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Two-loop renormalization factors of dimension-six proton decay operators in the supersymmetric standard models

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ABSTRACT

The renormalization factors of the dimension-six effective operators for proton decay are evaluated at two-loop level in the supersymmetric grand unified theories. For this purpose, we use the previous results in which the quantum corrections to the effective Kähler potential are evaluated at two-loop level. Numerical values for the factors are presented in the case of the minimal supersymmetric SU(5) grand unified model. We also derive a simple formula for the one-loop renormalization factors for any higher-dimensional operators in the Kähler potential, assuming that they are induced by the gauge interactions.

1. Introduction

Discovery of the Higgs boson [1,2] may suggest the existence of supersymmetry (SUSY). The supersymmetric theories may accommodate the hierarchical structure with the great desert naturally. Searches for rare processes, such as proton decay, would be one of methods to access the physics beyond the supersymmetric standard models (SUSY SMs). The processes are dictated with the effective higher-dimensional operators. When comparing the prediction with the observation precisely, we need to include the radiative corrections correctly.

The realization of the gauge coupling unification strongly motivates us to study the supersymmetric grand unified theories (SUSY GUTs) [3–6]. In the theories, proton decay is induced by the exchanges of the colored Higgs multiplets and the X gauge bosons, which yield the baryon and lepton number non-conserving interactions. They are expressed in terms of the dimension-five and -six effective operators, respectively. It is found that the former interactions in general give rise to dominant channels for proton decay, such as $p \to K^+\bar{\nu}$ [7,8]. However, the current experimental limits on the channel, $\tau(p \to K^+\bar{\nu}) > 3.3 \times 10^{33}$ yrs [9], are so severe that the contribution of the dimension-five operators is required to be suppressed by a certain mechanism; otherwise the model is excluded just as the case of the minimal SUSY SU(5) GUT unless the SUSY particles in the SUSY SM are much heavier than the weak scale [10,11]. A variety of such mechanisms have been proposed. For example, the Peccei–Quinn symmetry [12] would be exploited for the purpose. The R symmetry also plays a role in suppressing the dimension-five proton decay in the models with extra dimensions [13]. With such a suppression mechanism imposed, the dimension-six operators in turn become dominant. In this case, the main decay mode is the $p \to \pi^0 e^+$ channel; the present experimental limit on its lifetime is given by $\tau(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ yrs [14]. Since in the SUSY GUTs the GUT scale $M_{\rm GUT}$ is relatively high, *i.e.*, $M_{\rm GUT} \sim 2 \times 10^{16}$ GeV, the predicted proton lifetime usually evades the experimental limit. However, the consequence might be altered if there exist extra particles in the intermediate scale. With such particles belonging to a representation of the grand unified group, the gauge coupling unification is still achieved, while its value at the unified scale turns out to be enhanced. Then, the proton lifetime is considerably reduced due to the large gauge coupling [15].

In order to study such possibilities based on the proton decay experiments, it is important to make a precise prediction for the decay rate. To that end, we need to determine the effects of the dimension-six operators, which are generated at the GUT scale, on the low-energy physics by using the renormalization group equations (RGEs). Indeed, there have been several literature in which the renormalization factors for the effective operators are evaluated. In Ref. [16], the long-distance QCD corrections are computed at two-loop level. For the short-distance factors, on the other hand, only the one-loop calculation is carried out in Ref. [17] in the SUSY SM.

In this Letter, therefore, we evaluate the renormalization factors of the dimension-six operators at two-loop level in the presence of the supersymmetry. In the calculation, we use the results for the two-loop corrections to the effective Kähler potential given in Ref. [18] Since in the SUSY GUTs, the most of the intermediate

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energy scales are supersymmetric, the short-distance renormalization factors are well approximated by those evaluated in purely SUSY theory. Thus, combined with the long-distance effects given in Ref. [16], our results offer a tool for making a prediction of the proton decay rate with accuracy of two-loop level.

We also derive a simple formula for the one-loop level renormalization factors of any higher-dimensional operators in the Kähler potential, when including only the gauge interaction contributions. It is applicable to other observables, such as the neutron–antineutron oscillation [19].

This Letter is organized as follows: in Section 2, we first write down the dimension-six effective operators in terms of the superfield notation. Our notations and conventions are also shown in the section. Then, in the subsequent section, we describe a way of calculating the renormalization factors of the operators by using the effective Kähler potential, and present the results for the computation. In Section 4, the comparison of the one- and two-loop renormalization factors is discussed in the minimal SUSY SU(5) GUT. Section 5 is devoted to conclusion and discussion.

