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Buoyancy induced MHD transient mass transfer flow with thermal radiation

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Abstract The problem of a transient MHD free convective mass transfer flow past an infinite vertical porous plate in presence of thermal radiation is studied. The fluid is considered to be a gray, absorbing-emitting radiating but non-scattered medium. Analytical solutions of the equations governing the flow problem are obtained. The effects of mass transfer, suction, radiation and the applied magnetic field on the flow and transport characteristics are discussed through graphs.

KEYWORDS
Gray fluid; Thermal radiation; Free convection; Hh functions

1. Introduction

MHD is the science of motion of electrically conducting fluids in presence of magnetic field. It concerns with the interaction of magnetic field with the fluid velocity of electrically conducting fluid. MHD generators, MHD pumps and MHD flow meters are some of the numerous examples of MHD principles. Dynamo and motor are classical examples of MHD principle. Convection problems of electrically conducting fluid in presence of magnetic field have got much importance because of its wide applications in Geophysics, Astrophysics, Plasma Physics, Missile technology, etc. MHD principles also find its applications in Medicine and Biology.

The natural flow arises in fluid when the temperature as well as species concentration change causes density variation leading to buoyancy forces acting on the fluid. Free convection is a process of heat or mass transfer in natural flow. The heating of rooms and buildings by use of radiator is an example of heat transfer by free convection.

On the other hand, the principles of mass transfer are relevant to the working of systems such as a home humidifier and the dispersion of smoke released from a chimney into the environment. The evaporation of alcohol from a container is an example of mass transfer by free convection. Radiation is also a process of heat transfer through electromagnetic waves. Radiative convective flows are encountered in countless industrial and environment processes such as heating and cooling chambers, evaporation from large open water reservoirs, astrophysical flows and solar power technology. Due to importance of the above physical aspects, several authors have carried out model studies on the problems of free convective hydrodynamic and magneto-hydrodynamic flows of incompressible viscous electrically conducting fluids under different flow geometries and physical conditions taking into account of thermal radiation. Some of them are Mansour [1], Ganesan and Loganathan [2], Mbeledogu et al. [3], Makinde [4], Samad and Rahman [5], Orhan and Ahmet [6], Prasad et al. [7], Takhar et al. [8], Gebhart et al. [9], Ali et al. [10], Hussain et al. [11], Hussain et al. [12], Ghaly [13], Muthucumaraswamy and Janakiraman [16], Muthucumaraswamy and Sivakumar [17], Ahmed and Dutta [18], Ahmed [19], Muthucumaraswamy et al. [20]. The effect of thermal radiation together with first order chemical reaction

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has been investigated by Rajesh [21] adopting an implicit finite difference scheme of Crank-Nicolson method. Several authors have adopted the finite element method for performing model studies to investigate the effects of thermal radiation on the extent and mass transfer characteristics in different hydrodynamical flows under different physical and geometrical conditions, the names of whom Sheri et al. [22], Sheri and Rao [23] and Siviah [24] are worth mentioning. Perdikis and Rapti [14] have obtained an analytical solution to the problem of an MHD unsteady free convective flow past an infinite vertical porous plate in the presence of radiation with time dependent suction. In the work of Perdikis and Rapti [14], the mass transfer effect is not taken into account. Further their investigation ignored the effects of the physical parameters on the transport characteristics which seem to be very important from the physical point of view.

In the present work, an attempt has been made to generalize the problem investigated by Perdikis and Rapti [14], to study the mass transfer effect together with the effects of different physical parameters on the flow and the transport characteristics. It is seen that the results of the present work for some limiting cases are in excellent agreement with those of Perdikis and Rapti [14].

2. Mathematical analysis

The equations governing the motion of an incompressible, viscous, electrically conducting and radiating fluid in the presence of a magnetic field having constant mass diffusivity are as follows:

Continuity equation:
$$\nabla \cdot \mathbf{q} = 0$$

(1)

Magnetic field continuity equation:
$$\nabla \cdot \mathbf{B} = 0$$

(2)

Ohm’s law:
$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{q} \times \mathbf{B})$$

(3)

MHD momentum equation with buoyancy force:
$$\rho \left[ \frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g} + \mu \nabla^2 \mathbf{q}$$

(4)

Energy equation:
$$\rho C_p \left[ \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T \right] = k \nabla^2 T + \phi + \frac{\mathbf{J} \cdot \mathbf{E}}{\sigma} - \nabla \cdot \mathbf{q}$$

(5)

Species continuity equation:
$$\frac{\partial C}{\partial t} + (\mathbf{q} \cdot \nabla) C = D \nabla^2 C$$

(6)

All the physical quantities are defined in the Nomenclature.

