# Path Optimization Study for Vehicles Evacuation Based on Dijkstra algorithm 

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#### Abstract

Emergency events, such as earthquakes, hurricanes, fires, chemical accidents, nuclear accidents, terror attacks and other events may led to injured or endanger the life and the health of human beings, and the large scale crowds have to evacuate from a danger area to a safe area by vehicles. In this paper, through observing real-time road network and analyzing three different emergency evacuation cases and the nodes, intersections delay and velocity of vehicles evacuation in the morning peak, common and evening peak. A dynamic road network model is built for vehicles evacuation based on Dijkstra algorithm. The optimal evacuation path is proposed in three different cases. The obtained outcomes provide well predictive method and theoretical basis for optimal emergency evacuation path selection and emergency rescue decision in public places, especially for those with high population density. So it seems very meaningful for us to cope with emergency situation or to prevent and mitigate disasters from the crisis events.


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## 1. Introduction

Frequent incidents or emergency events have caused great harm to the society. We need emergency evacuate the large populations from the incident area to a safe area when emergency happens, so it will cause the urban traffic congestion and even the system. Therefore, it has become an important issue of public safety field that how to use fastest speed evacuate the crowd to safe area after the incident. In the emergency evacuation, it is necessary to provide the shortest path from a specified origin node to other destination nodes and used the shortest time in the whole evacuation process. One of the most used methods to solve the shortest path problem is the Dijkstra algorithm[1]. Path selection problem is a shortest path or optimal path problem. The most commonly used algorithm of the optimal path are Dijkstra algorithm, Floyd algorithm, etc. And some improved optimal path algorithm for vehicle navigation, such as bidirectional search algorithms, hierarchical search algorithm, genetic algorithm and the shortest path algorithm based on neural network, and so on[2].

Voorhess et al. [3-5] have proposed some other models and put them into practice under different emergency conditions for estimating evacuation time in which a statistical aggregate method. It takes a considerable amount of computation to calculate the operating characteristics of individual vehicles. Various techniques have been proposed for speeding up Dijkstra's original label procedure. For undirected graphs a linear running time can be achieved for integral lengths [6] or on average in a randomized setting $[7,8]$ For many instances that arise in practice, the graphs have some underlying geometric structure which can also be exploited to speed up shortest path computations [9-11]. James B.Orlin[12] propose an efficient method for implementing Dijkstra's algorithm for the Single Source Shortest Path Problem (SSSPP) in a graph whose edges

[^0]have positive length, and where there are few distinct edge lengths.
This paper is organized as follows. A dynamic road network model based on Dijkstra's algorithm is built in section 2, and made a calculation for the model by inputting data of the urban route in section 3 . The results are analysed in section 4 and select the optimal path according to the different period, which in morning peak, common and evening peak, and Section 5 concludes the paper.

## 2. Modeling

### 2.1. Algorithm

Dijkstra algorithm is recognized as the best algorithm at present when all the weight number is $\mathrm{W}_{\mathrm{ij}} \geq 0$. And its basic thought is the process what from the start and gradually expand to outward. In the process of exploring, record the path each to a point, and called the label.

Specific label consists of two parts, the first part is a letter, which indicate a symbol of in front of a point that shows where is. And the second part is a number, it denote the distance from the starting point to the current position that state how far is it. Label are divided into temporary label and permanent label. All the label is temporary label in the beginning, and make one of a temporary label for a permanent label in every algorithm cycle. Therefore, it can be find out the shortest path from start to finish at most by $\mathrm{n}-1$.

Dijkstra algorithm steps as follows: set $\mathrm{v}_{0}$ as a start point and set v as an end point.
First, start point label is $\left(\mathrm{v}_{0}, 0\right)$, adjacent point label is $\left(v_{0}, L\left(v_{0}, v\right)\right)$, and the other label is $\left(v_{0},+\infty\right)$.In addition, set as $V=V-v_{0}$. Second, If $V=\varphi$, so end the algorithm. Third, use $v_{k} \in V$, it has the smallest label of the $L\left(v_{k}\right)=\min _{v_{i} \in V}\left\{L\left(v_{i}\right)\right\}$. If $v_{k}=v_{n}$, end the algorithm, and else make $v_{k}$ for a permanent label and set is $V=V-v_{k}$. Fourth, check the adjacent point $v_{k}$, if $L\left(v_{i}\right)>L\left(v_{k}\right)+L\left(v_{k}+v_{i}\right)$, set label $v_{i}$ as $\left[v_{k}, L\left(v_{i}\right)>L\left(v_{k}\right)+L\left(v_{k}+v_{i}\right)\right]$, so given label and return the second step.

