Studies and Applications of Three Kinds of Calculation Methods by Describing Damage Evolving Behaviors for Elastic-Plastic Materials

YU Yargui¹, LIU Xiang²

¹. Zhejiang GuangXin Academy of New Technology Application of Electromechanical and Chemical Engineering, Hangzhou Science Park of Chinese Academy of Science, Hangzhou 310012, China
². School of Information and Electronic Engineering, Zhejiang Science and Technology University, Hangzhou 310023, China

Abstract: By means of bidirectional combined coordinate system, three kinds of calculation methods are proposed with respect to the damage evolving rate and the life of elastic plastic material, which include the single parameter method, the ratio method and the multiplication method. In this work a lot of new calculation equations are given; a new concept on the all-around material constant is provided, which has functional relations with each of the typical material parameters: the fatigue strength coefficient \( f_1 \), the fatigue strength exponent \( f_1' \), the fatigue ductility coefficient \( f_6 \), the fatigue ductility exponent \( f_6' \), the average stress, the average strain, critical loading time and so on. In addition, an example of a car part is given, and some comparisons of calculation results are made. The calculation methods will have practical significance in avoiding the unnecessary fatigue tests, saving time, manpower and capital, as well as providing the convenience for engineering applications in a certain degree.

Keywords: single parameter; ratio; multiplication method; damage; calculation

Upto date, numerous scientists have suggested various kinds of the calculation expressions of fatigue damage of structure and material, which include the Dowling’s equation, the Landgraf’s \( e \) equation, the energy equation, and so on. Their works have made valuable contributions to experimental researches and engineering applications. The special features of these equations are that they all use such typical material constants as the fatigue strength coefficient \( f_1 \), the fatigue strength exponent \( f_1' \), the fatigue ductility coefficient \( f_6 \), the fatigue ductility exponent \( f_6' \), the cyclic strength coefficient \( K' \) and the strain hardening exponent \( n' \). And these material constants have been widely accepted and applied in every engineering domain. But these equations do not include the concrete physical parameters shown about a structure damage (for instance, the damage variable \( D \), the mi-
crack size \( a_0 \), the dislocation loop size \( \lambda \). On the other hand, many other scientific researchers have suggested the damage evolving equations in connection with a damage parameter \( D \) in modern fatigue damage discipline. Li Changchun provided the damage evolving equation connected to the dislocation loop size \([1] \); Murakami proposed an equation corresponding to the small crack size \( a \) on its initiation and propagation problems. However, these equations include new material constants, which are not used by general material handbooks. Therefore the new equations cannot be applied in engineering now, because they still require more tests, until these material constants are quite reliable. In this way, it has yet to give a great volume of the arm of flesh, materials and bankrolls. The authors of the present paper suggest some new damage evolving equations for describing the elastic-plastic behaviors of material based on the latter research results, these equations adopt both the former material constants used by engineering applications in wide range, and the damage parameter \( D \) (or the \( a, \lambda \), etc.) used in new damage evolving equations lately. It is conceivable that the equations suggested here can avoid the unnecessary fatigue test and will be of practical significance to save time, manpower and capital.

1 Studies and Analyses of Three Kinds of Calculation Equations

1.1 The single parameter method (\( \Delta \sigma \) or \( \Delta \varepsilon_p \)) calculation

In order to use curves to explain the new damage evolving rate and the relating life expressions suggested by each kind of methods, here the bidirectional double logarithmic coordinate system and bidirectional curves are given. In Fig. 1, the upward direction along the ordinate axis is presented as the damage evolving rate \( \frac{dD}{dN} \) and the downward direction, presented as the each history life \( N_{\text{ini}} \). The distance \( OO_1 \) between axis \( O1 \) and \( O1 \parallel \) is shown as the region from noncrack to microcrack initiation; and the distance \( O1O_2 \) between \( O1 \parallel \) and \( O2 \parallel \) as the region to be relative to life \( N_{\text{mic/mac}} \) from microcrack growth until macrocrack generation. In the positive direction coordinate system \( dD/dN=\Delta H / 2 \), the curve 1 \( (ABA_1) \) and 2 \( (A'B_1A_2) \) show the varying regularities of elastic-plastic material behaviors \( (\varepsilon_c > \varepsilon_p) \) under high cycle loading that it can be described by the following equation \([2] \)

