A model for the mass-number independence of the antiproton annihilation on nuclei at low energies

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Abstract

A simple model explaining the recently observed approximate independence of the annihilation cross section on light nuclei at low energies is proposed. The salient idea is based on the realization that the \( \pi \) from the annihilation on a nucleon have energies in the region of the \( \Delta(1232) \) resonance. The coherent propagation of these \( \pi \) through the excitation of several \( \Delta \) resonances results in a destructive interference explaining why the annihilation of antiprotons in nuclei is suppressed. This model suggests a very effective way to produce "\( \Delta \) matter" with several \( \Delta \) resonances in interaction.

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1. Introduction

A recently published result from the Low Energy Antiproton Ring LEAR at CERN showed a puzzling phenomenon for the annihilation cross sections of antiprotons on light nuclei. The OBELIX Collaboration measured the total annihilation cross section \( \bar{p}D \) and \( \bar{p}^4\text{He} \) at small momenta \( p_{\text{lab}} \lesssim 50 \text{ MeV/c} \) and found that these are smaller than the expected geometrical behavior \( \sigma_{\text{annih}} \approx \sigma_{\text{annih}}(\bar{p}p)A^{2/3}/k \) [1].

A similar effect is found for \( \bar{p}^{20}\text{Ne} \) [2,3]. However, before one can draw conclusions one has to subtract the influence of the Coulomb interaction whose acceleration of the \( \bar{p} \) in the nuclear field becomes very important at small momenta [2,4].

A related result is the determination of the imaginary part of the \( \bar{p}D \) and \( \bar{p}^4\text{He} \) scattering lengths and volume from the shifts and widths of antiprotonic atomic levels. They appear also smaller than those of the \( \bar{p}p \) interaction [5].

A combined analysis of these results by Protasov et al. [6] which includes a proper separation of the Coulomb interaction confirms the surprising result. They derive for the imaginary part \( b_0 \) of the complex s-wave-scattering length \( A_0 = a_0 + ib_0 \)

\[
\begin{align*}
\bar{p}p & \quad b_0 = -0.694 \pm 0.027 \text{ fm}, \\
\bar{p}D & \quad b_0 = -0.62 \pm 0.02 \text{(stat)} \pm 0.04 \text{(syst)}, \\
\bar{p}^4\text{He} & \quad b_0 = -0.36 \pm 0.03 \text{(stat) }^{+0.19}_{-0.11} \text{(syst)}. 
\end{align*}
\]

A similar result holds for the p-wave-scattering volumes for which \( b_1 = -0.75 \pm 0.06 \text{ fm}^3 \) for \( \bar{p}p \) and \( b_1 \approx \text{constant} \approx -4 \text{ fm}^3 \) for mass numbers \( 2 \leq A \leq 7 \) is obtained [6]. Since \( \sigma_{\text{annih},l} = \sigma_{r,l} \approx \pi \lambda^2(2l+1) \) at low momenta, it follows in contradiction to the expected geometrical behavior \( \sigma(\bar{p}p) > \sigma(\bar{p}D) > \sigma(\bar{p}^4\text{He}) \). This finding is considered to be
“mysterious” [7] and no explanation seems to be at hand.

A conflicting result for the annihilation of antineutrons on nuclei has been recently presented [8]. This investigation, also from a group of the OBELIX Collaboration, used a special low momentum antineutron beam produced by the \( \vec{p} \rightarrow \vec{n} \) charge exchange reaction [9]. They found that the cross sections were \( \propto A^{2/3} \) over the range from the lowest \( n \) momenta at threshold with \( p_n = 50 \text{ MeV/c} \) (\( p_p = 100 \text{ MeV/c} \)) 50 MeV/c to 500 MeV/c. However, a careful analysis of the antineutron spectra published [9] and used [10] shows that they have an \( \approx 1/p_n \) behavior from \( p_n = 50 \text{ MeV/c} \) to \( \approx 100 \text{ MeV/c} \) which can be excluded due to the \( (p_p - 100 \text{ MeV/c})/p_{p,0} \) dependence of the charge exchange cross section at threshold [11]. The low momentum antineutrons with 50 MeV/c < \( p_n < 100 \text{ MeV/c} \) in this experiment are possibly due to a contamination of high momentum antineutrons faking the \( \propto A^{2/3} \) dependence.

