instrumental and operational errors, so the corresponding set of $c_{M,i}$ values can be regarded as a sample from a statistical population of data. Hence each compressibility estimate is provided as an average value $\overline{c_{M,i}}$ with its standard deviation $\sigma_{c_{M,i}}$. Because of the large spatial variability of the original data, groups of adjacent measurements can be clustered together using the weighted moving average technique (Baù et al., 1999), thus allowing for narrower confidence intervals.

The analysis of the data distribution on an arithmetic plot reveals that the $\overline{c_{M,i}}$ values follow a non-linear trend. In particular the regression by a power law provides the best correlation index, so the constitutive relationship for the uniaxial compressibility takes on the general form

$$c_M = a\sigma_z^b \tag{2}$$

The regression by a power law is equivalent to a linear regression on a double log–log plot, i.e. a linear regression carried on the population of the logarithms of the data (Hald, 1952)

$$y(x) = \alpha + \beta(x - \bar{x}) \tag{3}$$

where y and x are the logarithmic transformation of c_M and σ_z, respectively, and $\bar{x}$ is the weighted mean of the x_i values. It should be noted that appropriate formulas must be used to transform the statistical population of the average $\overline{c_{M,i}}$ with the standard deviation $\sigma_{c_{M,i}}$ into the logarithms $\overline{y_i}$ and σ_{y_i} (Papoulis, 1965).

Eq. (3) provides the mean value for y. The empirical variance v^2 of the regressed y data is given by (Hald, 1952)

$$v^2 = \frac{\sum_{i=1}^{N}[\overline{y_i} - y(x_i)]^2}{N-2} \tag{4}$$

with N the total number of regressed values. Hence, the 95% confidence interval for y is obtained as

$$y = \alpha - \beta\bar{x} + \beta x \pm 2v \tag{5}$$

Back transformation of (5) to c_M provides

$$c_M = 10^{\alpha-\beta\bar{x}\pm 2v}\sigma_z^{\beta} \tag{6}$$

and comparison between Eqs. (2) and (6) shows that the power law coefficients a and b read

$$a = 10^{\alpha-\beta\bar{x}\pm 2v} \tag{7}$$

$$b = \beta \tag{8}$$

The above procedure has been used by Baù et al. (2002) to derive a constitutive relationship for c_M in the Northern Adriatic basin. The regression of the data obtained from 11 marker surveys in 3 instrumented boreholes during the period 1992–1999 provided the following constitutive model:

$$c_M = 1.0044 \times 10^{-2}\sigma_z^{-1.1347} \tag{9}$$

where c_M represents the expected (mean) value of the uniaxial compressibility. The 95% confidence interval, corresponding to $-2v$ and $+2v$ in Eq. (6), is defined by $a = 5.1602 \times 10^{-3}$ and 1.9546×10^{-2}, respectively. In Eq. (9) c_M is in [MPa^{-1}] and σ_z in [MPa]. A graphical representation of (9) is given in Fig. 2.

By the way it is calculated (see Eq. (7)), coefficient a of Eq. (2) is a log-normally distributed random variable with mean value and variance given by (Hald, 1952)

$$E(\log a) = \log\mu = \alpha - \beta\bar{x} \tag{10}$$

$$E\left[(\log a - \log\mu)^2\right] = \sigma^2 = v^2 \tag{11}$$

where α, β, and $\bar{x}$ are obtained from the regression (Eq. (3)) of the available sample of data, and v^2 is calculated by Eq. (4). The coefficient b (see Eq. (8)) is directly obtained from the regression procedure. For the Northern Adriatic basin, Eqs. (10) and (11) provide $\log\mu = -1.9981$, with $\beta = -1.1347$ and $\sigma^2 = 0.0209$.