2. Dimension-six effective operators

To begin with, we write the dimension-six effective operators for proton decay in a SUSY and gauge invariant manner with superspace notation:

$$\mathcal{O}^{(1)} = \int d^2\theta\, d^2\bar{\theta}\, \epsilon_{abc}\epsilon_{ij}\left(\overline{U}^\dagger\right)^a\left(\overline{D}^\dagger\right)^b e^{-\frac{2}{3}g_Y V_1}\left(e^{2g_3V_3}Q_i\right)^c L_j,$$
$$\mathcal{O}^{(2)} = \int d^2\theta\, d^2\bar{\theta}\, \epsilon_{abc}\epsilon_{ij}\overline{E}^\dagger e^{\frac{2}{3}g_Y V_1}\left(e^{-2g_3V_3}\overline{U}^\dagger\right)^a Q_i^b Q_j^c, \tag{1}$$

where all the chiral superfields correspond to left-handed fermions, and V_1 and V_3 are the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(3)_C$ vector superfields with the gauge coupling constants g_Y and g_3, respectively. The subscripts i, j, are the $\mathrm{SU}(2)_L$ indices, while a, b, c are the color indices. Furthermore, we omit the generation indices for simplicity.

The relationship between bare and renormalized operators is written in the following form:

$$\mathcal{O}_B^{(I)} = Z^{(I)}\mathcal{O}^{(I)} \quad (I = 1, 2), \tag{2}$$

where the subscript B indicates the operator is bare. Then, the Wilson coefficients $C^{(I)}$ for the operators $\mathcal{O}^{(I)}$ obey the differential equations,

$$\mu\frac{d}{d\mu}C^{(I)}(\mu) = \gamma_{\mathcal{O}^{(I)}}C^{(I)}(\mu), \tag{3}$$

with $\gamma_{\mathcal{O}^{(I)}}$ the anomalous dimensions for the operators defined as

$$\gamma_{\mathcal{O}^{(I)}} \equiv \mu\frac{d}{d\mu}\ln Z^{(I)}. \tag{4}$$

The anomalous dimensions are obtained by analyzing the vertex functions (or the effective action) in which the operators are inserted. Since we now deal with the dimension-six operators which contain four chiral or anti-chiral superfields, it is sufficient to consider the four-point vertex functions which include the corresponding external superfields. Their renormalization group equations (RGEs) are given as

$$\left[\mu\frac{\partial}{\partial\mu} + \beta_\alpha\frac{\partial}{\partial g_\alpha} - \sum_i \gamma_i + \gamma_{\mathcal{O}^{(I)}}\right]\Gamma_{\mathcal{O}^{(I)}} = 0. \tag{5}$$

Here, $\Gamma_{\mathcal{O}^{(I)}}$ are the four-point vertex functions with an insertion of the operators $\mathcal{O}^{(I)}$. The gauge coupling constants and their beta functions are denoted by g_α and β_α, respectively, and the sum over each gauge group is implicit. Further, γ_i shows the anomalous dimension of each superfield contained in the operators. From now on, we often omit the superscript (I) for brevity.

3. Renormalization factors

In this section, we present the formulae for the renormalization factors. They are derived from the effective Kähler potential given in Ref. [18]. In the calculation, the dimensional reduction scheme ($\overline{\mathrm{DR}}$) [20] is employed for the regularization. We first obtain the one-loop results and confirm the results in Ref. [17] in the former subsection. Then, in the latter subsection, we evaluate the two-loop contribution.

3.1. One-loop

Let us first evaluate the vertex functions at one-loop level. For this purpose, we use the results in Ref. [18], where the effective Kähler potential for generic four-dimensional $N = 1$ SUSY theories is computed up to two-loop level. According to the results, the one-loop correction[1] to the Kähler potential is given as

$$\Delta K_1 = -\sum_\alpha \frac{1}{16\pi^2}\,\mathrm{Tr}\, M_{C(\alpha)}^2\left(2 - \ln\frac{M_{C(\alpha)}^2}{\bar{\mu}^2}\right), \tag{6}$$

where $\bar{\mu}^2 \equiv 4\pi e^{-\gamma}\mu^2$ defines the $\overline{\mathrm{MS}}$ renormalization scale, and the mass matrix $M_{C(\alpha)}^2$ is defined by

$$\left(M_{C(\alpha)}^2\right)_{AB} \equiv 2g_\alpha^2\bar{\phi}_a\left(T_A^{(\alpha)}\right)^a{}_b G^b{}_c\left(T_B^{(\alpha)}\right)^c{}_d\phi^d, \tag{7}$$

with ϕ the background for the chiral superfield Φ and $G^a{}_b$ the Kähler metric

$$G^a{}_b \equiv \frac{\partial^2}{\partial\bar{\phi}_a\partial\phi^b}K(\bar{\phi}, \phi). \tag{8}$$

