ELSEVIER
Available online at www.sciencedirect.com
ScienceDirect
Chinese Journal of Aeronautics 21(2008) 335-340
Chinese Journal of Aeronautics
www.elsevier.com/locate/cja

An Efficient Method for Reliability-based Multidisciplinary Design Optimization

Fan Hui*, Li Weiji

School of Aeronautics, Northwestern Polytechnical University, Xi'an 710072, China

Received 8 October 2007; accepted 10 March 2008

Abstract

Design for modern engineering system is becoming multidisciplinary and incorporates practical uncertainties; therefore, it is necessary to synthesize reliability analysis and the multidisciplinary design optimization (MDO) techniques for the design of complex engineering system. An advanced first order second moment method-based concurrent subspace optimization approach is proposed based on the comparison and analysis of the existing multidisciplinary optimization techniques and the reliability analysis methods. It is seen through a canard configuration optimization for a three-surface transport that the proposed method is computationally efficient and practical with the least modification to the current deterministic optimization process.

Keywords: multidisciplinary design optimization (MDO); concurrent subspace optimization; reliability analysis; advanced first order second moment method

1 Introduction

Design for modern engineering system is becoming multidisciplinary and incorporates practical uncertainties; therefore, it is necessary to synthesize reliability analysis and the multidisciplinary design optimization (MDO) techniques[1-2] for the design of complex engineering system. Current reliability-based design optimization (RBDO) approaches may be broadly characterized as bi-level (in which the reliability analysis is nested within the optimization), sequential (in which iteration occurs between optimization and reliability analysis), and unilevel (in which the design and reliability analysis are combined into a single optimization[3]. Bi-level RBDO methods[3-4] are simple and general-purpose, but can be computationally demanding. Unilevel RBDO method develops a new formulation, which is completely different from the current deterministic optimization, which makes it difficult for the engineer to deal with. In the existing sequential RBDO framework[5], the deterministic optimization and the reliability analysis are decoupled from one another, and individual discipline feasible (IDF) method is used for multidisciplinary analysis (MDA). This makes it not very efficient. Such methods do not synthesize uncertainty analysis and MDO system very well and only introduce reliability analysis into the MDO process. This makes the whole process complex or time consuming. It is necessary to choose an appropriate reliability analysis method based on the formulation of the adopted MDO process to fully utilize the results of the deterministic design optimization, which will be efficient with acceptable accuracy.

2 MDO Method

MDO architectures[6] in the current phase in-

*Corresponding author. Tel.: +86-29-88491415.
E-mail address: fhnwpu502@163.com
Foundation item: National Natural Science Foundation of China (10377015)

clude multidisciplinary feasible (MDF), IDF, collaborative optimization (CO), concurrent subspace optimization (CSSO), and bi-level integrated synthesis system (BLISS). The key feature of CSSO is to decouple the relationship among disciplinary analyses by approximating state variables. This makes its efficiency not be influenced by the quantity of state variables and makes it possible to deal with design of the complex engineering system[7].

Typical CSSO framework includes both subspace level and system level optimization. Considering that subspace optimization only yields additional components of the design database, Doctor Li[8] proposed an improved CSSO method. Subspace "experts" only contribute to the design process by suggesting candidate designs in such improved CSSO method, and system approximations are used to provide the system optimum design. The subspace designers can suggest as many designs as the time and resources are permitted to construct the design database. Some designs are then extracted from the database to build system level response surface and perform system level optimization. The next step of the process is to extract more designs from the database to update system approximation and produce system coordination until the goal optimum converges. Performing deterministic MDO by the improved CSSO method involves: ① defining the optimization problem and designing variables and corresponding bounds; ② selecting design points based on the design of experiment (DOE); ③ making disciplinary and system analyses to develop the design database; ④ building surrogate models of objective and constraints functions; ⑤ conducting system level optimization and achieving optimum design; ⑥ updating the design points and repeating steps ③-⑤ until convergence is achieved.

Polynomial based response surface method (RSM) (second order or third order) and neural network (NN) techniques have been used to generate surrogate models. The NN model is not able to identify the effects of design variables on the final design. This lowers its credibility. Different from the NN models, polynomial based RSM can filter trivial terms and provide derivatives of object function and constraints with respect to the input variables while ensuring acceptable precision for most of the practical applications[9].