In this Letter it is proposed that a suppression of the antinucleon annihilation on nuclei can be understood by considering the coherent rescattering of the antinucleon annihilation on nuclei can be understood by considering the coherent rescattering of the antinucleon annihilation on nuclei can be understood by considering the coherent rescattering of the antinucleon annihilation on nuclei can be understood by considering the coherent rescattering of the antinucleon annihilation on nuclei. It is felt that this is necessary since there are frequent confusions of notions like “black disc”, large phase shifts and absorption parameters etc. in the literature which obscure the simple facts about annihilation at low energies. It follows the presentation of the model and a discussion of its consequences.

2. Basic antinucleon–nucleon annihilation

A rather good description of the antinucleon–nucleon cross sections at low momenta is given by the complex scattering length with \( a_0 = b_0 = -0.69 \text{ fm} \) and volume \( a_1 = 0, b_1 = 0.75 \text{ fm}^3 \) [6] for \( p_{lab} \leq 200 \text{ MeV/c} \) when the “zero-effective range” approximation holds. For the model discussed here the Coulomb interaction is “switched off” and, therefore, the cross sections are given by:

\[
\sigma_{e,l} = \pi k^2 (2l + 1) |e^{2i\Delta_l} - 1|^2, \tag{1}
\]

\[
\sigma_{r,l} = \pi k^2 (2l + 1) (1 - |e^{2i\Delta_l}|^2), \tag{2}
\]

where \( e^{2i\Delta_l} = e^{2(i\delta_l + y)} = \eta_l e^{2ib_l} \) with the usual absorption parameter \( \eta_l \) and phase shift \( \delta_l \). The connection between \( \eta_l, \delta_l \) and the complex scattering length \( A_l = a_l + ib_l \) is given by:

\[
\eta_l e^{2ib_l} = \frac{i + A_l k^{2l+1}}{i - A_l k^{2l+1}}. \tag{3}
\]

Using the values of \( a_l \) and \( b_l \) cited above the cross sections plotted in Fig. 1 result. It is interesting to depict these cross sections in the classical plot of the elastic \( \sigma_e \) versus the reaction, i.e., annihilation cross section \( \sigma_r \), and see what absorption parameters and phase shifts result. Fig. 2 shows this plot. From this follows that the antinucleon–nucleon scattering at low energies is characterized by small \( \delta_l \). This means that this scattering is a “diffractive scattering” from a gray disc of two partial waves only [13]. The reaction cross section \( \sigma_r \) is large due to the \( 1/k \) dependence, but
does not exhaust the unitarity limit. Of course, the
description of the reaction with a complex scattering
length and a “zero-effective range” breaks down for
$k \gtrsim 1$ fm$^{-1}$.

A further fact which will be used in the following is
the $\pi$ multiplicity of the annihilation cross section.
This multiplicity can be described reasonably well by
the distribution function

$$P(n) = \frac{1}{\sqrt{2\pi} D} e^{-(n-(n_\pi))^2/2D^2}, \quad (4)$$

where $\langle n_\pi \rangle = 5$ and $D^2 = 0.95$ [12]. This means that it
is sufficient to consider $\pi$ multiplicities of $2 \leq n \leq 7$.
Additionally the momentum spectra of the $\pi$ will be
needed. They are mostly not known for the each mul-
tiplicity separately. But it will turn out that it suffices to
use the sum spectrum over all multiplicities parameter-
ized by the Maxwell–Boltzmann distribution

$$dN/dp = C(p^2/\epsilon)e^{-p/\epsilon}, \quad (5)$$

where $C$ is a normalization constant, $\epsilon$ is the energy
of the $\pi$s and $\epsilon_0$ a “temperature” parameter [14]. For
$\epsilon_0 = 100$ MeV has been chosen.

