Improved $p(t)$-linear Average Method for Ground Thermal Properties Estimation during in-situ Thermal Response Test

Linfeng Zhang$^a$, Quan Zhang$^a$,*, José Acuña$^b$, Xiaowei Ma$^a$

$^a$College of Civil Engineering, Hunan University, Changsha, Hunan 410082, China
$^b$Department of Energy Technology, Royal Institute of Technology (KTH), Brinellvägen 68, 10044 Stockholm, Sweden

Abstract

One potential problem for the ground coupled heat pump is that the ground thermal properties is hardly to be known due to the complicated ground construction. The $p(t)$-linear average method has been proved that it can improve the accuracy of borehole thermal resistance. However, the $p(t)$-linear fluid temperature distribution approximation is not agree well with the temperature profile measured by the fiber cable. Thus, in this paper, an improved $p(t)$-linear average method in which the fluid temperature distribution approximation based on the vertical temperature profile is proposed. With the new vertical temperature profile simulation model, the accuracy for the borehole thermal resistance estimation will be improved comparing to the true value. Besides that, the estimation results are sensitive with the distance between two pipes, and together with the borehole thermal resistance, the distance will be optimized by the outlet fluid temperature. The life cycle cost analysis results of a case study for an office building in Hunan University show that, although the operation cost will be increased, the total cost during the whole lifetime will be reduced with a lower initial investment.

Keywords: Ground-Coupled Heat Pump, Thermal Response Test, Fluid Temperature profile, Improved $p(t)$-linear average method;

1. Introduction

The directed-use of geothermal energy is one of the oldest energy utilization forms, and the Ground-Coupled Heat Pump (GCHP) plays an important role as one of the directed-use of geothermal energy [1]. With a vertical U-pipe,
the heat of the building is rejected into the ground through the circulating fluid in summer and extracted from the ground in winter. Due to the stable ground temperature, the GCHP system has a higher energy efficiency compared to the conventional air-source heat pump system. However, as the input parameters in the GCHP design procedure, the ground thermal properties should be estimated firstly.

The ground thermal properties estimation method was firstly developed by Mogensen [2]. With a constant electrical heater, the heated circulating fluid was rejected into the U-pipe and cooled by the surrounding soil. The inlet and outlet temperature and the flow rate of the circulating fluid was record with a data logger. Based on the infinite line source model, the exponential integral equation can be approximated as a simplest linear form which is a function of natural log of time. After some basic algebraic rearrangement, the ground thermal conductivity can be estimated by the slope of the late-time temperature trend, and borehole thermal resistant can be estimated by the intercept of the late-time temperature trend. This method has been commonly considered as the design standards for TRT by International Ground Source Heat Pump Association[3] and developed by a lot of researchers in various countries[4-12]. With a similar approach, Li and Lai[13] proposed a new ground thermal properties estimation method based on infinite cylinder source model. Despite the strength, the linear fluid temperature distribution assumption will lead to an overestimated borehole thermal resistance.

In order to response to the limitations of the linear fluid temperature distribution assumption, some improvement methods are proposed. With a new experimental apparatus named as fiber optic cable, the Distributed Thermal Response Test (DTRT) was proposed by Acuña [14]. This apparatus gave us a final solution for the temperature profile approximation. As well as the temperature profile along the U-pipe was measured by as fiber optic cable located inside U-pipe of BHE, the ‘true’ ground thermal properties were obtained by the nonlinear curve fitting method.

However, due to the high investment for the fiber optic cable, the temperature profile along the U-pipe cannot be measured. Some temperature profile approximations were proposed. Using a numerical heat transfer model around the borehole, a $p$-linear average method proposed by Marcotte and Pasquier[15] with $p$ value which was proved to be a better approximation for the circulating fluid temperature profile inside the U-pipe. In this method, $p \rightarrow -1$ was selected to be best approximation parameter according to their numerical simulation study. Later, Du and Chen [16]offered a different recommendation that $p = 0$ and $p \rightarrow -1/2$ were the best choices for single U-pipe and double U-pipe, respectively. With a vertical temperature profile method, Beier [17, 18] proposed a new estimation method. In his study, $p \rightarrow -3$ was recommended. Our group has also found that the $p$ value was a time-varying parameter [19]. We proposed a new parameter estimation method named $p(t)$-linear average method to improve the accuracy of the ground thermal properties estimation and a lower borehole thermal resistance was estimated in our study.