The times of MDA is at least $(n+1)(n+2)/2$ to build a complete quadratic polynomial of n input variables to achieve all coefficients of linear, quadratic, and interaction terms. Considering that in engineering system design, there are many design variables with wide range and the computation cost of single MDA is very expensive, it is necessary to choose an efficient DOE method and the polynomial construction. Uniform design extremely reduces the number of experiment or simulation runs as compared to the traditional factorial design and the central composite design under the same number of design variables at the same levels while ensuring the sampling uniformity[10]. The stepwise regression method overcomes the problem of lacking enough test cycles in the uniform design, which is an automatic tool to build a model by systematically adding the most significant variables or removing the least significant variables during each step.

3 Reliability Analysis Method

Two types of methods have been pursued for reliability analysis. One is Monte Carlo simulation, and the other is analytical approximations. Monte Carlo simulation tends to be accurate with a large number of simulations, usually 10 000 to 100 000 trials. This makes it impractical for complex multidisciplinary systems. Analytical methods include first order second moment (FOSM), advanced first order second moment (AFOSM), and second order second moment (SOSM) approximation techniques. The FOSM method is very simple and requires minimal computation effort but sacrifices accuracy for nonlinear limit state or systems with non-normal input variables. The accuracy of the SOSM method is improved compared with that of the FOSM method, but its computation effort is also greatly increased and this makes it not frequently used in practices. The AFOSM method, a more accurate analytical approach than the FOSM method, is able to handle correlated, non-normal random variables, and nonlinear limit states[11-12] and is applied in most practical cases.

$x_1, x_2, \cdots, x_n$ are a set of random variables and each random variable is assumed as normal distributed for the sake of simplification (non-normal random variables can be transformed to normal through equivalent normal method). The merit function is shown as $Z = g_x(x_1, x_2, \cdots, x_n)$ and the limit state equation is expressed as $Z = g_x(x_1, x_2, \cdots, x_n) = 0$. At the most probable point (MPP), $x^* = (x_1^*, x_2^*, \cdots, x_n^*)$, the merit function is expanded to the following form:

$$Z_{\mathrm{L}} = g_x(x_1^*, x_2^*, \cdots, x_n^*) + \sum_{i=1}^{n} \frac{\partial g_x(x^*)}{\partial X_i}(X_i - x_i^*) \quad (1)$$

The reliability index is shown as[12]

$$\beta = \frac{\mu_{Z_{\mathrm{L}}}}{\sigma_{Z_{\mathrm{L}}}} = \frac{g_x(x_1^*, x_2^*, \cdots, x_n^*) + \sum_{i=1}^{n} \frac{\partial g_x(x^*)}{\partial X_i}(\mu_{x_i} - x_i^*)}{\sqrt{\sum_{i=1}^{n} [\frac{\partial g_x(x^*)}{\partial X_i}\sigma_{x_i}]^2}} \quad (2)$$

Supplementary equations are needed to achieve the reliability index because MPP is unknown[12]

$$x_i^* = \mu_{x_i} + \beta\sigma_{x_i} \cos\theta_{x_i} \quad (i = 1, 2, \cdots, n) \quad (3)$$

$$\cos\theta_{x_i} = -\frac{\partial g_x(x^*)}{\partial X_i}\sigma_{x_i} \Bigg/ \sqrt{\sum_{j=1}^{n} [\frac{\partial g_x(x^*)}{\partial X_j}\sigma_{x_j}]^2} \quad (i = 1, 2, \cdots, n) \quad (4)$$

The initial MPP is selected, then β and new MPP can be achieved by iteratively executing equations Eqs.(1)-(4) when the derivatives of merit function with respect to input variables are known. When convergence is achieved, $g_x(x_1^*, x_2^*, \cdots, x_n^*) = 0$ and Eq.(2) can be transformed to

$$\beta = \frac{\mu_{Z_{\mathrm{L}}}}{\sigma_{Z_{\mathrm{L}}}} = \frac{\sum_{i=1}^{n} \frac{\partial g_x(x^*)}{\partial X_i}(\mu_{x_i} - x_i^*)}{\sqrt{\sum_{i=1}^{n} [\frac{\partial g_x(x^*)}{\partial X_i}\sigma_{x_i}]^2}} \quad (5)$$