Finally, it is mentioned that the range of the annihi-
lation is $r_{\text{annih}} \approx 1.2$ fm [15] and the annihilation on
nuclei happens in their surface. Since the life length of
the $\rho$ is about 1.3 fm it will decay into $\pi$s in the anni-
hilation volume and no intermediate $\rho$s are considered
in the following model.

3. Model

Considering these characteristics the annihilation
on a nucleus can be described in the following way.
The antiproton annihilates with a very large cross
section on one nucleon emitting several $\pi$s with a
multiplicity distribution given by Eq. (4) leaving a
system of $(A-1)$ nucleons. We shall distinguish two
situations:

(a) $E_{\bar{p}} < \epsilon_B$. The kinetic energy of the antiproton
$E_{\bar{p}}$ is smaller than the binding energy $\epsilon_B$ of the
nucleon. In this case the whole nucleus has to
take the momentum of the antiproton and the
annihilation center moves with the off-shell Fermi
momentum $k_F$. However, no energy is transferred
to the annihilation $\pi$s. The $\pi$ wave functions
are spherical waves originating in the annihilation
center with a wave vector $\vec{k} = \vec{k}_n + \vec{k}_F$ where $\vec{k}_n$ is
the wave vector of the $\pi$ decay channel with
multiplicity $n$.

(b) $E_{\bar{p}} > \epsilon_B$. In this case the annihilation center
takes an on-shell momentum $k_{\bar{p}}/2$ and an kinetic
energy of $T = k_{\bar{p}}^2/4m_N$ which is available for
a distribution to the $\pi$ energies. Consequently
the on-shell momentum of the $\pi$s can now be
changed by the Fermi momentum according to
$\vec{k} = \vec{k}_n + \vec{k}_{\bar{p}} + \vec{k}_{\text{Fermi}}$. The $\pi$ wave functions
are now distorted spherical waves with this wave
vector.

The total transition rate $\dot{w}_{\text{annih}}$ of the annihilation of
one $\bar{p}$ on one nucleon $N$ and the subsequent interac-
tion of $n$ $\pi$s with the $(A-1)$ nucleus is given by:

$$\dot{w}_{\text{annih}} \propto \langle \bar{p}N|O|\prod_n \pi_n \rangle \cdot \prod_n \langle \pi_n | \pi_n \otimes (A-1) \rangle^2, \quad (6)$$

where $O$ is the elementary annihilation operator. The
free wave function of each of the $\pi$s after the anni-
hilation without the presence of a nucleus is given by
the spherical wave for case (a) or a distorted spherical wave for case (b):

$$|\pi_n\rangle = \frac{A e^{i(k\bar{r} - wt)}}{r},$$

(7)

where $\bar{r}$ is the spherical coordinate with the origin in the annihilation center. The $\vec{k}$s are the wave vectors of the $\pi$s constrained by kinematics. Due to the lack of multiplicity separated momentum spectra of $\bar{p}N$ annihilation two variants of these spectra have been used. In the first (variant I), the energy of the $\pi$s is given by the sharp energies $\epsilon_n = \sqrt{m_N^2 + p^2} = 2 \cdot m_N/n$ where $m_N$ and $m_\pi$ are the masses of the nucleon and $\pi$, respectively. In the second (variant II), the momentum distribution of the $\pi$s given by Eq. (5) has been applied. The energy distribution of the $\pi$s is in the range $0 < \epsilon < 1000$ MeV with a maximum at 250 MeV/c. This is is just the range of the $\pi$ absorption through the $\Delta(1232)$ from the maximum at $\omega_N = 297$ MeV to the resonance tails. Due to the large cross section of this resonance and the large geometrical probability of absorption, the annihilation $\pi$s will excite this resonance with a large probability before they escape. As will be shown in the following the phase shifts of the $\pi$ wave caused by the absorption and re-emission produce a destructive interference explaining the observed small cross section for antiproton annihilation on light nuclei.

In a hybrid model one could replace the $\bar{p}$ wave in the $|\bar{p}N\rangle$ state by a $\bar{p}$ wave calculated in an optical model for the $\bar{p}A$ system. However, since the coupled channels simulated by the optical model are explicitly considered in the second matrix element of Eq. (6) such an approach contains an element of double counting. It is the aim of the model of this Letter to propose a microscopic mechanism and not to produce a fit which would be badly founded given the still limited data anyhow. A study of the “saturation of low-energy antiproton annihilation on nuclei” in the framework of a pure optical model has been performed in Refs. [16,17].