In Eq. (6), Tr denotes the trace over the adjoint representation of a gauge group whose coupling constant is g_α and generators are given by $T_A^{(\alpha)}$. Moreover, in the following calculation, we only take the gauge interactions into account, *i.e.*, we neglect the superpotential.[2]

In order to obtain the renormalization factors for the higher-dimensional effective operators, we consider the Kähler potential

$$K = \bar{\phi}_a\phi^a + C\mathcal{O} + C\mathcal{O}^\dagger, \tag{9}$$

with C the Wilson coefficient of the operator $\mathcal{O}$. In this case, the Kähler metric reads

$$G^a{}_b = \delta_b^a + C\mathcal{O}^a{}_b + C\mathcal{O}^{\dagger a}{}_b, \tag{10}$$

with $\mathcal{O}^a{}_b \equiv \partial^2\mathcal{O}/\partial\bar{\phi}_a\partial\phi^b$. By substituting the above equations to Eq. (6), we have

$$\Delta K_1 = -\sum_\alpha \frac{g_\alpha^2}{16\pi^2}2\left(1 + \ln\bar{\mu}^2\right)\Big[C_\alpha(a)\bar{\phi}_a\phi^a$$
$$+ \left\{C\left(\bar{\phi}T_A^{(\alpha)}\right)_a\mathcal{O}^a{}_b\left(T_A^{(\alpha)}\phi\right)^b + \text{h.c.}\right\}\Big], \tag{11}$$

where $C_\alpha(i)$ are the quadratic Casimir group theory invariants for the superfield Φ_i, defined in terms of the Lie algebra generators T_A by $(T_A^{(\alpha)}T_A^{(\alpha)})_b^a = C_\alpha(i)\delta_b^a$. Further, we keep only the terms up to the first order with respect to the Wilson coefficient, C, and do

[1] This one-loop result is first derived in Ref. [21].

[2] Experimental constraints on the effective operators in Eq. (1) are particularly severe when the external lines of the operators are of the first and/or second generations. In such a case, the size of the Yukawa couplings are negligible.

not show the terms including the logarithmic dependence on the background fields, which are not relevant to the present calculation. At the first order in the perturbation theory, the RGE (5) then leads to

$$\gamma_{\mathcal{O}}^{(1)}\mathcal{O}=\sum_i \gamma_i^{(1)}\mathcal{O}+\sum_\alpha \frac{g_\alpha^2}{16\pi^2}4\big(\bar\phi T_A^{(\alpha)}\big)_a\mathcal{O}^a{}_b\big(T_A^{(\alpha)}\phi\big)^b. \tag{12}$$

Here, the superscript (1) of the anomalous dimensions denotes that they are evaluated at one-loop level. In supersymmetric theories, $\gamma_i^{(1)}$ is given as[3]

$$\gamma_i^{(1)}=-2\sum_\alpha C_\alpha(i)\frac{g_\alpha^2}{16\pi^2}. \tag{13}$$

Now we evaluate the second term in Eq. (12). To that end, we analyze the structure of the term on a general basis in order to derive the formula for the one-loop renormalization factor of any operator. Consider the following operator which contains an arbitrary number of both chiral and anti-chiral superfields and is singlet under a given global symmetry G as a whole:

$$\mathcal{O}=\bar\lambda_a^{i_1\ldots i_m}\lambda^a_{j_1\ldots j_n}\overline{\Phi}_{i_1}\cdots\overline{\Phi}_{i_m}\Phi^{j_1}\cdots\Phi^{j_n}. \tag{14}$$

Here, the coefficients $\lambda^a_{j_1\ldots j_n}$ and $\bar\lambda_a^{i_1\ldots i_m}$ make the set of superfields G singlet. When G is localized (gauged), the operator invariant under both supersymmetry and the gauge symmetry is

$$\int d^2\theta\, d^2\bar\theta\, \big(\bar\lambda_a^{i_1\ldots i_m}\overline{\Phi}_{i_1}\cdots\overline{\Phi}_{i_m}\big)\big[e^{2gV_G^AT^A}\big]^a{}_b\big(\lambda^b_{j_1\ldots j_n}\Phi^{j_1}\cdots\Phi^{j_n}\big), \tag{15}$$

where g and V_G^A are the coupling constant and the gauge vector superfields of the gauge group G, respectively. Moreover, T^A are assumed to be the generators for an irreducible representation, which are relevant to the transformation properties of the composite chiral superfield $\Phi_{j_1}\cdots\Phi_{j_n}$; under the gauge transformation, $\Phi_{j_1}\cdots\Phi_{j_n}$ is transformed as