The vertical effective stress σ_z in undisturbed conditions can be calculated as a function of the depth z. Using Terzaghi's effective stress principle (Terzaghi and Peck, 1967), we have

$$\sigma_z = \hat{\sigma}_z - p \tag{12}$$

where $\hat{\sigma}_z$ is the total vertical stress (compressive stresses are assumed to be positive). The total vertical stress can be calculated by means of the overburden gradient function obg(z), defined such that $\hat{\sigma}_z = z \cdot \mathrm{obg}(z)$. Assuming the basin to be normally pressurized and fully saturated with groundwater, the pore fluid pressure is calculated as $\gamma_w z$, where γ_w is the groundwater specific weight. Finally, Eq. (12) becomes

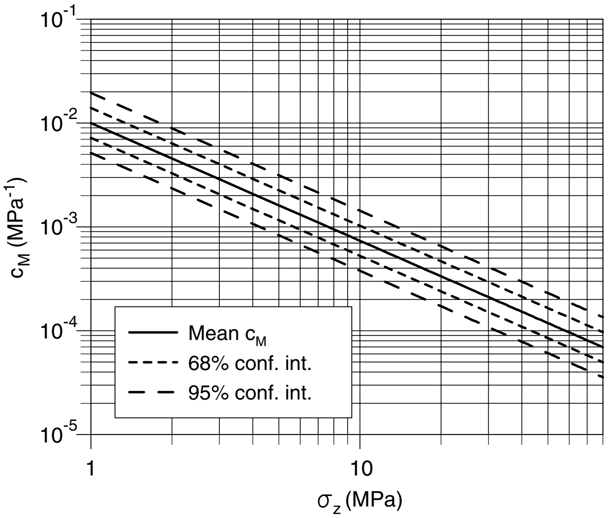

Fig. 2. Constitutive model of the vertical uniaxial compressibility for the Northern Adriatic basin (after Baù et al. (2002)). The dashed profiles show the 68% and 95% confidence intervals on a double log–log plot.

$$\sigma_z = [\mathrm{obg}(z) - \gamma_w]z \tag{13}$$

For the Northern Adriatic basin an average overburden gradient function has been estimated by density log surveys yielding the following expression for Eq. (13) (Baù et al., 2002):

$$\sigma_z = \left(1.2218 \times 10^{-2} \cdot z^{0.0766} - 9.8 \times 10^{-3}\right)z \tag{14}$$

In Eq. (14) z is in [m] and σ_z in [MPa]. Using σ_z from Eq. (14) in (9) allows for the calculation of c_M as a function of depth z only, so that c_M can be assumed to be constant within any horizontal layer.

In the analysis that follows, an ensemble of equally likely spatial distributions of c_M is obtained by assuming a as a linear (function of z only), stationary, and ergodic stochastic process (de Marsily, 1986), characterized by a log-normal distribution with mean value and variance as defined in Eqs. (10) and (11). The stochastic realizations of a are generated using the method of Rice (1954) as modified by Shinozuka and Jan (1972), and assuming an exponential covariance function

$$\mathrm{cov}[\log a(z_1), \log a(z_2)] = \sigma^2 \cdot \exp\left(-\frac{|z_1 - z_2|}{\lambda}\right) \tag{15}$$

where λ is the vertical correlation length.

3. Numerical experiments

The ensemble of spatial realizations of c_M generated with the correlated random line process are used in a test case to simulate land subsidence by a Monte Carlo simulation. The problem concerns a cylindrical porous volume with a radius of 8000 m and a height of 5000 m consisting of a sequence of alternating sandy and clayey layers with hydro-geological properties typical of the Northern Adriatic basin. Hydraulic conductivity of sand (k_{sand}) and clay (k_{clay}) is assumed to decrease with depth from 10^{-5} to 10^{-7} m/s, and from 10^{-9} to 10^{-11} m/s, respectively (Ferronato et al., 2004). The Poisson ratio ν is set to 0.3 (Teatini et al., 2000) and the grain compressibility to 1.63×10^{-5} MPa^{-1} (Geertsma, 1973). The lower boundary is fixed and impervious; the lateral boundary is fixed, too, with a zero pore pressure variation enforced on it. At the ground surface zero external stress and zero incremental pore pressure are prescribed (Baù et al., 2004).

A pumping of 1000 m^3/day, evenly distributed over a cylindrical volume with radius equal to 500 m, is assumed to take place from a sandy layer 20 m thick and 1000 m deep. In the present study we address land subsidence at the steady state, so that the problem is uncoupled with the pore pressure solution independent of c_M. Hence the incremental pressure field p is provided by the deterministic solution to the classical steady state flow equation

$$\nabla\left[K_{ij}\left(\frac{\nabla p}{\gamma_w}\right)\right] = f \tag{16}$$

where K_{ij} are the components of the hydraulic conductivity tensor and f the forcing source/sink function. The stochastic solution to the poromechanical problem is obtained in terms of the displacements $\mathbf{u} = (u_x, u_y, u_z)^T$ by solving the equations of equilibrium for an isotropic, generally heterogeneous, porous medium