We now consider the unsteady MHD two-dimensional free convective mass transfer flow of an incompressible viscous and electrically conducting fluid bounded by an infinite vertical porous plate. A magnetic field of constant flux density is
assumed to be applied normal to the plate directed into the fluid region. The fluid is a gray, emitting and absorbing radiating, but non-scattered medium, and the Rosseland approximation is used to describe the radiative heat flux in the energy equation.

In order to make the mathematical model of the present work idealized, the present investigation is restricted to the following assumptions.

I. All the fluid properties are considered constants except the influence of the density with temperature and concentration in the buoyancy force.

II. The viscous and Ohmic dissipations of energy are negligible.

III. The magnetic Reynolds number is small.

IV. The plate is electrically non-conducting.

V. The radiation heat flux in the direction of the plate velocity is considered negligible in comparison with that in the normal direction.

VI. No external electric field is applied for which the polarization voltage is negligible leading to \( \mathbf{E} = \mathbf{0} \).

We introduce a rectangular Cartesian coordinate system \((x, y, z)\) with \(X-\text{axis}\) along the plate in the upward vertical direction, \(Y-\text{axis}\) normal to the plate and directed into the fluid region and \(Z-\text{axis}\) along the width of the plate. Let \(\mathbf{q} = (u, v, 0)\) denote the fluid velocity, \(\mathbf{B} = (0, B_0, 0)\) be the magnetic flux density and \(\mathbf{q}_r = (0, q_r, 0)\) be the radiation flux at the point \((x', y', z', t')\) in the fluid.

Eq. (1) yields

\[
\frac{\partial v}{\partial y} = 0
\] (7)

On the basis of the assumption (VI), infinite plate assumption, and by the virtue of Eq. (3), the momentum Eq. (4) splits to the following equations:

\[
\rho \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial x} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \rho g - \frac{\partial p}{\partial x} - \sigma B_0 u
\]  
(8)

\[
\frac{\partial p}{\partial y} = 0
\] (9)

Eq. (9) shows that \(\rho\) is independent of \(y\) indicating the fact that the pressure inside the boundary layer is the same as the pressure outside the boundary layer along a normal to the plate and due to this fact, Eq. (8) in the free stream takes the form:

\[
\frac{\partial p}{\partial x} = -\rho \omega g
\]  
(10)

The elimination of the term \(\frac{\partial u}{\partial x}\) from Eqs. (8) and (10) gives,

\[
\rho \left( \frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (\rho \omega - \rho) g - \sigma B_0 u
\]  
(11)

The equation of state on the basis of classical Boussinesq approximation is

\[
\rho \omega = \rho \left[ 1 + \beta (T - T_\infty) + \beta_0 (C - C_\infty) \right]
\]  
(12)

On unification of Eqs. (11) and (12), we establish the following non-linear partial differential equation:

\[
\frac{\partial u}{\partial y} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_\infty) + \beta_0 (C - C_\infty) \nabla B_0 u
\]  
(13)

The assumption (II) leads the energy Eq. (5) to reduce to the form

\[
\rho C_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} \nabla \frac{\partial T}{\partial y}
\]  
(14)

The reduced form of the mass diffusion Eq. (6) is as given below:

\[
\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}
\]  
(15)

Eq. (7) leads that \(v\) is function of time only and we take the suction velocity \(v\) as follows:

\[
v = -a \left( \frac{\partial T}{\partial y} \right)^{1/2}
\]  
(16)

In Eq. (16), the negative sign signifies the fact that the direction of the suction velocity is toward the plate.