4 AFOSM-based CSSO Process

The uncertainties of RBDO are modeled as random design variables and random operation parameters. A typical RBDO problem can be formulated as follows

$$\begin{aligned} &\min \ f(d, p, y(d, p)) \\ &\text{s.t.} \ \ g^{\mathrm{rc}}(X, \eta) \geq 0 \\ &\qquad g_j^{\mathrm{D}}(d, p, y(d, p)) \geq 0 \quad (j = 1, \cdots, N_{\mathrm{soft}}) \\ &\qquad d^{\mathrm{l}} \leq d \leq d^{\mathrm{u}} \end{aligned} \quad (6)$$

where d is the design variable, p the operation parameter, X the random variable, y the state variable, g^{rc} the reliability constraint, and g^{D} the deterministic constraint. Reliability constraints can be formulated as

$$g_i^{\mathrm{rc}} = \beta_i - \beta_{\mathrm{reqd},\,i} \quad (7)$$

where β_i is the reliability index due to the ith failure mode at the given design, and $\beta_{\mathrm{reqd},\,i}$ the required reliability level of this failure mode.

Reliability constraints can be set up using Eq.(5) since analytical approximation in CSSO can provide the derivatives of constraints with respect to the input variables. The initial design point for RBDO is selected as the result of the deterministic optimization to avoid premature convergence and spurious optimal design. A new RBDO framework is developed by integrating AFOSM into CSSO multidisciplinary design system. It involves: ① performing deterministic optimization by CSSO to achieve initial optimal design, the derivatives of constraints with respect to random variables, and approximations of object function/constraints; ② finding MPP by AFOSM; ③ setting up reliability constraints at the MPP using Eq.(5); ④ finding the optimal design as the mean values of random variables using the polynomial based RSM; ⑤ updating the MPPs using new values of the design variables and operation parameters. Steps ③-⑤ are repeated until convergence is achieved. The whole process is shown as Fig.1. MATLAB optimization toolbox is used as commercial optimizer to improve the operation efficiency in the process.

The new approach is based on deterministic design optimization and realizes smooth transition from the current process to the RBDO; secondly, the results of deterministic design optimization can be fully utilized to reduce the computational cost. Such results include initial design point of RBDO, the derivatives of constraints with respect to random variables, which are needed in reliability analysis, and approximations of object function/constraints; finally, reliability analysis is decoupled from the optimization process. As seen in Fig.1, the computation of MPPs is not performed inside the optimization loop. This makes it extremely efficient.

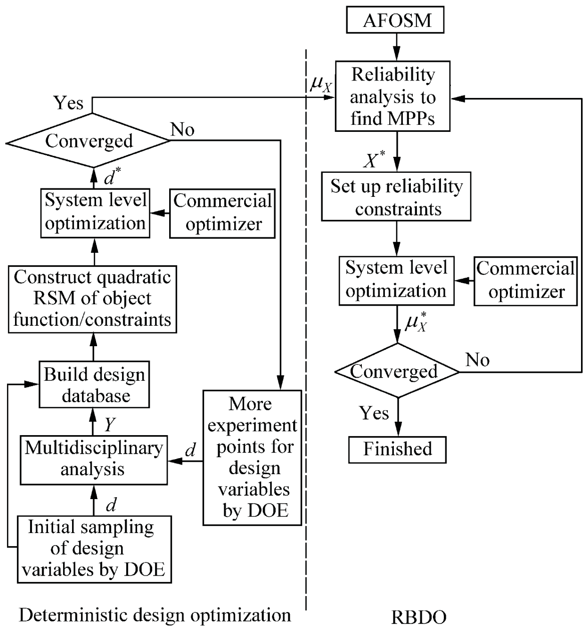

Fig.1 Flowchart of RBDO-CSSO framework.

5 Application Example

A canard configuration optimization for a three-surface transport is taken as an example to verify the new AFOSM-based CSSO method. The gross take-off weight of the transport is chosen as an objective function for this study while the fuselage, wing (wingspan is 47 m and reference area reaches 310 m^2), and vertical tail designs are fixed. The goal is to evaluate the influence of the canard on the conventional transport.

5.1 Design optimization formulation

The CAD model of the transport built by Pro/Engineer includes fuselage, canard, wing, vertical tail, horizontal tail, and engines. The design variables of the canard and their corresponding bounds are listed in Table 1. The definitions of the related parameters are shown in Fig.2.