$$G\nabla^2 u_i + (\Lambda + G)\frac{\partial(\mathrm{div}\,\mathbf{u})}{\partial i} = \frac{\partial p}{\partial i} \quad i = x, y, z \tag{17}$$

where the shear modulus G and the Lamé constant Λ are random variables related to c_M through the following:

$$G = \frac{1-2v}{2(1-v)c_M} \tag{18}$$

$$\Lambda = \frac{v}{(1-v)c_M} \tag{19}$$

The test problem is solved using an axi-symmetric configuration. The porous volume is discretized into annular elements with a 3-node triangular cross-section, totalizing 8096 nodes, 15834 elements, and 87 horizontal layers with thickness ranging between 10 and 250 m. The mesh structure is shown in Fig. 3 along with the boundary conditions imposed on displacements. The pore pressure field at steady state is obtained

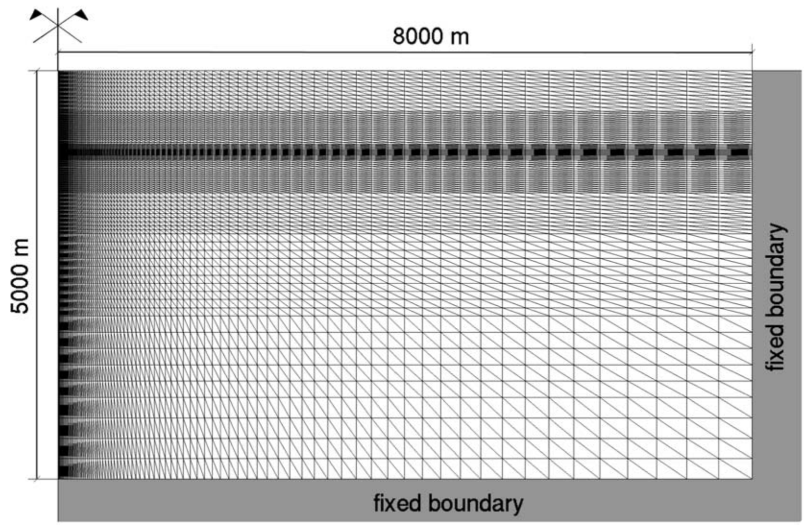

Fig. 3. Vertical cross-section of the FE mesh with the displacement boundary conditions.

by solving the flow equation (16) in a cylindrical reference frame via the FE method. The average pore pressure drawdown within the pumped volume turns out to be about 1.5 MPa. The pore pressure field thus obtained is used as an external source of strength in Eqs. (17) solved by FE with the total stress formulation (Gambolati et al., 2001) for each of the generated spatial realizations of c_M.

The stochastic simulation is performed using an ensemble of 1000 c_M realizations, with each c_M distribution generated over a vertical step of 5 m. For layers thicker than 5 m c_M is computed as the mean of the compressibility values comprised within those layer. An example of the difference between the original distribution of c_M generated every 5 m and the actual values used in the discretized FE layers of Fig. 3 is shown in Fig. 4. The number of c_M spatial realizations (1000) proves sufficiently large so as to reproduce at the mid-depth of each layer the average (expected) value and the 95% confidence interval prescribed by the constitutive law (9).

Since the c_M values are log-normally distributed because of the definitions of a and b (Eqs. (7) and (8)), and the displacement in a poroelastic model is linearly dependent on c_M, land subsidence is also expected to be log-normally distributed. The frequency distribution obtained with the Monte Carlo simulation is compared to the theoretical log-normal Probability Distribution Function (PDF) (Hald, 1952)

$$\phi(\eta) = \frac{M}{\sigma\eta}\varphi(u) \tag{20}$$

with $M = \log e = 0.4343$, η land subsidence, and $\varphi(u)$ the normal distribution function

$$\varphi(u) = \frac{1}{\sqrt{2\pi}\sigma}\mathrm{e}^{-\frac{u^2}{2}} \quad u = \frac{\log\eta - \log\mu}{\sigma} \tag{21}$$

Fig. 5 shows this comparison, assuming a correlation length λ (Eq. (15)) equal to 50 m, for two points located on the top surface, one at the symmetry axis ($r = 0$, maximum subsidence) and the other over the outer boundary of the depleted cylindrical volume ($r = 500$ m). As expected, land subsidence fits quite well into a log-normal distribution and the size of the confidence intervals decreases as we move farther from the symmetry axis.