\[
dD/dN = A_1 \Delta H^{m_1} = A_1 \Delta \sigma^{m_2} D \tag{1}
\]

where \( \Delta H \) is defined to be the damage stress factor range, \( \Delta H = \Delta \sigma / \sigma_0 \), \( \Delta \sigma \) is a local stress range value, \( A_1 \) is a comprehensive property parameter of a material which is a function value to bear relations to fatigue strength coefficient \( \sigma'_f \), fatigue strength exponent \( b_1 \) and loading history \( (\ln D_{\text{mac}} - \ln D_{\text{0}}) \), here \( m_1 = \frac{1}{b_1} \).

\[
\begin{align*}
A_1 &= 2(2 \frac{\sigma_{ij}^{\text{mac}}}{\sigma_{ij}^{\text{0}}} - 1)^{-m_1 \left( \ln D_{\text{mac}} - \ln D_{\text{0}} \right)} \quad \text{for} \quad \sigma = 0 \quad \tag{2} \\
A_1 &= 2\left( \frac{\sigma_{ij}^{\text{mac}}}{\sigma_{ij}^{\text{0}}} - 1 - \sigma_{ij}^{\text{mac}} / \sigma_{ij}^{\text{0}} \right)^{-m_1 \left( \ln D_{\text{mac}} - \ln D_{\text{0}} \right)} \quad \text{for} \quad \sigma_{ij}^{\text{mac}} \neq 0 \quad \tag{3}
\end{align*}
\]

Where \( D_{\text{mac}} \) is a damage value corresponding to the macrocrack forming size \( a_{\text{mac}} \) of a specimen material, as \( a_{\text{mac}} = 0 \). \( ? \). 1.0 mm, \( D_{\text{0}} = 0 \), \( ? \). 1.0 \( \delta^{[4]} \) (for \( \sigma_{ij}^{\text{mac}} = 0 \) at point \( A_1 \) or \( \ln \left( \frac{\sigma_{ij}^{\text{0}}}{E} \right) \); for \( \sigma_{ij}^{\text{mac}} \neq 0 \), at point \( A_2 \) or \( \ln \left( \frac{\sigma_{ij}^{\text{mac}}}{E} \right) \). \( D_{\text{0}} \) is a baseline damage value corresponding to the microcrack forming size \( a_0 \). When its surface or interior grain

of a material commences damaging and forming microcrack a0, then it may be for average grain size d** instead of the a0. So each history life Noi from baseline damage to macrocrack forming should be such that
\[ N_{oi} = \frac{(\ln D_{a} - \ln D_{0})(\Delta\sigma)^{-m_1}}{2(\dot{\epsilon}_0)^{1 - m_1}(\ln D_{mac} - \ln D_{0})} \text{ (for } \sigma_m = 0) \] (4)

and
\[ N_{oi} = \frac{(\ln D_{a} - \ln D_{0})(\Delta\sigma)^{-m_1}}{2(\dot{\epsilon}_0)^{1 - m_1}(\ln D_{mac} - \ln D_{0})} \text{ (for } \sigma_m \neq 0) \] (5)

Here the relating curves of the Eqs. (4) and (5) are the inverted curves A1BA and A2B1A3 respectively.

On the other hand, the positive direction curve 3 (CBC1) shows the varying regularities of plastic material behaviors (\( \varepsilon_i > \varepsilon_c \)) under low cycle loading, it can be described by the following equation
\[
d\bar{D}/dN = B_1\Delta l^{\nu_1} = B_1\Delta l^{\nu_1}, D \] (6)