At first the momentum case (a) defined above is considered. In the presence of the nucleus $(A - 1)$ the $\pi$ propagates and is rescattered by the $\Delta(1232)$ resonance yielding a wave function:

$$|\pi_n\rangle = \frac{B e^{i(k\bar{r} - wt)}}{r} + C \frac{e^{i(k|\vec{r} - \bar{r}_n| - wt)}}{|\vec{r} - \bar{r}_n|} |L(\omega)| e^{i\phi(\omega)}.$$  

(8)

where $\bar{r}_n$ defines the position of the scatterer in the nucleus, $|L(\omega)|$ is the absolute value of the Breit–Wigner amplitude of the $\Delta(1232)$, and $\phi(\omega)$ the phase advance by the absorption and re-emission through the resonance with $\omega = \epsilon$. Since the wave length of the annihilation $\pi$s is of the order $\lambda = 2\pi/k \approx 5$ fm and $r_s = |\bar{r}_s| \approx 1.8$ fm the average distance of the nucleons in a nucleus, no asymptotically free $\pi$ wave is developing. Therefore, the phase advance through the resonance

$$\phi(\omega) = \arctan \left( \frac{\omega_0 - \omega}{\Gamma/2} \right)$$

(9)

and not the free scattering phase shift has to be taken. It is noted that the measured phase shift would not apply for the scattering of a spherical wave given here.

For very large $|\vec{r}| \gg r_s$ one can neglect the different positions of the scatterers, i.e., nucleons, and gets

$$|\pi_n\rangle \rightarrow B \frac{e^{i(k\bar{r} - wt)}}{r} \left( 1 + a_\pi e^{-ikr_s} |L(\omega)| e^{i\phi(\omega)} \right),$$

(10)

where $a_\pi$ is the $\pi$ absorption probability. The phase factor $e^{-ikr_s}$ reflects the phase advance due to the pathlength difference from one scatterer to the other. The second term in the brackets can be understood as the rescattering probability.

Before the rescattering amplitudes can be summed up the absorption probability $a_\pi$ and the number of rescatters $n_0$ have to be determined. Since the absorption of the $\pi$s happens in the near field of the spherical wave the measured cross sections can be used as an educated guess only. Starting from such estimates, $a_\pi$ and $n_0$ have to be regarded as model parameters. The measured cross section for $\pi^0$ absorption on nucleons is $\sigma_{abs} = 14$ fm$^2$ [18]. If one considers a sphere with radius $r_s = 1.8$ fm one can estimate $a_\pi \approx 14$ fm$^2/4\pi r_s^2 \approx 0.34$. The value $a_\pi = 0.3$ has been used in the calculations. The model results change somewhat with $a_\pi$ as will be discussed further below.

The number of rescatters $n_0$ before the $\pi$ escapes can be estimated by the random walk of the $\pi$ through the $(A - 1)$ residual nucleus [19]. Using $R = 1.1$ fm $(A - 1)^{1/3}$ for the nuclear radius one gets $n_0 = \langle R \sigma_{abs} \rangle^2 \approx 6 \cdot (A - 1)^{2/3}$. Since the annihilation happens in the surface of the nucleus only about half of this number can be taken. Additionally for very light
nuclei the average density \( \rho \) is smaller. Therefore, for the calculations \( n_0 = 1 \cdot (A - 1)^{2/3} \) as a sensible approximation for light nuclei has been adopted. For heavier nuclei \( n_0 \) is large and the result is insensitive to this number.