An additional piece of information provided by the stochastic approach is the probability that at some points of particular interest the expected subsidence will exceed a limit threshold value η_t. As is well known, the probability that $\eta > \eta_t$ is obtained by integrating over the point of interest the PDF between η_t and $+\infty$

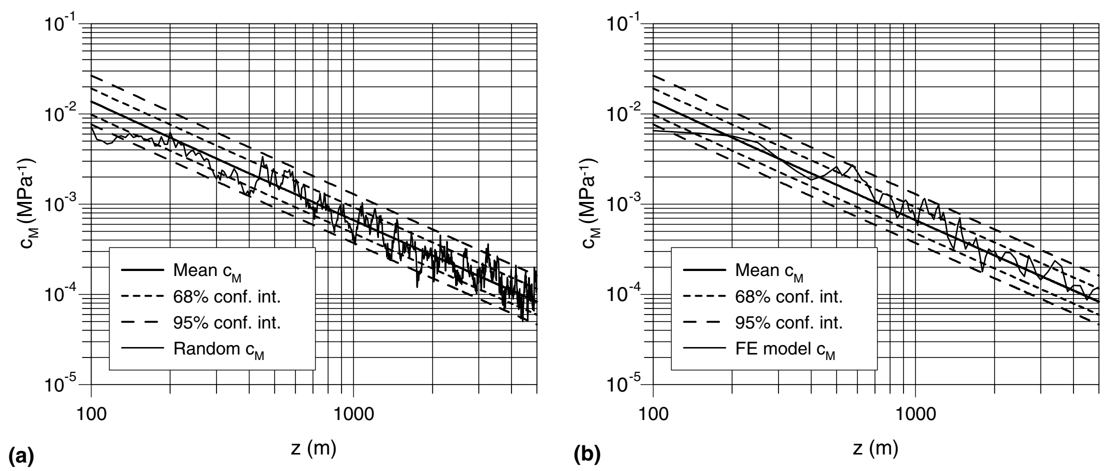

Fig. 4. Example of a stochastic realization for c_M obtained with $\lambda = 50$ m: (a) c_M values generated every 5 m; (b) c_M values prescribed in each mesh layer.

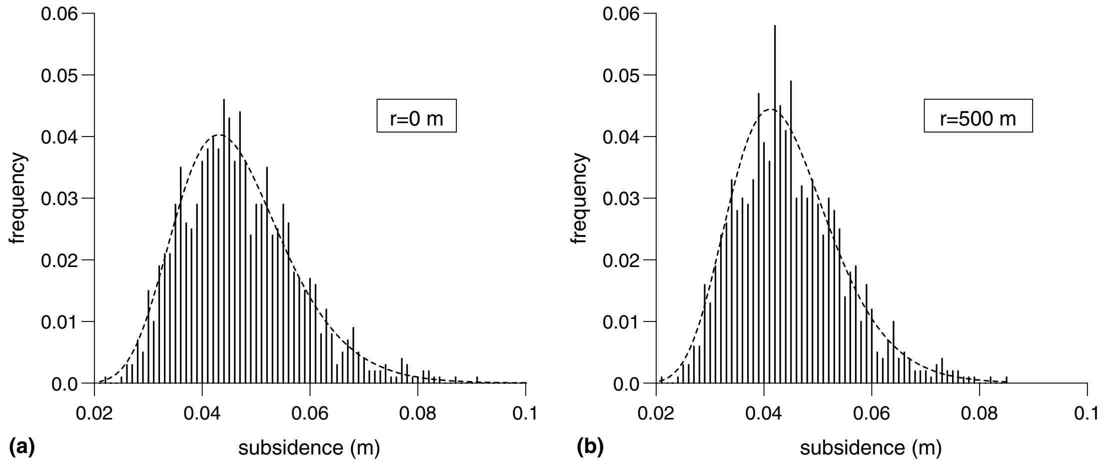

Fig. 5. Frequency plot (linebars) and theoretical (dashed profile) PDF of land subsidence at: (a) surface point with maximum subsidence ($r = 0$ m); (b) surface point over the outer boundary of the depleted cylindrical volume ($r = 500$ m).