Here \( \Delta l \) is defined to be the damage strain factor range. \( \Delta l = \Delta \varepsilon_{pl}D^{\nu_1} \), \( \Delta \varepsilon_{pl} \) is a local strain range value, and \( B_1 \) is also a comprehensive property parameter of material which is a function value concerned with fatigue ductility coefficient \( \varepsilon_f \), experiment \( \nu_1 \) and \( m_1 = 1/\nu_1 \), Similarly its function values are as follows,
\[
B_1 = \frac{2(\dot{\epsilon}_0)^{1 - m_1}(\ln D_{mac} - \ln D_{0})}{(\ln \varepsilon_f) - (\ln \varepsilon_c)} \text{ (for } \varepsilon_m = 0) 
\] (7)

and
\[
B_1' = \frac{2(2\dot{\epsilon}_0)(1 - \varepsilon_m/\varepsilon_c)\varepsilon_f}{(\ln D_{mac} - \ln D_{0})} \text{ (for } \varepsilon_m \neq 0) 
\] (8)

Here it should be pointed out that \( \varepsilon_m \) is usually equal to 0 in Eq. (8). \( D_{mac} \) is a damage value corresponding to \( \dot{\epsilon}_f \) at point c1). Eqs. (7) and (8) are substituted into the Eq. (6), to obtain each life \( N_{oi} \), relative to(\( \ln D_{mac} - \ln D_{0} \), respectively.
\[
N_{oi} = \frac{\ln D_{a} - \ln D_{0}}{2(\dot{\epsilon}_0)^{1 - m_1}(\ln D_{mac} - \ln D_{0})(\Delta \varepsilon_{pl})^{-m_1}} \text{ (for } \sigma_m = 0) \] (9)

\[
N_{oi} = \frac{\ln D_{a} - \ln D_{0}}{2\left[1 - \frac{\varepsilon_m}{\varepsilon_c}\right]^{1 - m_1}(\ln D_{mac} - \ln D_{0})(\Delta \varepsilon_{pl})^{-m_1}} \text{ (for } \sigma_m \neq 0) \] (10)

It is well known that the Dowling’s equations are
\[
D = \frac{1}{N} = 2\left(\frac{\dot{\epsilon}_0}{\varepsilon_c}\right)^{1/\nu_1}, \text{ (} \varepsilon_c > \varepsilon_p \) (11)
\]
\[
D = \frac{1}{N} = 2\left(\frac{\dot{\epsilon}_0}{\varepsilon_c}\right)^{1/\nu_1}, \text{ (} \sigma_m = 0 \) (12)
\]

and
\[
D = \frac{1}{N} = 2\left(\frac{\dot{\epsilon}_0}{\varepsilon_c}\right)^{1/\nu_1}, \text{ (} \varepsilon_c < \varepsilon_p \) (13)
\]

Due to the above mentioned \( m_1 = 1/\nu_1 \) and \( m_1' = 1/\nu_1' \), if the variation of the damage parameter \( D_{oi} \) in loading process is not considered, and only both \( D_{oi} \) and \( D_{mac} \) of origination and termination are calculated, then it will be as follows,
\[
\ln D_{a} - \ln D_{0} = \ln D_{mac} - \ln D_{0} \] (14)

So the Eqs. (1), (4), (5), (6), (9) and (10) are in agreement with the Dowling’s Eqs. (11), (12) and (13). But, in the method of the single parameter \( \Delta \sigma \) or \( \Delta \varepsilon_{pl} \), the new concepts of all-around material constants are provided definitely, which have the function relations with \( \dot{\epsilon}_f, \nu_1, (\ln D_{mac} - \ln D_{0}) \) under high cycle loading, and with \( \dot{\epsilon}_f, c_1, (\ln D_{mac} - \ln D_{0}) \) under low cycle loading.

1.2 The ratio method \( \varepsilon_{pl}/\varepsilon_c(\Delta \varepsilon_{pl}/\Delta \sigma) \) calculation

The authors studied and considered that the elastic-plastic behavior of a material in damage evolving process can be described with the ratio \( \varepsilon_{pl}/\varepsilon_c(\Delta \varepsilon_{pl}/\Delta \sigma) \). And \( d\bar{D}/dN \) related to the curves 4 and 5 (AC1 and A3C1) should be as follows \(^{[6]}\),
\[
d\bar{D}/dN = C(\Delta \varepsilon_{pl}/\Delta \sigma)^{m_1/m_1'}, D \] (15)