### 2.2. Establishing road network model

Use buses or other vehicles evacuating crowds from source $S_{i}$ to destination $D_{i}$ by the guidance of path network when in the emergency event. Road network relate to road impedance $L_{i j}$ and intersection impedance $\mathrm{J}_{\mathrm{ij}}$. Assigned each of the $\mathrm{B}_{\mathrm{i}}$ to correspond an evacuation path, and the evacuation time is $t_{i}$. The construction of the model is to look for the optimal evacuation path, which set vector set and make the minimum $Z(t)$.

Transform multi-source network problem of more start point $S_{i}$ and end point $D_{j}$ to network problem of single start s and end d by use the method of add virtual nodes.

Build the model system represented by $\mathrm{G}=(\mathrm{V}, \mathrm{E}, \mathrm{L}, \mathrm{J}, \mathrm{s}, \mathrm{d})$, including set $\mathrm{V}=\{\mathrm{i} \mathrm{i}=1,2, \ldots, \mathrm{n}\}$ as the network intersection set, $E=\{(i, j) \mid i, j=1,2, \ldots, n\}$ as the network path set, $L=\left\{L_{i j} \mid i, j=1,2, \ldots, n\right\}$ as the road impedance set and $J=\left\{J_{i} \mid i=1,2, \ldots, n\right\}$ as the road impedance set of intersection. In addition, set $s$ as a start point and $d$ as the end point.

In the equation, set $F=\left\{f_{i j} \mid i, j=1,2, \ldots, n\right\}$ as the flow of path $(i, j), f_{i j k}\left(t_{j}\right)$ as the flow of the node $i$ to $k$ and by $j, D=$ $\left\{\mathrm{d}_{\mathrm{ij}} \mid \mathrm{i}, \mathrm{j}=1,2, \ldots, \mathrm{n}\right\}$ as the length of path $(\mathrm{i}, \mathrm{j}), \mathrm{C}=\left\{\mathrm{c}_{\mathrm{ij}} \mid \mathrm{i}, \mathrm{j}=1,2, \ldots, \mathrm{n}\right\}$ as the traffic capacity of path $(\mathrm{i}, \mathrm{j}), \mathrm{c}_{\mathrm{ijk}}\left(\mathrm{t}_{\mathrm{j}}\right)$ as the traffic capacity of the node from i to k and by $\mathrm{j}, \mathrm{O}=\left\{\mathrm{O}_{\mathrm{ij}}\left(\mathrm{t}_{\mathrm{i}}\right) \mid \mathrm{i}, \mathrm{j}=1,2, \ldots, \mathrm{n}\right\}$ as the of vehicle working from the leave time $t_{i}$ in path ( $i, j$ ), and so on.

The model is formulated as:

$$
\begin{align*}
& \min Z(t)=\min \left(\sum_{i, j=1}^{n} X_{i j s} l_{i j}+\sum_{i, j=1}^{n} X_{i j s} j_{i j}\right)  \tag{1}\\
& X_{i j s}\left\{\begin{array}{l}
1, \text { vehicle k is evacuating from i to } \mathrm{j} \\
0, \text { vehicle k is not evacuating from i to } \mathrm{j}
\end{array}\right. \tag{2}
\end{align*}
$$

Constraint conditions are edges capacity constraint and point capacity constraint. So edge capacity constraint is $0 \leq \mathrm{f}_{\mathrm{ij}} \leq \mathrm{c}_{\mathrm{ij}}$, $(\mathrm{i}, \mathrm{j}) \in \mathrm{E}$, and point capacity constraint is $0 \leq \mathrm{f}_{\mathrm{ijk}} \leq \mathrm{c}_{\mathrm{ijk}},(\mathrm{i}, \mathrm{j}),(\mathrm{j}, \mathrm{k}) \in \mathrm{E}$.

$$
\sum_{t_{i}} \sum_{j} f_{i j}\left(t_{i}\right)=\sum_{t_{j}} \sum_{k} f_{i j k}\left(t_{j}\right),(i, j),(j, k) \in E, \text { i neither start point nor end point and the flow of path and intersection with a }
$$

balance.