$$P(\eta > \eta_t) = \int_{\eta_t}^{+\infty} \phi(\eta)\mathrm{d}\eta \tag{22}$$

As an example, consider as a threshold value $\eta_t = 5$ cm. At $r = 0$, the mean value for η is 4.65 cm and the 95% confidence interval is $2.88 < \eta < 7.14$ cm. The probability of exceedance $P(\eta > \eta_t)$ turns out to be 34%. At $r = 2000$ m, for instance, where the mean value for η is 3.00 cm and the 95% confidence interval is $2.02 < \eta < 4.32$ cm, $P(\eta > \eta_t)$ turns out to be only 0.1%.

3.1. Sensitivity analysis

To gain a better insight on the impact of the c_M uncertainty on the land subsidence prediction in different field conditions, a sensitivity analysis has been performed for the following parameters:

(1) the correlation length λ;
(2) the permeability contrast $\kappa = k_{\mathrm{sand}}/k_{\mathrm{clay}}$;
(3) the pumped aquifer depth h.

3.1.1. Correlation length

The influence of the correlation length λ is studied for the limiting cases $\lambda \to 0$ and $\lambda \to \infty$. An example of the generated random c_M vs. z for various λ values is provided in Fig. 6 which helps understand how the correlation length affects the c_M realizations. The 95% confidence intervals for the predicted land subsidence are given in Fig. 7 along with the subsidence profiles obtained from the c_M constitutive law of Eq. (9) with $a = 1.0044 \times 10^{-2}$ (mean value), and $a = 5.1602 \times 10^{-3}$ and 1.9546×10^{-2} (95% confidence interval limits). When $\lambda \to 0$, practically obtained by setting $\lambda = 1$ m, the c_M in adjacent layers is not correlated and the 95% confidence interval turns out to be quite narrow. By contrast, as $\lambda \to \infty$, practically obtained by setting $\lambda > 10\,000$ m, all c_M are correlated and the stochastic simulation provide almost the same 95% confidence interval as the one obtained with the extreme profiles of Fig. 2. The latter turns out to be the most conservative assumption, as it provides the largest confidence interval.