As research continues, one potential problem appeared. Although the sensitive analysis indicated that the $p(t)$-linear average method have a much smaller relative error area comparing to the conventional method, the $p$-linear average approximation is not well agreed with the true temperature profile, and the borehole thermal resistance is not validated by real value. In this paper, an improved $p(t)$-linear average method was proposed. The DTRT dataset was employed as a benchmark. The relative error for the improved $p(t)$-linear average method was calculated and compared with the existed estimation methods. Besides, the temperature profile for a selected time was calculated and compared. Finally, the economic impact of overestimating the borehole thermal resistance is analyzed and ascertained based on life cycle analysis of a case study in Hunan University.

2. Improved $p(t)$-linear average method

According to previous studies [15, 19], the $p$-linear function is a good approximation for the temperature profile along the borehole. However, for a better choice, the temperature profile should be simulated by an analytical model which is called vertical temperature profile model proposed by Beier [17]. The improved $p(t)$-linear average method improved our previous research of the $p(t)$-linear average method and vertical temperature profile model. The details of this method is introduced as follows.
2.1. Mathematical model

For a single U-pipe, there are two pipes in the borehole, the downward pipe and upward pipe. The energy balance model is employed to analyze the heat transfer process of the two pipes when the circulating fluid was entering and leaving. It is written as:

\[
\begin{align*}
&\left\{ \begin{array}{l}
\frac{d}{dz} \left[ a_1 T_1 \right] = \frac{T_2 - T_1}{R_{12}^{s}} + \frac{T_0 - T_1}{R_{s1}^{s} + R_{s1}^{b}} \\
-\frac{d}{dz} \left[ a_2 T_2 \right] = \frac{T_1 - T_2}{R_{12}^{s}} + \frac{T_0 - T_2}{R_{s2}^{s} + R_{s2}^{b}}
\end{array} \right. \\
\end{align*}
\]

Where the subscript 1 refers to the downward pipe, while the subscript 2 refers to the upward pipe. \( R_{12} \) is the interaction thermal resistance between the two pipes, \( R_{b1} \) is the borehole thermal resistance for the first pipe, and \( R_{s2} \) is the soil thermal resistance outside the borehole for the first pipe. To simplify the study, an assumption is made that the U-pipe is located in a symmetric position. Under this assumption, we can obtain that:

\[ R_{b1} = R_{b2} = 2R_s, R_{s1} = R_{s2} = 2R_s \]  

To calculated the soil thermal resistance outside the borehole, the infinite line source model is employed due to its pinpoint accuracy after 5-10h for a typical Ground Heat Exchanger, the temperature distribution around the U-pipe at a sampling time \( t \) and a selected radius \( r \) is written as:

\[
\Delta T(r,t) = T(r,t) - T_0 = \frac{q}{4\pi \kappa_s} \int_{r^{s}/4\pi, t}^{r} \frac{e^{-u}}{u} \, du = \frac{q}{4\pi \kappa_s} \left( \ln \frac{4\alpha_t t}{r^2} - \gamma \right)
\]

The borehole wall temperature can be calculated using Eq. (4) when the radius \( r = r_b \), the soil thermal resistance is defined as Eq. (5). The borehole thermal resistance is defined as Eq. (6).

\[
T_b = T_0 + \frac{q}{4\pi \kappa_s} \left( \ln \frac{4\alpha_t t}{r_b^2} - \gamma \right)
\]

\[
R_s = \frac{T_b - T_0}{q} = \frac{1}{4\pi \kappa_s} \left( \ln \frac{4\alpha_t t}{r_b^2} - \gamma \right)
\]

\[
R_{b} = \frac{T_1 - T_b}{q}
\]

Accompany with Eq. (4) and Eq. (6), the circulating fluid temperature inside the U-pipe can be calculated as

\[
T_f = T_0 + q \left( R_b + \frac{1}{4\pi \kappa_s} \left( \ln \frac{4\alpha_t t}{r_b^2} - \gamma \right) \right)
\]
Based on Eq. (7), the borehole thermal resistance, the ground thermal conductivity and the ground thermal diffusivity can be estimated with nonlinear curve fitting method.