$$
\sum_{t_{i}} \sum_{j} f_{i j}\left(t_{i}\right)=\sum_{t_{j}} \sum_{(j, i) \in E} f_{j i}\left(t_{j}\right),(i, j),(j, k) \in E \text {, i neither start point nor end point and the flow of middle nodes with a }
$$

balance.
$\sum_{t_{1}} \sum_{j} f_{1 j}\left(t_{1}\right)-\sum_{t_{j}} \sum_{j} f_{\mathrm{nj}}\left(t_{j}\right)=v(f),(l, j),(j, l) \in E$, the flow of start point with a balance.
$\sum_{t_{i}} \sum_{i} f_{i n}\left(t_{i}\right)-\sum_{t_{j}} \sum_{j} f_{\mathrm{nj}}\left(t_{j}\right)=v(f),(i, n),(n, j) \in E$, the flow of end point with a balance.
The problem belongs to NP complete class, which is no polynomial time algorithm and just adopted a heuristic algorithm. The design of a polynomial algorithm is follows.

Part one includes four steps.
First step is given an initial value $f$ of road network, and calculate road impedance of the between adjacent nodes and intersection at any time. Second step is establish a sequence of $N$ and put start point $s$, and pick up a node $v_{i}$ from the $N$ as the current node when N point is not null. Third is calculate all adjacent nodes $\mathrm{v}_{\mathrm{i}}$ of the road network node $\mathrm{v}_{\mathrm{i}}$, if
$l\left(s, v_{j}\right)>l\left(s, v_{i}\right)+l_{i j}+j_{i j}$ so, $l\left(s, v_{j}\right)=l\left(s, v_{i}\right)+l_{i j}+j_{i j}$. Fourth step is add the adjacent point is modified with the shortest distance to the sequence of N , and maintain orderly increasing according to the length of each road section.

Part two includes five processes.
First step is initialize the N , and add destination d to the sequence. Second step is repeat the following steps, until the source point output K times. Third step is take out the first point $\mathrm{v}_{\mathrm{i}}$ from the sequence, which become a node of its precursor according to the positions of it in the output number add to the number of output. Fourth step is calculate the length increment of vj in all adjacent points. Fifth step is take the length increment of these points and the positions of in the output number in precursor added to N , and maintain orderly according to the length increment. Build the road network as shown in Fig. 1.


Fig. 1. Road network planar graph.

### 2.3. Setting parameters

Considering the real situation, road traffic has different traffic characteristics at different periods. So take three time periods that in morning peak, common and evening peak as example to study path choice problem when with evacuation requirements.

In the network, some parameters about traffic are shown in table 1, 2 and 3 in morning peak, common and evening peak.

## 3. Calculating

All tables should be numbered with Arabic numerals. Headings should be placed above tables, center justified. Leave one line space between the heading and the table. Only horizontal lines should be used within a table, to distinguish the column headings from the body of the table, and immediately above and below the table. Tables must be embedded into the text and not supplied separately. Below is an example which authors may find useful.

Input the initial flow, and calculate weight values of edges and nodes by using the dynamic road impedance function model.

On the basis of the calculation to weight values, establish a sequence of $N$ and put starting point $s$. Pick up a node $v_{i}$ from the N as the current node when N point is not null.

Calculate all adjacent nodes $\mathrm{v}_{\mathrm{j}}$ of the road network node $\mathrm{v}_{\mathrm{i}}$. If $l\left(s, v_{j}\right)>l\left(s, v_{i}\right)+l_{i j}+j_{i j}$, so $l\left(s, v_{j}\right)=l\left(s, v_{i}\right)+l_{i j}+j_{i j}$.

Add the adjacent point is modified with the shortest distance to the sequence of N , and maintain orderly increasing according to the length of each road section.

On the basis of above using the algorithm of part two backtracking one time from the destination d and get the optimal path at different periods, which are in morning peak, common and evening peak. This case considered traffic flow, saturation and intersection delay are different in morning, noon and evening in real life.