$$\big(\lambda^a_{j_1\ldots j_n}\Phi^{j_1}\cdots\Phi^{j_n}\big)\to\big(e^{ig\Lambda^AT^A}\big)^a{}_b\big(\lambda^b_{j_1\ldots j_n}\Phi^{j_1}\cdots\Phi^{j_n}\big), \tag{16}$$

with Λ^A any chiral superfields. Further, we write the generators for each chiral superfield Φ as t^A, *i.e.*, $\Phi^j\to(e^{ig\Lambda^At^A})^j{}_{j'}\Phi^{j'}$. Then, since the coefficients $\lambda^a_{j_1\ldots j_n}$ and $\bar\lambda_a^{i_1\ldots i_m}$ assemble the transformation properties of each chiral superfield into that of the composite operator $\lambda^a_{j_1\ldots j_n}\Phi^{j_1}\cdots\Phi^{j_n}$, it follows that

$$\big(T^A\big)^a{}_b\lambda^b_{j_1\ldots j_n}=\lambda^a_{j_1'j_2\ldots j_n}\big(t^A\big)^{j_1'}{}_{j_1}+\cdots+\lambda^a_{j_1\ldots j_{n-1}j_n'}\big(t^A\big)^{j_n'}{}_{j_n}, \tag{17}$$

and similarly for the anti-chiral superfields,

$$\bar\lambda_b^{i_1\ldots i_m}\big(T^A\big)^b{}_a=\big(t^A\big)^{i_1}{}_{i_1'}\bar\lambda_b^{i_1'i_2\ldots i_m}+\cdots+\big(t^A\big)^{i_m}{}_{i_m'}\bar\lambda_b^{i_1\ldots i_{m-1}i_m'}. \tag{18}$$

These expressions imply that $\lambda^a_{j_1\ldots j_n}$ and $\bar\lambda_a^{i_1\ldots i_m}$ are invariant tensors under G. By using the relations, we now evaluate the second term in Eq. (12). It goes as follows:

$$\begin{aligned}
&\big(\bar\phi t^A\big)_a\mathcal{O}^a{}_b\big(t^A\phi\big)^b\\
&\quad=\big[\big(t^A\big)^{i_1}{}_{i_1'}\bar\lambda_b^{i_1'i_2\ldots i_m}+\cdots+\big(t^A\big)^{i_m}{}_{i_m'}\bar\lambda_b^{i_1\ldots i_{m-1}i_m'}\big]\\
&\qquad\times\big[\lambda^a_{j_1'j_2\ldots j_n}\big(t^A\big)^{j_1'}{}_{j_1}+\cdots+\lambda^a_{j_1\ldots j_{n-1}j_n'}\big(t^A\big)^{j_n'}{}_{j_n}\big]\\
&\qquad\times\bar\phi_{i_1}\cdots\bar\phi_{i_m}\phi^{j_1}\cdots\phi^{j_n}\\
&\quad=\bar\lambda_b^{i_1\ldots i_m}\big(T^A\big)^b{}_a\big(T^A\big)^a{}_c\lambda^c_{j_1\ldots j_n}\bar\phi_{i_1}\cdots\bar\phi_{i_m}\phi^{j_1}\cdots\phi^{j_n}\\
&\quad=C_G^{\text{comp}}\mathcal{O},
\end{aligned} \tag{19}$$

where C_G^{comp} is defined by $T^AT^A=C_G^{\text{comp}}\mathbb{1}$; it corresponds to the Casimir invariant for the composite chiral superfield $\lambda^a_{j_1\ldots j_n}\Phi^{j_1}\cdots\Phi^{j_n}$. Substituting the expression into Eq. (12), we finally obtain a generic formula for the one-loop renormalization factors of arbitrary operators:

$$\gamma_{\mathcal{O}}^{(1)}=\sum_\alpha\frac{g_\alpha^2}{16\pi^2}\bigg[4C_\alpha^{\text{comp}}-2\sum_i C_\alpha(i)\bigg], \tag{20}$$

with C_α^{comp} the Casimir invariants of the gauge group α for the chiral part of the operators.

So far we have assumed that the set of chiral (anti-chiral) superfields forms an irreducible representation. When it is reducible, independent operators are formed. They are not mixed with each other at one-loop level if only gauge interactions are effective.