Considering the estimated ground thermal properties is the average values for the whole ground, the effective mean fluid temperature should be calculated under the condition that only the inlet and outlet fluid temperature is recorded. Thus, the fluid temperature profile along the U-pipe should be simulated. With Spiegel’s method\[20\], the fluid temperature profile can be simulated with the solutions for Eq. (1), and it is shown in Eq. (8) under the symmetric assumption. The details of the solutions have been reported by Beier \[17\].

\[
\frac{T(l) - T_0}{T_{in} - T_0} = C_1 e^{\alpha l} + C_2 e^{-\alpha l}
\]  

(8)

Where \( l \) is the length of the U-pipe, and \( l = 2z \). Here, the unknown variables \( C_1, C_2, \alpha \) can be replaced by vector \( p \),

\[
p(1) = C_1, p(2) = \alpha, p(3) = C_2, \Theta(L_D) = \frac{T(L_D) - T_0}{T_{in} - T_0}, L_D = \frac{l}{L}
\]  

(9)

Substituting Eq. (9) into Eq. (8), Eq. (8) can be rewritten as:

\[
\Theta(L_D) = p(1)e^{p(2)L_D} + p(3)e^{-p(2)L_D}
\]  

(10)

When \( L_D = 1 \), the dimensionless outlet fluid temperature can be calculated as:

\[
\Theta_{out} = p(1)e^{p(2)} + p(3)e^{-p(2)}
\]  

(11)

The dimensionless integrated mean fluid temperature can be written as:

\[
\Theta = \int_0^1 (p(1)e^{p(2)L_D} + p(3)e^{-p(2)L_D})dL_D = \frac{\Theta_{out}}{p(2)} + \frac{p(3)}{p(2)} - \frac{2p(3)}{p(2)}e^{-p(2)}
\]  

(12)

2.2. Computation algorithm

To estimate the ground thermal properties, the computation algorithm was summarized as follows:

Step 1: assume the arithmetic average of inlet and outlet fluid temperature as the effective mean fluid temperature, based on Eq. (7), estimate the ground thermal properties with nonlinear curve fitting method.

Step 2:
   Step 2.1 calculated the vector \( p \) with estimated ground thermal properties.
   Step 2.2 simulate the vertical temperature profile with Eq. (8)
   Step 2.3 calculate the mean temperature based on calculated \( p \) values with Eq. (12)

Step 3: estimated the new ground thermal properties with the new effective mean fluid temperature based on Eq. (7).

Step 4: Compare the new ground thermal properties to the previous one, if the accuracy tolerance is meet, stop the iterative process and output the ground thermal properties. If not, go to Step 2.
3. Thermal response test setup

To validate the improved $p(t)$-linear average method, an in-situ TRT operated in Sweden was employed[21]. The borehole was 251.5m, and the borehole was filled by ground water. With an adjustable electrical heater which is 9450kW, the heated circulating fluid was pumped into the U-pipe and cooled by the surround soil. The test was last for 160h, and the third 48h was selected for the ground thermal properties estimation with a constant heat. The fluid temperature profile along the borehole was measured by fiber optic cable installed inside the U-pipe, and the flow rate was measured by the flow meter. The details of the TRT have been reported by Acuña and Beier[14, 18]. The primary parameters are list in Table 1.

Table 1. Primary parameters for the DTRT.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borehole radius</td>
<td>$r_b$</td>
<td>70mm</td>
</tr>
<tr>
<td>Borehole length</td>
<td>$L$</td>
<td>251.5m</td>
</tr>
<tr>
<td>Pipe outer radius</td>
<td>$r_{po}$</td>
<td>20mm</td>
</tr>
<tr>
<td>Pipe inner radius</td>
<td>$r_{pi}$</td>
<td>17.6mm</td>
</tr>
<tr>
<td>Pipe wall thermal conductivity</td>
<td>$k_{pw}$</td>
<td>0.40W/(K m)</td>
</tr>
<tr>
<td>Thermal conductivity of water in borehole</td>
<td>$k_w$</td>
<td>0.57W/(K m)</td>
</tr>
<tr>
<td>Fluid volumetric heat capacity</td>
<td>$C$</td>
<td>4260kJ/(k m³)</td>
</tr>
<tr>
<td>Fluid volumetric flow rate</td>
<td>$m$</td>
<td>0.50 kg/s</td>
</tr>
<tr>
<td>Convective heat transfer coefficient inside pipes</td>
<td>$h_i$</td>
<td>1200 W/(K m²)</td>
</tr>
<tr>
<td>Heat input rate</td>
<td>$Q$</td>
<td>9450 W</td>
</tr>
<tr>
<td>Reference ground temperature</td>
<td>$T_r$</td>
<td>9.1°C</td>
</tr>
</tbody>
</table>