According to the above study of the mathematical model, get some different results from morning peak, common and
evening peak as shown in table 4.
Table 1. Traffic parameters

| Edge number | Road length (km) | Traffic capacity (pcu/h) |
| :---: | :---: | :---: |
| $(i, j)$ | $d_{i j}$ | $c_{i j}$ |
| $(1,2)$ | 1.5 | 4500 |
| $(1,4)$ | 2 | 3500 |
| $(2,3)$ | 1 | 4500 |
| $(2,5)$ | 1 | 3000 |
| $(3,6)$ | 1.5 | 2400 |
| $(4,5)$ | 0.5 | 2000 |
| $(4,7)$ | 1.6 | 3500 |
| $(5,6)$ | 0.5 | 2000 |
| $(5,8)$ | 1 | 3000 |
| $(6,9)$ | 0.8 | 2400 |
| $(7,8)$ | 1.8 | 2000 |
| $(7,10)$ | 0.7 | 3500 |
| $(8,9)$ | 2.1 | 2000 |
| $(8,11)$ | 1.5 | 3000 |
| $(9,12)$ | 0.8 | 2400 |
| $(10,11)$ | 0.9 | 1800 |
| $(10,13)$ | 0.7 | 3500 |
| $(11,12)$ | 1.1 | 1800 |
| $(11,14)$ | 1.3 | 3000 |
| $(12,15)$ | 2400 |  |
| $(13,14)$ | $15)$ | 4000 |
|  |  |  |

Table 2. Road intersection parameters

| Node number | The signal cycle of intersection (s) | green ratio |
| :---: | :---: | :---: |
| $v_{i}$ | $T_{i}$ | $g / c$ |
| 1 | 90 | 0.6 |
| 2 | 75 | 0.55 |
| 3 | 60 | 0.6 |
| 4 | 60 | 0.6 |
| 5 | 75 | 0.55 |
| 6 | 60 | 0.6 |
| 7 | 90 | 0.6 |
| 8 | 60 | 0.55 |
| 9 | 75 | 0.5 |
| 10 | 60 | 0.5 |
| 11 | 60 | 0.55 |
| 12 | 75 | 0.6 |
| 13 | 90 | 0.6 |
| 14 | 75 | 0.55 |
| 15 | 90 | 0.6 |

Table 3. Traffic flow and saturation at different periods

| Edge number$(i, j)$ | Morning peak |  | Common |  | Evening peak |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Flow | Saturation | Flow | Saturation | Flow | Saturation |
| $(1,2)$ | 3250 | 0.72 | 2115 | 0.47 | 3915 | 0.87 |
| $(1,4)$ | 2740 | 0.78 | 1785 | 0.51 | 2835 | 0.81 |
| $(2,3)$ | 3100 | 0.69 | 2385 | 0.53 | 3375 | 0.75 |
| $(2,5)$ | 2060 | 0.69 | 1860 | 0.62 | 2460 | 0.82 |
| $(3,6)$ | 1820 | 0.76 | 1248 | 0.52 | 1776 | 0.74 |
| $(4,5)$ | 1570 | 0.79 | 920 | 0.46 | 1520 | 0.76 |
| $(4,7)$ | 2660 | 0.76 | 1925 | 0.55 | 2975 | 0.85 |
| $(5,6)$ | 1610 | 0.81 | 1200 | 0.6 | 1520 | 0.76 |
| $(5,8)$ | 2370 | 0.79 | 1710 | 0.57 | 2310 | 0.77 |
| $(6,9)$ | 1750 | 0.73 | 1320 | 0.55 | 2016 | 0.84 |
| $(7,8)$ | 1260 | 0.63 | 980 | 0.49 | 1640 | 0.82 |
| (7, 10) | 2590 | 0.74 | 1715 | 0.49 | 2835 | 0.81 |
| $(8,9)$ | 1320 | 0.66 | 1080 | 0.54 | 1720 | 0.86 |
| $(8,11)$ | 2070 | 0.69 | 1530 | 0.51 | 2370 | 0.79 |
| $(9,12)$ | 1848 | 0.77 | 1104 | 0.46 | 2040 | 0.85 |
| $(10,11)$ | 1458 | 0.81 | 774 | 0.43 | 1476 | 0.82 |
| $(10,13)$ | 2240 | 0.64 | 1820 | 0.52 | 3010 | 0.86 |
| $(11,12)$ | 1314 | 0.73 | 810 | 0.45 | 1404 | 0.78 |
| $(11,14)$ | 2040 | 0.68 | 1170 | 0.39 | 2430 | 0.81 |
| $(12,15)$ | 1896 | 0.79 | 1032 | 0.43 | 1848 | 0.77 |
| $(13,14)$ | 3240 | 0.81 | 2000 | 0.5 | 3360 | 0.84 |
| $(14,15)$ | 3040 | 0.76 | 1760 | 0.44 | 3280 | 0.82 |