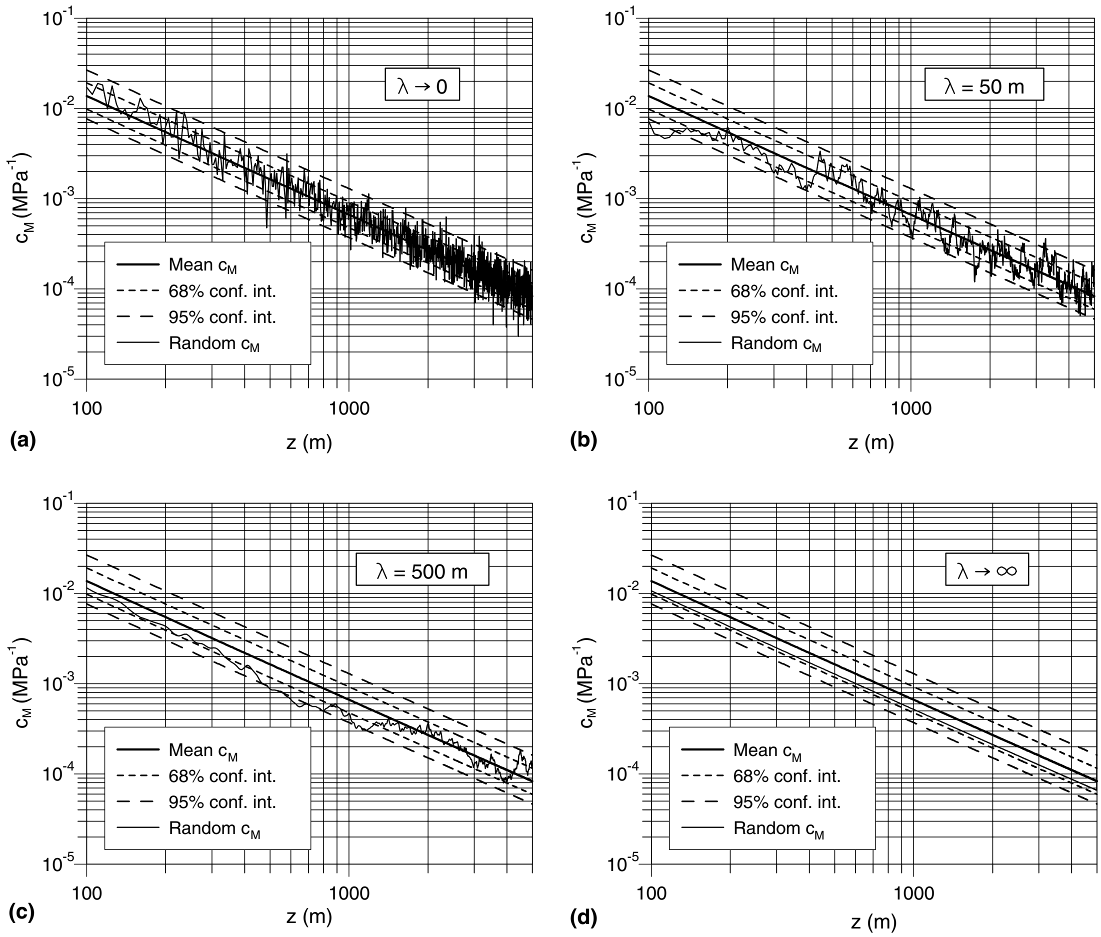

Fig. 6. Examples of c_M stochastic realizations with: (a) $\lambda \to 0$ m; (b) $\lambda = 50$ m ; (c) $\lambda = 500$ m; (d) $\lambda \to \infty$.

3.1.2. Permeability contrast

The hydraulic conductivity assumed for sand and clay in each test problem is summarized in Table 1 along with the ratio κ. Groundwater withdrawal is calibrated so as to achieve in any case a maximum pore pressure drawdown approximately equal to 1.5 MPa within the pumped sandy formation. Hence, the effect of decreasing κ is that of increasing the volume of the depleted porous medium, as is shown in Fig. 8 which provides the extent of the steady state drawdown in three test cases around the pumped aquifer.

The results obtained from the Monte Carlo simulation with $\lambda = 50$ m are shown in Fig. 9. To perform a meaningful comparison, land subsidence $\eta(r)$ is normalized with respect to $\bar{\eta}(r)$, i.e. the mean value at the radial distance r. Fig. 9a suggests that the prediction uncertainty, as quantified by the amplitude of the 95% confidence interval, decreases as a larger porous volume is depleted. This is further evidenced in Fig. 9b, where narrower PDFs of the normalized land subsidence at $r = 0$ correspond to smaller κ values. This behavior is due to the fact that land subsidence primarily depends on the compaction of the depleted medium. For smaller κ, the overall compaction is controlled by the c_M of a larger set of adjacent layers and, as is known from statistics, the size of the corresponding confidence interval decreases as the number of random values within the group increases.

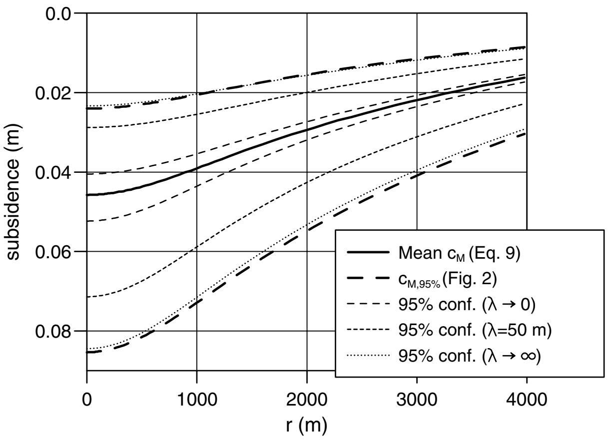

Fig. 7. Land subsidence obtained with mean c_M and the most extreme profiles of Fig. 2 (thick profiles), and land subsidence obtained with the Monte Carlo simulation (thin profiles) and different λ.