With this preparation, the sum of rescattering amplitudes can now be written down

\[
G(\omega, A) = (1 + q + q^2 + q^3 + \cdots + q^{n_0})
\]

where

\[
q = q(\omega) = \alpha_{\pi} e^{-i k_{\pi} r_s} |L(\omega)| e^{i \phi(\omega)}.
\]

The wave vector \( k = |\vec{k}_n + \vec{k}_\| \) has to be averaged over the internal Fermi momenta \( k_I \) with \( |k_I| < k_{\text{Fermi}} = 1.25 \text{ fm}^{-1} \). Finally, \( |G(\epsilon, A)|^2 \) is either summed over \( \epsilon_\pi \) (variant I) or averaged over \( \epsilon \) with the momentum distribution in Eq. (5) (variant II). The cross section is then calculated with

\[
\sigma_{\text{annih}} = \sigma_0 \left( \sum_{n=2}^{7} P(n) |G(A)|^{2 n_\Delta} \right) A^{2/3},
\]

where \( n_\Delta = n_\Delta(n, A) \) is the number of \( \Delta \)s produced and is calculated using the \( \pi \) absorption probability and the combinatorics of available nucleons. Again only one half of the \( \pi \)s are considered. It is assumed that each \( \pi \) belonging to a \( \Delta \) resonance follows the same rescattering chain and, consequently, the probability amplitudes of each chain have to be multiplied. As mentioned for a strongly absorbing disc one expects a dependence \( \sigma_{\text{annih}} \propto R^2 \propto A^{2/3} \). The reduction of the cross section with reference to this expectation is clearly seen in Fig. 3.

Finally, one has to consider the case (b) when \( E_{\bar{p}} > \epsilon_R \). For this situation the primary spherical wave from the annihilation moves on-shell with \( \vec{k}_{\bar{p}} + \vec{k}_{\text{Fermi}} \) with respect to the scatterers, i.e., nucleons, in the nucleus. Therefore, the second matrix element in Eq. (6) will be 1 since the spherical waves of the rescattering will destructively interfere with the primary wave at large distances. Consequently, no suppression is expected for momenta above the given condition.

4. Discussion

The cross section for the deuteron is somewhat to large since the rescattering mechanism from the one nucleon left after the annihilation is not yet very effective. However, Wycech et al. [20] have shown that in the deuteron a sufficient reduction is produced by the interference of the amplitudes of the initial rescattering of the \( \bar{p} \) from the proton and neutron.

The only sensitivity of the model is in the pion absorption probability \( \alpha_{\pi} \). For \( ^4\text{He} \) the normalized cross section changes as follows \( \alpha_{\pi} \), normalized cross section for variant I: \( \{0.2, 2.0\}, \{0.3, 1.6\}, \{0.4, 1.2\} \). Since the model proposed here is “schematic” a fit to data does not make sense. A more sophisticated study including other rescattering diagrams and nucleus excitations would be necessary. The \( \pi N \) knock-out and \( \Delta N \rightarrow NN \) reactions are, however, suppressed due to form factors in the cold nucleus of case (a) whereas they get important in case (b). Cases (a) and (b) can be identified with the “recoil less” and “quasi free” cases in Ref. [19], respectively. A less schematic study of the coherent propagation is complicated since it cannot be based on the frequently used “infinite matter” calculations with plane waves. The effects of the finite nucleus and the spherical waves are essential.

The model shows that a small annihilation cross section on light nuclei could be understood. More con-
clusive experiments with low energy antiprotons are, therefore, called for. The salient point is the coherent propagation of the hadrons in the cold nucleus. It appears that this situation should be considered in general as more favorable for searching for medium effects of hadrons in nuclei.

Finally, it should be mentioned that the mechanism of the model suggests the production of systems of several interacting $\Delta$s. They can be "mixed" with a choosable number of nucleons. The situation is more favorable than for the quasi free $pA$, $AA$ or $\bar{p}A$ reactions at higher energies since, as argued, the off-shell Fermi smearing of the decay $\pi$s vanishes for small $\bar{p}$ momenta outside the nucleus. Additionally, the coherent process of the $\pi^-$ rescattering in the final state suggested by this model implies that other final state interactions are not important. Therefore, the $\pi$ and nucleon multiplicities, their energy spectra and the invariant $\Delta$ mass spectra at low $\bar{p}$ momenta will give a new access to this highly intriguing system. An experimental set up with close to $4\pi$ acceptance for charged and neutral $\pi$s and nucleons, but with only moderate energy resolution, would be adequate for such a study.

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