Table 1 Design variables and optimization results

Design variables	Lower bound	Upper bound	Deterministic optimization	RBDO
Canard span/m	14.00	18.00	14.28	15.68
Canard aspect ratio	6.00	8.50	7.78	7.40
X position of the canard/m	3.00	6.00	4.53	3.85
Canard tip ratio	0.15	0.40	0.23	0.22
Z position of the canard/m	1.00	4.00	1.22	1.20

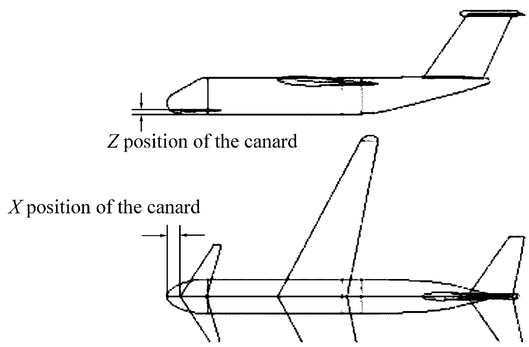

Fig.2 Parameter definitions of canard.

The fixed parameters are predefined as follows referring to current typical transport[13]: crew is 3; payload 48 t; cruise speed $Ma = 0.74$ at altitude of 11 000 m; four engines, single engine specific fuel consumption (SFC) during cruise set as 0.52 kg/(kg·h^{-1}) and 2 100 kg by weight. The performance constraints include: the probabilities of longitudinal static margin no less than 7% and range no less than 4 500 km are both larger than 99.3%, and equivalent reliability index $\beta = 2.5$[12]. The optimization goal is to minimize the gross take-off weight W_0 (including empty weight W_{empty}, fuel weight W_{fuel}, payload weight W_{payload}, and crew weight W_{crew}).

The lift to drag ratio (*L*/*D*) and the gross take-off weight are required to obtain the range. Rigid longitudinal trim should be performed in advance to get the weight of the horizontal tail. Thus, there is coupling among aerodynamics, weight, and performance disciplines in the canard configuration design problem. The RBDO problem is expressed as follows

$$\begin{aligned} &\min W_0 = W_{\text{crew}} + W_{\text{payload}} + W_{\text{fuel}} + W_{\text{empty}} \\ &\text{s.t.} \quad \beta(1 - 4\,500 / R > 0) > 2.5 \\ &\qquad \beta(K_n \geq 7\%) > 2.5 \end{aligned} \tag{8}$$

Payload, cruise velocity, and engine SFC are characterized as random variables. These are given in Table 2.

Table 2 Random variables

Variable	Mean	Coefficient of variable	Distribution
Payload/kg	48 000	0.05	Normal
Cruise velocity/($\text{m}\cdot\text{s}^{-1}$)	200-240	0.05	Uniform
Engine SFC/($\text{kg}\cdot \text{kg}^{-1}\cdot\text{h}$)	0.52	0.06	Normal

5.2 Optimization results and discussion

Uniform experimental scheme is chosen for 5 factors with 4 levels in the order of U_{12} (4^5), U_{20} (4^5), U_{28} (4^5), and U_{36} (4^5). CAD model of the transport is constructed by Pro/TOOLKIT[14] programming of Pro/Engineer system. Aerodynamic analysis exports the total lift, drag, and aerodynamic center location of the specific layout design by computing the wing incidence angle (angle of attack during cruise is zero to ensure cabin floor is horizontal) and the horizontal tail area. Additional canard changes the load distribution of the lift surface. This indicates that horizontal tail area can be achieved only by the longitudinal trim method (it is rigid trim without considering aeroelasticity in this article) but not by the frequently used tail-volume coefficient method. The elevator deflection angle is assumed to be constant for simplification. Aerodynamic analysis is done by applying the engineering methods and equations listed in Ref.[15]. These methods have been demonstrated to be very effective in the conceptual design of subsonic aircraft. Range estimation is done using Brequet range equation. Weight distribution and the gross take-off weight are estimated through the empirical and statistical method. Finally, the location of the center of gravity and the longitudinal static stability margin are calculated[15].

The quadratic response surface of the object function/constraints is constructed based on the design database. Then, the deterministic design optimization and RBDO are executed in sequential order according to Fig.1. The optimal results are listed in Table 1. The analyses show that a canard in low position, with high aspect ratio, low taper ratio, and moderate canard span promises optimum performance, which is compatible with the research result of Strohmeyer[16].