Table 1
Hydraulic conductivity of sand and clay in the numerical test problems. The ratio k_{sand}/k_{clay} is denoted by κ

	k_{sand} [m/s]	k_{clay} [m/s]	κ
Base case	10^{-5}–10^{-7}	10^{-9}–10^{-11}	10000
k_1	10^{-5}–10^{-7}	10^{-8}–10^{-10}	1000
k_2	10^{-5}–10^{-7}	10^{-7}–10^{-9}	100

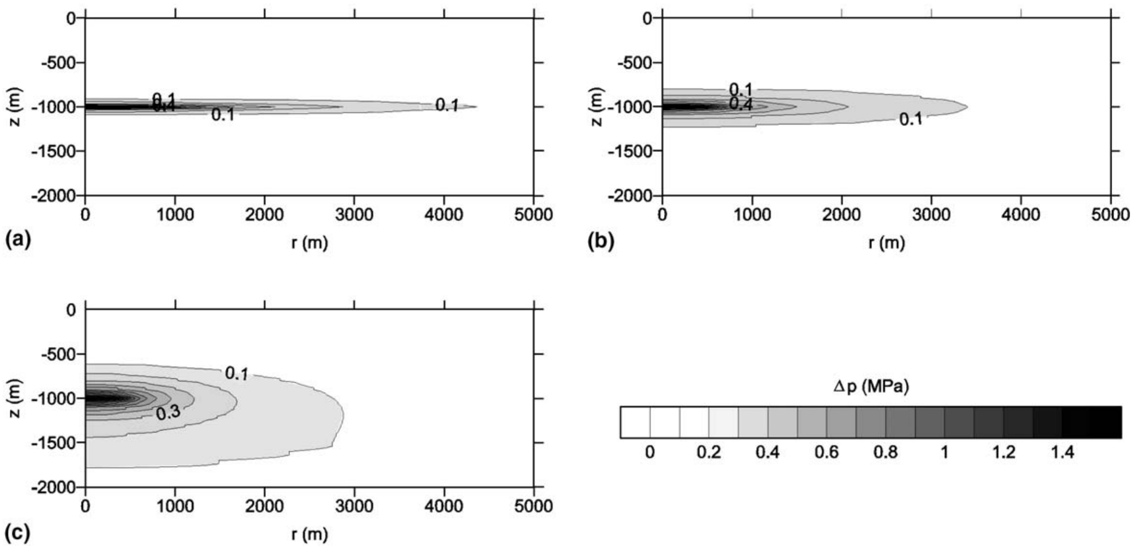

Fig. 8. Pore pressure drawdown (in MPa) for the test cases: (a) base case; (b) k_1; (c) k_2. Groundwater pumping is calibrated so as to attain a maximum pore pressure drawdown of 1.5 MPa.

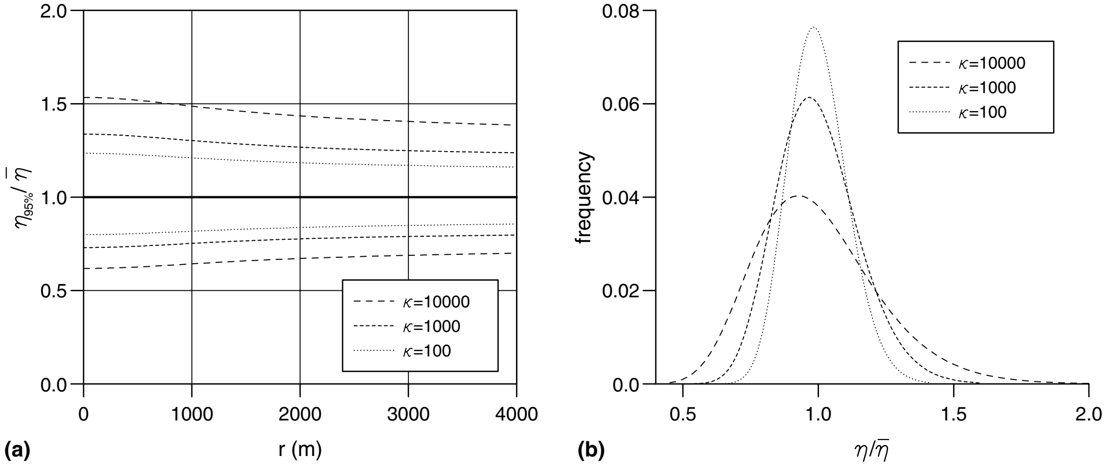

Fig. 9. (a) 95% confidence intervals of the normalized land subsidence obtained with the Monte Carlo simulation and different κ values; (b) PDF of the normalized land subsidence at the symmetry axis.