4. Result and discussion

4.1. Ground thermal properties estimation

Accompany with the temperature and fluid rate data recorded by the data logger, the ground thermal properties such as the ground thermal conductivity and borehole thermal resistance were estimated. Although the fluid temperature profiles was measured by the fiber optic cable, only the inlet and outlet fluid temperature was employed to satisfy the conventional test apparatus. The mean temperature calculated by the fluid temperature profiles was set as benchmark for ground thermal properties estimation, and various temperature profile approximations were calculated and compared with the benchmarked temperature.

The ground thermal properties was estimated by the curve fitting method, which is illustrated in Fig. 1. Take the mean temperature measured by the fiber optic cable as the benchmark temperature, three other average temperature were calculated and compared based on arithmetic average method, $p(t)$-linear average method and improved $p(t)$-linear average method, respectively. Comparing to the benchmark temperature, the mean temperature difference for the three methods were 0.734°C, 0.035°C and -0.002°C, respectively.
According to the various sequences of mean temperature, the ground thermal conductivity and the borehole thermal resistance can be estimated by the nonlinear curve fitting method are listed in Table 2. Compared to the true borehole thermal resistance by the real mean fluid temperature, the relative error for the conventional method, \( p(t) \)-linear average method and improved \( p(t) \)-linear average method were 25.4%, 5.24% and 4.29%, respectively. The borehole thermal resistance for the conventional method was overestimated. Take 5% as a criterion for the upper limit relative error value, it was indicated that the improved \( p(t) \)-linear average method was an effective method.

Table 2. Ground thermal properties estimation results.

<table>
<thead>
<tr>
<th>Estimation Methods</th>
<th>Rb</th>
<th>k_s</th>
<th>Tm((^\circ)C)</th>
<th>Rb difference</th>
<th>Tm difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>DTRT method by Acuna</td>
<td>0.063</td>
<td>3.1</td>
<td>16.10</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Conventional method</td>
<td>0.079</td>
<td>3.08</td>
<td>16.86</td>
<td>25.40%</td>
<td>12.48%</td>
</tr>
<tr>
<td>( p(t) )-linear average method</td>
<td>0.0663</td>
<td>3.10</td>
<td>16.16</td>
<td>5.24%</td>
<td>0.87%</td>
</tr>
<tr>
<td>Improved ( p(t) )-linear average method</td>
<td>0.0657</td>
<td>3.04</td>
<td>16.12</td>
<td>4.29%</td>
<td>0.21%</td>
</tr>
</tbody>
</table>

In order to further analyze the accuracy of the improved \( p(t) \)-linear average method, the temperature profile for a selected time was simulated and compared to the measured vertical temperature profile. For our case study, \( t=46h \) was selected. The fluid temperature profile was calculated and illustrated in Fig. 2. The fluid mean temperature based on the simulated temperature profiles at \( t=46h \) were also calculated and listed in Table 2. Although fluid mean temperature for the \( p(t) \)-linear average method and the improved \( p(t) \)-linear average method was very close, the fluid vertical temperature profiles for these two methods were different. The fluid temperature profile for the improved \( p(t) \)-linear indicated a better agreement with the measured temperature profile compared to other methods.

4.2. The uncertain factor for improved \( p(t) \)-linear average method

Although the improved \( p(t) \)-linear average method has shown its benefit for the borehole thermal resistance estimation, the borehole thermal resistance were sensitive with the distance between the pipe which is the only parameter cannot be maintained in the test. Fig. 3 indicated the borehole thermal resistance is changing with various distance between two pipes. The lager distance results in a lager borehole thermal resistance. However, as shown in Fig. 4, the distance between two pipes as well as the borehole thermal resistance can be optimized by the outlet fluid temperature. For our case study, the range for half distance between two pipes was 0.02-0.05m. Fig. 4 indicates that the best half distance between two pipes was 0.025m, and the best borehole thermal resistance was 0.0657.
4.3. The economic impact for the estimation results

Avoid hyphenation at the end of a line. Symbols denoting vectors and matrices should be indicated in bold type. Scalar variable names should normally be expressed using italics. Weights and measures should be expressed in SI units. All non-standard abbreviations or symbols must be defined when first mentioned, or a glossary provided.