Now we apply the formula to the dimension-six effective operators for proton decay in Eq. (1). We find $C_3^{\text{comp}}=C_3(\Box)=4/3$ in the case of $SU(3)_C$ and $C_2^{\text{comp}}=0$ in the case of $SU(2)_L$ for both $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(2)}$. Here, $\Box$ denotes the fundamental representation of the corresponding group, and we have used $C_3(\Box)=C_3(\overline{\Box})$. Note that the latter equation for $SU(2)_L$ follows from the fact that the $SU(2)_L$ non-singlet superfields in the effective operators have the same chirality and form an $SU(2)_L$ singlet. For $U(1)_Y$ contributions, on the other hand, we obtain different results for the operators $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(2)}$: $C_Y^{\text{comp}}=(Y_Q+Y_L)^2$ for $\mathcal{O}^{(1)}$ and $C_Y^{\text{comp}}=(2Y_Q)^2$ for $\mathcal{O}^{(2)}$. As a result, by using these factors we obtain that

$$\gamma_{\mathcal{O}^{(i)}}^{(1)}=\sum_{\alpha=Y,2,3}\frac{g_\alpha^2}{16\pi^2}\big[\gamma_{\mathcal{O}^{(i)}}^{(1)}\big]_\alpha, \tag{21}$$

where

$$\big[\gamma_{\mathcal{O}^{(1)}}^{(1)}\big]_3=\big[\gamma_{\mathcal{O}^{(2)}}^{(1)}\big]_3=-\frac{8}{3}, \tag{22}$$

$$\big[\gamma_{\mathcal{O}^{(1)}}^{(1)}\big]_2=\big[\gamma_{\mathcal{O}^{(2)}}^{(1)}\big]_2=-3, \tag{23}$$

$$\begin{aligned}
&\big[\gamma_{\mathcal{O}^{(1)}}^{(1)}\big]_Y=-\frac{11}{9},\\
&\big[\gamma_{\mathcal{O}^{(2)}}^{(1)}\big]_Y=-\frac{23}{9}.
\end{aligned} \tag{24}$$

These results are totally consistent with those in Ref. [17].

3.2. Two-loop

Next, we discuss the two-loop level contribution. Again, we use the results in Ref. [18]. The radiative corrections to the Kähler potential at two-loop level are described by

$$\begin{aligned}
\Delta K_2={}&\frac{1}{2}R^b{}_a{}^d{}_cJ^a{}_b{}^c{}_d\big(M^2\big)-\sum_\alpha f_{ABC}^{(\alpha)}f_{DEF}^{(\alpha)}I^{BDEAFC}\big(M^2_{V(\alpha)}\big)\\
&-\sum_\alpha\big(GT_A^{(\alpha)}\phi\big)^b{}_{;c}\big(\bar\phi T_B^{(\alpha)}G\big)_a{}^{;d}H^a{}_b{}^c{}_d{}^{AB}\big(M^2,M^2_{V(\alpha)}\big),
\end{aligned} \tag{25}$$

with $f_{ABC}^{(\alpha)}$ the structure constants of the gauge group α. The mass functions and the geometric factors appear in Eq. (25) are displayed in Appendix A. By using them, we readily obtain the two-loop corrections to the vertex functions. We found from explicit

[3] The anomalous dimension of fields, γ_i, may be also derived at one- and two-loop levels from the effective Kähler potential derived in Ref. [18] in the similar way. See the first term in Eq. (11).

calculation that the two-loop correction is not given simply by the gauge transformation properties of the composite chiral superfield in the operator and anomalous dimension of the external fields, which is different from the one-loop ones. At present, however, since the explicit derivations are quite complicated, we simply give the final results and defer full details [19].

The RGE in Eq. (5) at two-loop level is given as

$$\mu\frac{\partial\Gamma^{(2)}_{\mathcal{O}}}{\partial\mu}+\sum_\alpha\frac{1}{16\pi^2}b_\alpha g_\alpha^3\frac{\partial}{\partial g_\alpha}\Gamma^{(1)}_{\mathcal{O}}-\sum_i\gamma_i^{(1)}\Gamma^{(1)}_{\mathcal{O}}$$
$$-\sum_i\gamma_i^{(2)}\Gamma^{(0)}_{\mathcal{O}}+\gamma^{(1)}_{\mathcal{O}}\Gamma^{(1)}_{\mathcal{O}}+\gamma^{(2)}_{\mathcal{O}}\Gamma^{(0)}_{\mathcal{O}}=0. \tag{26}$$

Here, the subscripts (0–2) indicate the quantities are evaluated at tree, one-loop, and two-loop level, respectively. One-loop anomalous dimensions $\gamma_i^{(1)}$ are shown in Eq. (13), while the two-loop ones are given as [22]

$$\gamma_i^{(2)}=\frac{1}{(16\pi^2)^2}\sum_{\alpha,\beta}2g_\alpha^2C_\alpha(i)\big[g_\alpha^2b_\alpha\delta_{\alpha\beta}+2g_\beta^2C_\beta(i)\big]. \tag{27}$$

Here, b_α are the one-loop beta function coefficients for gauge coupling constants, given as $b_\alpha=\sum_i I_\alpha(i)-3C_\alpha(G)$ with $C_\alpha(G)$ and $I_\alpha(i)$ the quadratic Casimir invariant for the adjoint representation of the group α and the Dynkin index of the chiral multiplet Φ_i, respectively.