3.1.3. Aquifer depth

Two further examples are discussed by using a pumped aquifer depth h of 500 m and 2000 m. As was done previously, the normalized land subsidence $\eta(r)/\bar{\eta}(r)$ is considered in order to compare meaningfully the different test cases. Fig. 10 points to a very small difference among the test cases, with an almost negligible decrease of the 95% confidence interval size as h increases. This is accounted for by the general form of Eq. (2), where a is the actual stochastic parameter with constant mean and variance. As h increases, σ_z increases too and σ_z^b decreases, being the exponent b negative. The most notable consequence is that the size of the confidence interval associated with c_M decreases with depth, hence a reduced uncertainty is expected as the depth of the depleted formation increases. Because of the exponential form of (2), however, such a variation is very small for the depth of usual interest. Thus we can conclude that the depth h plays a

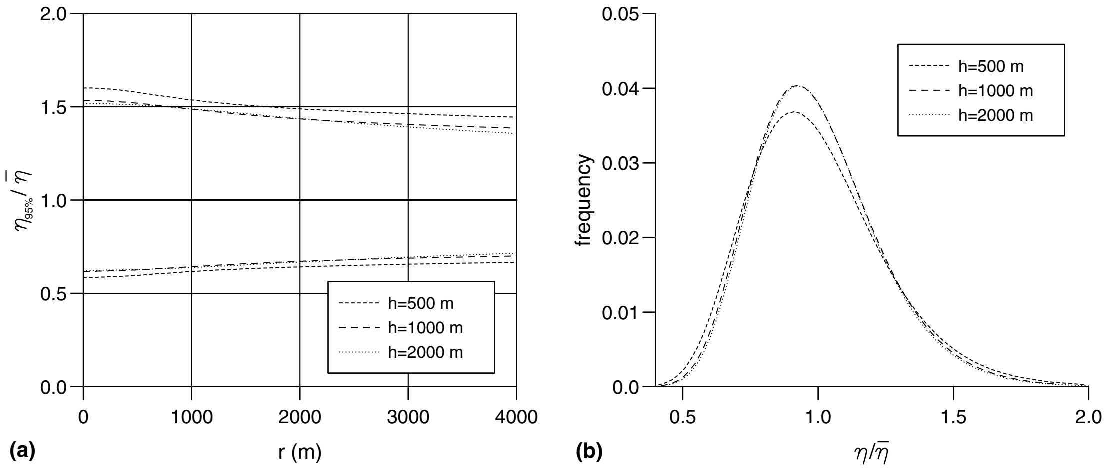

Fig. 10. (a) 95% confidence intervals of the normalized land subsidence obtained with the Monte Carlo simulation and different h values; (b) PDF of the normalized land subsidence at the symmetry axis.

negligible role on the uncertainty of η as is related to the stochastic geomechanical properties of the porous medium.

4. Conclusions

The present paper addresses the influence of the geomechanical uncertainty of the porous medium on the prediction of anthropogenic land subsidence due to subsurface fluid withdrawal. The study focuses on the vertical uniaxial rock compressibility c_M and uses a stochastic approach where c_M is regarded as a random spatial process. An axi-symmetric aquifer system is simulated with a stratified hydro-geological setting typical of a sedimentary basin with the geomechanical properties taken from the Northern Adriatic, Italy. A basin-scale constitutive law for c_M, log-normally distributed with depth-dependent mean and constant variance, is derived from the statistical analysis of available in situ marker measurements (Baù et al., 2002) and implemented into a poroelastic stochastic FE model based on a Monte Carlo simulation of an ensemble of 1000 realizations.

A sensitivity analysis to the c_M vertical correlation length, permeability contrast between sand and clay, and depleted aquifer depth is performed. The following results are worth summarizing:

- the correlation length λ has a significant impact on the amplitude of the 95% confidence interval, which increases as also λ increases. In the limiting case with $\lambda \to \infty$ the largest confidence interval is obtained, i.e. the one derived by Baù et al. (2002);
- a larger permeability contrast helps reduce the volume of the depleted porous medium, and hence increase the uncertainty of the resulting land subsidence;
- the uncertainty of land subsidence prediction proves almost insensitive to the aquifer depth.

Finally, the results point out that the c_M stochastic characterization may help define the quality of the simulation and represent a first contribution to the evaluation of the geomechanical uncertainty on the reliability of the predicted land subsidence. New on-going research is addressing the influence of the c_M uncertainty on the transient land subsidence where the flow field is also affected by the geomechanical response of the porous medium.

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