Table 4. Results of evacuation path

| Evacuation period | Evacuation path | Delay of intersection <br> (s) | Time of path (s) | Evacuation <br> time(s) |
| :---: | :---: | :---: | :---: | :---: |
| morning peak | s-13-10-7-8-5-2-3-d | 247 | 975 | 1222 |
|  | s-13-14-15-12-9-6-3-d | 226 | 998 | 1224 |
|  | s-13-10-11-8-5-6-3-d | 253 | 984 | 1237 |
| common | s-13-14-11-12-9-6-3-d | 153 | 819 | 972 |
|  | s-13-10-7-4-5-6-3-d | 167 | 811 | 978 |
|  | s-13-10-11-12-9-6-3-d | 146 | 843 | 989 |
| evening peak | s-13-10-11-12-9-6-3-d | 308 | 1097 | 1405 |
|  | s-13-14-11-8-9-6-3-d | 302 | 1109 | 1411 |
|  | s-13-10-7-4-1-2-3-d | 335 | 1082 | 1417 |

## 4. Analysis

The analysis of intersection delay and velocity for vehicles at different evacuation periods as follows.
(1) Because of the large amount of traffic flow to work in morning peak, and the road saturation is relatively high, so the traffic condition are bad that with low speed and high intersection delay. The analysis figure of vehicles evacuation velocity with intersections delay in morning peak is shown in Fig. 2.


Fig. 2. The average evacuation velocity and intersections delay of vehicles on paths in morning peak.
From the Fig. 2 as shown that the average velocity of traffic flow is about $25 \mathrm{~km} / \mathrm{h}$ in morning peak. So it has a small difference with actual data that more in accord with the actual situation. In addition, there is certain relation between the velocity and delay of nodes, and the node with a bigger delay has a relative lower velocity.
(2)There is less traffic flow in common that the number of vehicles is decreased, and a less intersection delay occurs compared with in morning peak, so the traffic tends to become a good condition. The analysis figure of vehicles evacuation velocity with intersections delay in common is shown in Fig. 2.


Fig. 3. The average evacuation velocity and intersections delay of vehicles on paths in common.
From the Fig. 3 as shown that the average velocity of traffic flow is up to about $33 \mathrm{~km} / \mathrm{h}$ in common, and it fit the real velocity. So in common period, the velocity of traffic flow is stable relatively that make for emergency evacuation.
(3) So the traffic flow for work and no-commuter vehicles are overlapped that lead to a bad traffic in evening peak. Intersection delay is more serious and the velocity of vehicles is lower than other periods. The analysis figure of vehicles evacuation velocity with intersections delay in evening peak is shown in Fig. 4.


Fig. 4. The average evacuation velocity and intersections delay of vehicles on paths in evening peak.

It can be seen from the above three compared figure, which there are big difference that velocity and delay in same paths and intersection at different periods due to the time changed. And there are also different delay in different intersection nodes at the same time. Real-time traffic conditions have to be considered as an important factor when calculating the emergency evacuation model of dynamic traffic network, so makes the model more realistic and easier to apply in practice.

So, we obtained the optimal route from the result at different periods respectively are s-13-10-7-8-5-2-3-d in morning peak, s-13-14-11-12-9-6-3-d in common and s-13-10-7-4-5-6-3-d in evening peak.

## 5. Conclusion

(1) In emergency situations, the dynamic network with evacuation time, intersection delay and evacuation paths are closely related. The evacuation results shown that morning peak period has more vehicles to work that need longer evacuation time and higher total cost than in the common.
(2) So the traffic flow is more in peak period and it will be lead to greater impact in the intersection that evacuation time is longer than non-peak period. In common period, the intersection delay is only 153 s , which far less 247 s in morning peak and 308s in evening peak. It can be seen from above three figures, the influence of traffic flow to intersection is more apparent than in the path. Pay attention to intersection factor when makes emergency evacuation plan, and put forward a solution to the intersection to ensure emergency evacuation is smoothly.
(3) The evacuation time of paths have a more difference at different periods, but the difference degree is less than the intersection. Because of the traffic flow is very larger in morning and evening peak, and the interference of vehicles is serious each other, so traffic speed is limited and led to the evacuation efficiency is decreased.
(4) We can know from the research of this paper, the resistance of road traffic network is very important for the efficiency of emergency evacuation at different periods.

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