Parameters comparison between the three-surface aircraft and the conventional transport is listed in Table 3. It is seen that the gross take-off weight of the three-surface aircraft is considerably lower than that of the conventional aircraft while keeping the same payload by re-allocating the load on the horizontal tail. This actually keeps the structural weight and greatly reduces fuel by decreasing the size of the horizontal tail and increasing the lift to drag ratio. It should be noted that the gross take-off weight, the size of the horizontal tail and canard of the RBDO are larger than those of the deterministic design optimization since uncertainties are being considered. Thus, RBDO can achieve a more compromising design, which balances operation risks and performance.

Table 3 Comparison of designs

Parameters	Conventional	Deterministic design optimization	RBDO
Gross take-off weight/kg	220 000	197 850	202 300
L/D in cruise	20.5	22.1	21.8
Area of horizontal tail/m^2	45.20	37.80	39.42
Area of canard/m^2	Not applicable	26.20	33.22

6 Conclusions

Current RBDO frameworks do not integrate reliability analysis into the MDO system very well and are not efficient enough to be applied in engineering applications. A new RBDO framework is developed by combining the AFOSM method with the concurrent subspace technique to deal with random operation parameters and design variables. Compared with the current RBDO methods, the new approach brings no change to the formulation of the deterministic design optimization and fully utilizes the results of the former. It also decouples reliability analysis with the optimization process. As mentioned, the computation of MPPs is not performed inside the optimization loop. This makes it extremely efficient. The proposed method is verified through the design optimization of a canard configuration for a three-surface transport.

References

[1] Huang J, Wu Z, Sun H Z, et al. Recent developments in conceptual/preliminary design optimization of aircraft. Acta Aeronautica et Astronautica Sinica 2000; 21(6): 481-487. [in Chinese]

[2] Batill S M, Renaud J E, Gu X Y. Modeling and simulation uncertainty in multidisciplinary design optimization. AIAA-2000-4803, 2000.

[3] Adams B M, Eldred M S, Wittwer J W. Reliability-based design optimization for shape design of compliant micro-electro-mechanical systems. AIAA-2006-7000, 2006.

[4] Agarwal H, Renaud J E. A unilevel method for reliability based design optimization[J]. AIAA-2004-2029, 2004.

[5] Ahn J, Lee J, Kim S, et al. Sequential reliability analysis framework for multidisciplinary systems. AIAA-2004-4517, 2004.

[6] Yu X Q, Ding Y L. Multidisciplinary design optimization: a survey of its algorithms and application to aircraft design. Acta Aeronautica et Astronautica Sinica 2000; 21(1): 1-6. [in Chinese]

[7] Stelmack M A, Batill S M. Application of the concurrent subspace design framework to aircraft brake component design optimization. AIAA-98-2033, 1998.

[8] Li X. Multidisciplinary design optimization method and its application in aircraft design. PhD thesis, Xi'an: Northwestern Polytechnical University, 2003. [in Chinese]

[9] Rodriguez D L. Response surface based optimization with a cartesian CFD method. AIAA-2003-0465, 2003.

[10] Liu W Q. Design of experiments. Beijing: Tsinghua University Press, 2005. [in Chinese]

[11] Wang Y P, Lu Z Z, Yue Z F. Reliability analysis for implicit limit state equation. Applied Mathematics and Mechanics 2005; 26(9): 1158-1164.

[12] Gong J X. Methods for reliability assessment in engineering systems. Dalian: Dalian University of Technology Press, 2003. [in Chinese]

[13] China Aero-information Center *Word Aircraft Handbook* Editorial Team. World aircraft handbook. Beijing: Aviation Industry Press, 2001. [in Chinese]

[14] Zhang J C. Tutorial of programming with Pro/Engineer system. Beijing: Peking University Press, 2003. [in Chinese]

[15] Roskam J. Airplane design: Parts I-VIII. Lawrence, Kansas: DAR-corporation, 1997.

[16] Strohmeyer D, Heinze W, Österheld C, et al. Three surface aircraft–a concept for future transport aircraft. AIAA-2000-0566, 2000.

Biographies:

Fan Hui Born in 1976, he received B.S. and M.S. from Northwestern Polytechnical University in 1997 and 2000, respectively. His main research interest is multidisciplinary design optimization.
E-mail: fhnwpu502@163.com

Li Weiji Born in 1939, he is a professor and Ph.D. supervisor in Northwestern Polytechnical University. His main research area covers aircraft conceptual design, and optimization theory and methods.
E-mail: lwjnwpu@126.com