where \( C = 2(\dot{\epsilon}_f)^{1 - m_1/m_1'}(\ln D_{mac} - \ln D_{0}) \text{ (for } \sigma_m = 0) \)

\[
C^* = 2(\dot{\epsilon}_f^{1 - \varepsilon_m/\varepsilon_c})^{1 - m_1/m_1'}(\ln D_{mac} - \ln D_{0}) \text{ (for } \sigma_m \neq 0, \varepsilon_m \neq 0) \] (17)

using C in Eq. (16) and \( C^* \) in Eq. (17) instead of the C in Eq. (15), the equations of life \( N_{oi} \) can be obtained as Eqs. (18) and (19),
The elastic-plastic behaviours of a material in damage evolving process can also be described with the multiplication $E_\varepsilon \varepsilon_e (\Delta \varepsilon_0 \Delta \varepsilon)$ [8]. And the $dN$’s relation with curve $AC_1$ should be
\[
dD/dN = A^* (\Delta \varepsilon_0 \Delta \varepsilon)^{m_{11} \over m_{11} m_{11}^*} \] (24)
in case of $\sigma_m=0$,
\[
A^* = 2 (4 \varepsilon_1^i \varepsilon_1^j)^{m_{11} \over m_{11} m_{11}^*} (\ln D_{mac} - \ln D_0) \] (25)
For $\sigma_m \neq 0$, using $A^*$ instead of $A'$,
\[
A^* = 2 (4 \varepsilon_1^i \varepsilon_1^j)^{m_{11} \over m_{11} m_{11}^*} (\ln D_{mac} - \ln D_0) \] (26)

Then the following equations are obtained,
\[
N_{ai} = { (\ln D_{ai}) - (\ln D_0) } (\Delta \varepsilon_0 \Delta \varepsilon)^{m_{11} \over m_{11} m_{11}^*} \] (27)
\[
N_{ai} = { (\ln D_{ai}) - (\ln D_0) } (\Delta \varepsilon_0 \Delta \varepsilon)^{m_{11} \over m_{11} m_{11}^*} \] (28)

And the equations of energy method are
\[
D = 1 \over N = 2 \left( 4 \varepsilon_1^i \varepsilon_1^j \right) \varepsilon_1^i \varepsilon_1^j \] (29)
\[
D = 1 \over N = 2 \left( 4 \varepsilon_1^i \varepsilon_1^j \right) \varepsilon_1^i \varepsilon_1^j \] (30)

Therefore, the multiplication method Eqs. (24), (27) and (28) are in agreement with the Eqs. (29) and (30) of energy method.

2 Example

A part in a car is made of rolled steel, and its curves of nominal stress vs time and local stress vs local strain are shown in Fig. 2, when it is loaded.

The local stress and strain can be calculated. Here the Neuber’s Eq. (31) and the cyclic stress-strain Eq. (32) are adopted [9].
\[
\Delta \sigma \Delta \varepsilon = K \Delta \varepsilon^2 / E \] (31)
\[
\Delta \varepsilon = \Delta \sigma / 2E + (\Delta \sigma / 2k)^{1 \over n} \] (32)

where $\Delta \sigma$ is the range of the nominal stress, $K$ is the effective stress concentration coefficient, $E$ is the elastic modulus of the material. On the other
From a different perspective, the damage of the part can also be calculated by the above-mentioned methods in Fig. 2. For instance, by means of the Dowling’s Eqs. (12) and (13), the Landgraf’s Eq. (23) and the energy method Eq. (30) and the Eq. (1), Eq. (6) of single parameter method, the Eq. (15) of the ratio method and the Eq. (24) of multiplication method, respectively. Then the calculated results of various methods are all put in the Table 1 and 2, so that they can be compared easily.