From the RGE in Eq. (26), we now obtain the two-loop anomalous dimensions for the effective operators. Again, we parametrize them as follows:

$$\gamma^{(2)}_{\mathcal{O}^{(I)}}=\frac{g_3^4}{(16\pi^2)^2}\big[\gamma^{(2)}_{\mathcal{O}^{(I)}}\big]_{33}+\frac{g_2^4}{(16\pi^2)^2}\big[\gamma^{(2)}_{\mathcal{O}^{(I)}}\big]_{22}$$
$$+\frac{g_Y^4}{(16\pi^2)^2}\big[\gamma^{(2)}_{\mathcal{O}^{(I)}}\big]_{YY}+\frac{g_2^2g_3^2}{(16\pi^2)^2}\big[\gamma^{(2)}_{\mathcal{O}^{(I)}}\big]_{23}$$
$$+\frac{g_Y^2g_2^2}{(16\pi^2)^2}\big[\gamma^{(2)}_{\mathcal{O}^{(I)}}\big]_{Y2}+\frac{g_Y^2g_3^2}{(16\pi^2)^2}\big[\gamma^{(2)}_{\mathcal{O}^{(I)}}\big]_{Y3}. \tag{28}$$

Then, we have

$$\big[\gamma^{(2)}_{\mathcal{O}^{(1)}}\big]_{33}=\big[\gamma^{(2)}_{\mathcal{O}^{(2)}}\big]_{33}=\frac{64}{3}+8b_3, \tag{29}$$

$$\big[\gamma^{(2)}_{\mathcal{O}^{(1)}}\big]_{22}=\big[\gamma^{(2)}_{\mathcal{O}^{(2)}}\big]_{22}=\frac{9}{2}+3b_2, \tag{30}$$

$$\big[\gamma^{(2)}_{\mathcal{O}^{(1)}}\big]_{YY}=\frac{113}{54}+\frac{5}{3}b_Y,$$
$$\big[\gamma^{(2)}_{\mathcal{O}^{(2)}}\big]_{YY}=\frac{91}{18}+3b_Y, \tag{31}$$

$$\big[\gamma^{(2)}_{\mathcal{O}^{(1)}}\big]_{23}=12,$$
$$\big[\gamma^{(2)}_{\mathcal{O}^{(2)}}\big]_{23}=20, \tag{32}$$

$$\big[\gamma^{(2)}_{\mathcal{O}^{(1)}}\big]_{Y2}=2,$$
$$\big[\gamma^{(2)}_{\mathcal{O}^{(2)}}\big]_{Y2}=\frac{2}{3}, \tag{33}$$

$$\big[\gamma^{(2)}_{\mathcal{O}^{(1)}}\big]_{Y3}=\frac{68}{9},$$
$$\big[\gamma^{(2)}_{\mathcal{O}^{(2)}}\big]_{Y3}=\frac{76}{9}. \tag{34}$$

4. Results

In this section, we give the numerical results of the renormalization factors in the minimal SUSY SU(5) GUT. The short-distance renormalization factors $A_S^{(I)}$ are defined as the ratios of the coefficients $C^{(I)}$ for the effective operators at the SUSY scale $M_{\rm SUSY}$ to those at the GUT scale $M_{\rm GUT}$:

$$A_S^{(I)}\equiv\frac{C^{(I)}(M_{\rm SUSY})}{C^{(I)}(M_{\rm GUT})}\quad(I=1,2), \tag{35}$$

where we assume $M_{\rm SUSY}=1$ TeV and $M_{\rm GUT}=1.5\times10^{16}$ GeV. The numerical results at one-loop level are given as