4.4. File naming and delivery

The conventional method will result in the overestimated borehole thermal resistance. In order to identify the economic impact for the overestimated borehole thermal resistance, the life cycle cost (LCC) simulation for a real building was operated in Hunan University.

Table 3. Economic comparison for various borehole thermal resistance.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$R_b = 0.063\text{K}\text{m/W}$</th>
<th>$R_b = 0.066\text{K}\text{m/W}$</th>
<th>$R_b = 0.079\text{K}\text{m/W}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borehole length (m)</td>
<td>2120</td>
<td>2154</td>
<td>2300</td>
</tr>
<tr>
<td>Initial cost (RMB)</td>
<td>169,600</td>
<td>172,320</td>
<td>184,000</td>
</tr>
<tr>
<td>Electricity Power (kWh)</td>
<td>3,146,944</td>
<td>3,145,419</td>
<td>3,138,477</td>
</tr>
<tr>
<td>Operation cost (RMB)</td>
<td>1,888,167</td>
<td>1,887,251</td>
<td>1,883,086</td>
</tr>
<tr>
<td>Total cost (RMB)</td>
<td>2,057,767</td>
<td>2,059,571</td>
<td>2,067,086</td>
</tr>
</tbody>
</table>

The hourly heating and cooling load for the building was simulated by DesignBuilder software, and it is illustrated in Fig. 5. With the standard ASHRAE design method[22], the length of the borehole can be determined by Eq. (13) and (14), the details for these equations can be refer to Chapter 34 of ASHRAE Handbook 2011.

\[
I_h = \frac{q_w R_{\text{gw}} + (q_{\text{in}} - W_c)(R_{\text{g}} + \text{PLF}_m R_{\text{gm}} + R_{\text{cd}} F_{\text{sc}})}{t_g - t_{\text{ai}} + t_{\text{wi}} - t_p}
\]

\[
I_c = \frac{q_w R_{\text{gw}} + (q_{\text{in}} - W_c)(R_{\text{g}} + \text{PLF}_m R_{\text{gm}} + R_{\text{cd}} F_{\text{sc}})}{t_g - t_{\text{ai}} + t_{\text{wi}} - t_p}
\]
Three borehole thermal resistances estimated by the conventional method, the improved \( p(t) \)-linear average method and the true mean temperature were employed for this simulation and comparison. Based on the information for the building and heat pump, with the borehole length calculated by Eq. (13) and (14), the LCC simulation with 20 years’ operation was operated with TRNSYS environment, and the results are shown in Table 3. It was obvious that the overestimated borehole thermal resistance resulted in overestimated borehole length. It also can be seen that the overestimated borehole thermal resistance will lead to low operation cost. However, the operation cost saving for the overestimated borehole thermal resistance is too small to meet the over borehole initial investment. The economic payback time for the oversized borehole length is 56.68 year, which is too long for real case study. Based on our LCC case study, the accurate borehole thermal resistance shows its own economic benefit on both initial investment and total investment for 20 years’ operation.

5. Conclusions

With a more suitable vertical temperature profile simulation model, an improved \( p(t) \)-linear average method was proposed in this paper. To validate the proposed model, the well prepared DTRT operated in Sweden was employed as a benchmark. Based on the benchmark, the conventional method, the \( p(t) \)-linear average method and the improved \( p(t) \)-linear average method were calculated and compared. It was found that:

- The proposed improved \( p(t) \)-linear average method is an effective estimation method, the mean temperature calculated by the improved \( p(t) \)-linear average method matched very well with the measured value.
- The improved \( p(t) \)-linear average method was very sensitive with the distance between two pipes, and the distance between two pipe as well as the estimated borehole thermal resistance can be optimized by the outlet temperature.
- The overestimated borehole thermal resistance will resulted in higher initial borehole investment and longer borehole length. The operation cost could also be reduced lightly, but it will take 56.68 years to meet the oversized initial investment according to our case study for a small office building.

In consequence, the improved \( p(t) \)-linear average method will improved the accuracy of the borehole thermal resistance. Besides that, both the total cost and the initial cost for the GCHP system will be reduced, too.

Acknowledgements

This work was supported by the Project on the Integration of Industry, Education and Research of Guangdong Province and Ministry of Education (Grant No. 2010B090400301), Science and Technology Planning Project of Changsha (Grant No. K1403142-11), and Interdisciplinary Program of Hunan University in 2014.
References