**Table 1  The comparison of various methods of damage calculation**

<table>
<thead>
<tr>
<th>Material</th>
<th>σ₀ / MPa</th>
<th>φ</th>
<th>E / MPa</th>
<th>K / MPa</th>
<th>n’</th>
<th>d’ / MPa</th>
<th>b’</th>
<th>m₁</th>
<th>c₁</th>
<th>m’₁</th>
<th>Kₙ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolled steel</td>
<td>320</td>
<td>0.67</td>
<td>1.92×10⁵</td>
<td>1125.9</td>
<td>0.193</td>
<td>935.9</td>
<td>−0.095</td>
<td>10.526</td>
<td>0.26</td>
<td>−0.07</td>
<td>2.22</td>
</tr>
<tr>
<td>Range of stress</td>
<td>Δσ₁ / MPa</td>
<td>Δσ₂ / MPa</td>
<td>Δσ₂ / MPa</td>
<td>Δσ₄ / MPa</td>
<td>Δσ₆ / MPa</td>
<td>Δσ₄ / MPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stress</td>
<td>395.5</td>
<td>699</td>
<td>521</td>
<td>791</td>
<td>434</td>
<td>240</td>
<td>656.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculation</td>
<td>Stress cycle</td>
<td>Number of equations</td>
<td>Δσ₀ / MPa</td>
<td>Δσ₁ / MPa</td>
<td>Δσ₂ / MPa</td>
<td>Δσ₄ / MPa</td>
<td>Δσ₆ / MPa</td>
<td>Δσ₄ / MPa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>of damage</td>
<td>stress and strain</td>
<td>(31)</td>
<td>780</td>
<td>0.0122</td>
<td>−21.7</td>
<td>0.0017</td>
<td>0.0882</td>
<td>0.0041</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10)</td>
<td>520</td>
<td>0.0038</td>
<td>3.8</td>
<td>0.0047</td>
<td>0.0010</td>
<td>0.0027</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(32)</td>
<td>910</td>
<td>0.024</td>
<td>3.3</td>
<td>0</td>
<td>0.0183</td>
<td>0.0047</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Calculation | Stress cycle | Number of equations | Number of equations | Calculation results |
| of damage | Dowling’s equation | Calculation results | Calculation results |
| Cumulated damage | D₁ = D₁ + D₂ + D₃ = 2.087×10⁻³ | D₁ = 1/N = 3.95×10⁻⁶ | (15) | (dD/dN)₁ = 3.43×10⁻⁴ |
| Life B = 1/D = 479.2 (number of cyclic loading segments) |

| Calculation | Stress cycle | Number of equations | Number of equations | Calculation results |
| of damage | Landgraf’s equation | Calculation results | Calculation results |
| Cumulated damage | D₁ = D₁ + D₂ + D₃ = 2.142×10⁻³ | D₁ = 1/N = 3.565×10⁻⁶ | (15) | (dD/dN)₁ = 3.38×10⁻⁶ |
| Life B = 1/D = 466.98 (number of cyclic loading segments) |

| Calculation | Stress cycle | Number of equations | Number of equations | Calculation results |
| of damage | The ratio method | Calculation results | Calculation results |
| Cumulated damage | D₁ = D₁ + D₂ + D₃ = 1.76418×10⁻³ | D₁ = 1/N = 1.50×10⁻³ | (24) | (dD/dN)₁ = 1.50×10⁻³ |
| Life, N = hB, h is the cyclic loading time for each segment, B = 1/D = 566.8 |

3 Discussions

The peculiarities suggested from the above methods consist in:

(1) They give definitely the expressions for calculating the damage evolving rate \(dD/dN\) and the life \(N_{\text{ao}}\) of corresponding various damage values \(D_{\text{ao}}\).

(2) They suggest a new concept of the all-around material constant, which is functionally related with each staple material constants \(\bar{a}, b'_{1}, \bar{c}, c'_{1}\) and average stress \(\sigma_{m}\), average strain \(\varepsilon_{m}\), and critical loading history (\(\ln D_{\text{mac}}-\ln D_{0}\)). So when the values of \(D_{\text{mac}}\) and \(D_{0}\) are accurately stipulated, the calculated values of damage and life may be more exact.