$$A_S^{(1)}(\text{1-loop})=1.959,$$
$$A_S^{(2)}(\text{1-loop})=2.058, \tag{36}$$

while at two-loop level, we have found

$$A_S^{(1)}(\text{2-loop})=1.961,$$
$$A_S^{(2)}(\text{2-loop})=2.052. \tag{37}$$

Here, we calculate the one-loop (two-loop) short-distance factors with the one-loop (two-loop) renormalization equations for the gauge coupling constants in the SUSY SM [22]. The numerical values of the unified gauge coupling constant at the one- and two-loop level are given as $\alpha_5(\text{1-loop})=0.03906$ and $\alpha_5(\text{2-loop})=0.03968$, respectively, where α_5 is defined as $\alpha_5\equiv g_3^2(M_{\rm GUT})/4\pi$. The results are hardly affected by the uncertainty of the input parameters, *e.g.*, the SU(3) gauge coupling constant, $\alpha_s(m_Z)=0.1184(7)$ [23]. There is a cancellation among the two-loop corrections since the signs of $[\gamma^{(2)}_{\mathcal{O}^{(1)}}]_{33}$ and $[\gamma^{(2)}_{\mathcal{O}^{(2)}}]_{33}$ are opposite to those of the other two-loop anomalous dimensions. Therefore, the numerical values at two-loop level hardly differ from the one-loop ones. Without cancellations, the significance of the two-loop contributions to the short-distance factors reaches a few percent of the one-loop ones.

5. Conclusion and discussion

We have evaluated the short-distance renormalization factors for the dimension-six proton decay operators at two-loop level with the effective Kähler potential. The procedure described in this Letter is generic and applicable to any higher-dimensional operators. We get the results $A_S^{(1)}(\text{2-loop})=1.961$ and $A_S^{(2)}(\text{2-loop})=2.052$ in the minimal SUSY SU(5) GUT. We have found that the two-loop contributions hardly change the renormalization factors evaluated at one-loop level.

Finally, we briefly comment on the extensions of the minimal SUSY GUT. The gauge coupling constants at the GUT scale increase if there exist extra particles in the intermediate scale. The two-loop effects may be more significant in such cases. In addition, let us note that our results are only for the $SU(3)_C\times SU(2)_L\times U(1)_Y$ gauge interactions. If some new gauge interactions exist below the GUT scale, we also need to evaluate the contributions of the gauge interactions. Even for such theories, however, it is possible to execute the prescription describe above to estimate the renormalization factors by means of the effective Kähler potential.

In this Letter, we neglect the possible effects of the threshold corrections from particles whose masses are around the GUT scale. Although the effects are model-dependent, to complete the two-loop level calculation, we also need to evaluate such corrections. We will discuss the issue on another occasion [19].

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Appendix A

Here, we show the explicit form of the mass functions as well as the geometric factors given in Eq. (25):

$$J^a{}_b{}^c{}_d\big(M^2\big) = \frac{2}{(16\pi^2)^2}\big(\ln\bar{\mu}^2\big)\sum_{\alpha,\beta}\big(M_\alpha^2 G^{-1}\big)^a{}_b\big(M_\beta^2 G^{-1}\big)^c{}_d, \quad (38)$$

$$\begin{aligned} I^{ABCDEF}\big(M^2_{V(\alpha)}\big) = &-\frac{1}{2}\frac{g_\alpha^2}{(16\pi^2)^2}\big(\ln\bar{\mu}^2\big)\big[4\big(M^2_{V(\alpha)}\big)_{AB}\delta_{CD}\delta_{EF} \\ &- \delta_{AB}\big(M^2_{V(\alpha)}\ln M^2_{V(\alpha)}\big)_{CD}\delta_{EF} \\ &- \delta_{AB}\delta_{CD}\big(M^2_{V(\alpha)}\ln M^2_{V(\alpha)}\big)_{EF}\big] + \text{cycl.}, \quad (39)\end{aligned}$$

where the "cycl." denotes the cyclic permutations of the labels AB, CD, EF, and

$$\begin{aligned} &H^a{}_b{}^c{}_d{}^{AB}\big(M^2, M^2_{V(\alpha)}\big) \\ &= -\frac{g_\alpha^2}{(16\pi^2)^2}\big(\ln\bar{\mu}^2\big)\bigg[\sum_\beta \delta_{AB}\big\{2\big(M_\beta^2 G^{-1}\big)^a{}_b\big(G^{-1}\big)^c{}_d \\ &\quad + 2\big(G^{-1}\big)^a{}_b\big(M_\beta^2 G^{-1}\big)^c{}_d - \big(G^{-1}\big)^a{}_b\big(M_\beta^2\ln\big\{M_\beta^2\big\}G^{-1}\big)^c{}_d \\ &\quad - \big(M_\beta^2\ln\big\{M_\beta^2\big\}G^{-1}\big)^a{}_b\big(G^{-1}\big)^c{}_d\big\} \\ &\quad + 2\big(G^{-1}\big)^a{}_b\big(G^{-1}\big)^c{}_d\big(M^2_{V(\alpha)}\big)_{AB} \\ &\quad + \big(G^{-1}\big)^a{}_b\big(G^{-1}\big)^c{}_d\big(M^2_{V(\alpha)}\ln M^2_{V(\alpha)}\big)_{AB}\bigg]. \quad (40)\end{aligned}$$