(3) Base on the standpoint to account for the crack size \(a\) also as a damage variable like the damage variable \(D\), the damage parameter \(D\) in each \(\sigma\)-equation for calculating damage rate and various history lifes \(N_{\text{ao}}\) may be yet converted into another physical parameter besides the calculation approach mentioned above. It is necessary and possible to describe concretely the damage of a material (for example, using a micro-crack size \(a^{[10,11]}\) and a contraction of area \(\Phi\), etc.). But here must be ordaining that \(D_{\text{ao}} < D < D_{\text{mac}}\), their units are all values of dimensionless, \(D_{\text{mac}} = 0.7-1.0\). And \(a_{0} < a < a_{\text{mac}}\), their units are all millimeter, \(a_{\text{mac}} = 0.7-1.0\) mm. So \(D\) and \(a\) can be treated as a relation of equivalent values.

(4) It is suggested in Ref.\([12]\) that there is a consanguineous relation between the damage parameter \(D\) and the contraction of area of a specimen, where the \(D\) is defined as the Eq. (33)

\[
D = 1 - \frac{A_{1}}{A_{\text{ld}}} \tag{33}
\]

here, \(A_{1}\) and \(A_{\text{ld}}\) are the sectional areas of statically tensional fracture for undamaged and damaged specimen respectively. All appearance, Eqs. (1), (6), (15) and (24) are only substituted by Eq. (33), so it is not difficult to derive out the various formal converted expressions.

(5) The calculation results from Table 1 and 2 can also be seen that the Dowling’s equation and the single parameter method, as well as the energy method and the multiplication method, are all accordant; and the ratio method equation and the Landgraf’s equation are also almost coincidental. But the calculation precision by the ratio method equations is more rigorous, because the influence of \(\varepsilon_{m} \neq 0\) is considered and Eq. (17) is used in calculation for all-around material constant \(C^{*}\).

4 Conclusions

(1) For symmetric and un-symmetric cyclic loading, the equations of damage evolving rate and the life estimation expression calculated by above three kinds of methods are full coincidental with the Dowling’s, the Landgraf’s and the energy equations under \(\ln D_{\text{ao}} - \ln D_{0} = \ln D_{\text{mac}} - \ln D_{0}\). But the significance of these three kinds of methods consists in: it not only suggests definitely the life estimation expression relative to various damage value \(D_{\text{ao}}\), but also gives out a new concept of the all-around material constant having functional relation with the \(\bar{a}, \bar{c}, \sigma_{m}, \varepsilon_{m}, b'_{1}, c'_{1}\) and \(\ln D_{\text{mac}}-\ln D_{0}\).

(2) In the new damage calculation, it considers both the component of \(\Delta \varepsilon_{p}\) and the component of \(\Delta \varepsilon_{p}\) in equations, so it may be more comprehensive and more exact; and for \(\varepsilon_{m} \neq 0\), calculation precision may be more rigorous than other methods.

(3) The equations given by three kinds of methods all can avoid the shortcomings of the unnecessary fatigue test and the difficulty of being short of new material constants, and assimilate the advantage using the six typic material constants \((K', n', \bar{a}, b'_{1}, \bar{c}, c'_{1})\) and the modernistic damage parameter \(D\).

(4) The damage parameters \(D\) in each of \(\sigma\)-equations for calculating the damage rate and the various history lifes \(N_{\text{ao}}\) can be converted into another physical parameter (or \(a\) or \(\lambda\), or \(A_{\text{ld}}\) etc.). Therefore it will be of practical significance to save test times, manpower and capital.
References


Biographies:

YU Yaurui Male, born in 1936, he is a dean of Zhejiang GuangXin Academy of New Technology Application of Electro Mechanical and Chemical Engineering, professor of Wenzhou University, pluralistic professor of Zhejiang Science & Technology University, pluralistic professor of Zhenzhou Teachers College, honorary director of Computer Application Institute. He studied the engineering applications of subjects on fatigue damage fracture, the technology of assistant calculation and auto control by computer on compressors and pressure vessel, and has published over 80 pieces of scientific papers and accomplished 12 items in the scientific research and engineering technology fields in Zhejiang and Liaoning Province. Tel: 86 577 88102037; 13067777178 E-mail: gx_yyg@126.com

LIU Xiang He is an instructor in School of Information and Electronic Engineering of Zhejiang Science and Technology University. Tel: + 86 577 88102037; E-mail: hn_lxa@126.com