Here, we drop the terms independent of the scale μ or containing two logarithms. The latter terms give rise to the logarithmic terms after differentiation, which cancel other logarithmic terms in the RGEs. The mass parameters are defined as

$$\big(M_\alpha^2\big)^a{}_b \equiv 2g_\alpha^2\big(T_A^{(\alpha)}\phi\big)^a\big(\bar{\phi}T_A^{(\alpha)}G\big)_b, \quad (41)$$

and

$$\big(M^2_{V(\alpha)}\big)_{AB} \equiv \frac{1}{2}\big[\big(M^2_{C(\alpha)}\big)_{AB} + \big(M^2_{C(\alpha)}\big)_{BA}\big]. \quad (42)$$

Further, G^{-1} is inverse of the Kähler metric $G^a{}_b$ defined in Eq. (8), and the curvature $R^a{}_b{}^c{}_d$ is given by

$$R^a{}_b{}^c{}_d \equiv \frac{\partial^2}{\partial\bar{\phi}_a\partial\phi^b}G^c{}_d - \bigg(\frac{\partial}{\partial\bar{\phi}_a}G^c_e\bigg)\big(G^{-1}\big)^e{}_f\bigg(\frac{\partial}{\partial\phi^b}G^f{}_d\bigg). \quad (43)$$

The third term in Eq. (25) includes the shorthand notations, $(GT_A\phi)^b{}_{;c}$ and $(\bar{\phi}T_B G)_a{}^{;d}$, which are defined as

$$\begin{aligned}(GT_A\phi)^a{}_{;b} &\equiv G^a{}_c(T_A)^c{}_b + \bigg(\frac{\partial}{\partial\phi^c}G^a{}_b\bigg)(T_A\phi)^c \\ &= (T_A)^a{}_c G^c{}_b + (\bar{\phi}T_A)_c\bigg(\frac{\partial}{\partial\bar{\phi}_c}G^a{}_b\bigg) \equiv (\bar{\phi}T_A G)_b{}^{;a}. \quad (44)\end{aligned}$$

Here, the second line follows from the gauge invariance of the Kähler potential.

References

[1] G. Aad, et al., ATLAS Collaboration, Phys. Lett. B 716 (2012) 1.
[2] S. Chatrchyan, et al., CMS Collaboration, Phys. Lett. B 716 (2012) 30.
[3] E. Witten, Nucl. Phys. B 188 (1981) 513.
[4] S. Dimopoulos, S. Raby, F. Wilczek, Phys. Rev. D 24 (1981) 1681.
[5] S. Dimopoulos, H. Georgi, Nucl. Phys. B 193 (1981) 150.
[6] N. Sakai, Z. Phys. C 11 (1981) 153.
[7] N. Sakai, T. Yanagida, Nucl. Phys. B 197 (1982) 533.
[8] S. Weinberg, Phys. Rev. D 26 (1982) 287.
[9] M. Miura, PoS ICHEP 2010 (2010) 408.
[10] T. Goto, T. Nihei, Phys. Rev. D 59 (1999) 115009.
[11] H. Murayama, A. Pierce, Phys. Rev. D 65 (2002) 055009.
[12] R.D. Peccei, H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440.
[13] L.J. Hall, Y. Nomura, Phys. Rev. D 64 (2001) 055003.
[14] H. Nishino, et al., Super-Kamiokande Collaboration, Phys. Rev. D 85 (2012) 112001.
[15] J. Hisano, D. Kobayashi, N. Nagata, Phys. Lett. B 716 (2012) 406.
[16] T. Nihei, J. Arafune, Prog. Theor. Phys. 93 (1995) 665.
[17] C. Munoz, Phys. Lett. B 177 (1986) 55.
[18] S. Nibbelink Groot, T.S. Nyawelo, JHEP 0601 (2006) 034.
[19] J. Hisano, D. Kobayashi, Y. Muramatsu, N. Nagata, in preparation.
[20] W. Siegel, Phys. Lett. B 84 (1979) 193.
[21] A. Brignole, Nucl. Phys. B 579 (2000) 101.
[22] S.P. Martin, M.T. Vaughn, Phys. Rev. D 50 (1994) 2282;
S.P. Martin, M.T. Vaughn, Phys. Rev. D 78 (2008) 039903 (Erratum).
[23] J. Beringer, et al., Particle Data Group, Phys. Rev. D 86 (2012) 010001.