The radiation heat flux by using the Rosseland approximation, is given by

\[
q_r = -\frac{4\sigma T^4}{3k} \frac{\partial T}{\partial y}
\]  
(17)

We further impose the following restrictions:

I: The plate temperature and the temperature away from the plate are proportional to \(T^m\)

II: The plate concentration and the concentration away from the plate are proportional to \(C^m\), where \(m\) is a non-negative integer constant.

III: The plate as well as the fluid far away from the plate is at rest.

IV: The difference between the fluid temperature \(T\) and \(T_\infty\) is very small.

Under these assumptions, the appropriate boundary conditions for the velocity, temperature and concentration fields are defined as

\[
\begin{align*}
  u(0, t) &= 0, \quad u(\infty, t) = 0 \\
  T(0, t) &= T_\infty + L T_1, T(\infty, t) \to T_\infty \\
  C(0, t) &= C_\infty + L C_1, C(\infty, t) \to C_\infty
\end{align*}
\]  
(18)

On the basis of the restriction (IV), we may expand \(f(T) = T^4\) in Taylor’s series about \(T_\infty\) as described below:

\[
f(T) = T^4 = f(T_\infty) + (T - T_\infty) f'(T_\infty) + \cdots
\]  
(19)

By neglecting the higher powers \(T - T_\infty\) in (19), we obtain

\[
T^4 = 4T_\infty^4 - 3T_\infty^2
\]  
(20)

Now Eq. (17), accomplished by Eq. (20) transforms to

\[
q_r = -\frac{16\sigma T_\infty^3}{3k} \frac{\partial T}{\partial y}
\]  
(21)

On the use of the relation (21), Eq. (14) becomes

\[
\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k} \frac{\partial T}{\partial y}
\]  
(22)

On substitution of the expression for \(v\) from Eq. (16), Eqs. (13), (22) and (15) respectively transforms to
\[
\frac{\partial u}{\partial t} - a \left( \frac{v}{u} \right)^{1/2} \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 \theta}{\partial y^2} + g\beta \theta + g\beta \varphi - \frac{\sigma B_{0}^2 u}{\rho} \tag{23}
\]

\[
\frac{\partial \theta}{\partial t} - a \left( \frac{v}{u} \right)^{1/2} \frac{\partial \theta}{\partial y} = \frac{1}{\rho C_{p}} \left( k + 16\alpha^{*} T^{3} \right) \frac{\partial^2 \theta}{\partial y^2} \tag{24}
\]

\[
\frac{\partial \varphi}{\partial t} - a \left( \frac{v}{u} \right)^{1/2} \frac{\partial \varphi}{\partial y} = D \frac{\partial^2 \varphi}{\partial y^2} \tag{25}
\]

where \( \theta = T - T_{\infty} \) and \( \varphi = C - C_{\infty} \).

The boundary conditions to be satisfied by Eqs. (23), (24), and (25) now become

\[
\begin{align*}
\begin{cases}
u(0,t) = 0, & u(\infty, t) = 0 \\
\theta(0, t) = L\theta^m, & \theta(\infty, t) \rightarrow T_{\infty} \\
\varphi(0, t) = L^m \varphi, & \varphi(\infty, t) \rightarrow C_{\infty}
\end{cases}
\end{align*} \tag{26}
\]

Proceeding with the analysis, we introduce the following non-dimensional variables and similarity parameters to normalize the flow model.

\[
Pr = \frac{\mu C_{p}}{k}, \quad M = \frac{\sigma B_{0}^2 l}{\rho}, \quad Sc = \frac{v}{D}, \quad N = \frac{kk^*}{4\alpha^* T^3_{\infty}}, \tag{27}
\]

\[
\lambda = \frac{\beta L^m}{\beta L}, \quad \eta = \frac{y}{2\lambda^{1/2}}
\]

All the above quantities are defined in the Nomenclature as mentioned earlier.

We define the velocity, temperature and concentration fields as

\[
\begin{align*}
u(\eta, t) &= Lg\beta \theta^m + \bar{u}(\eta, t) \\
\theta(\eta, t) &= L^m \theta(\eta) \\
\varphi(\eta, t) &= L^m \varphi(\eta)
\end{align*}
\]

where \( \bar{u}(\eta, t) = u_0(\eta) + Mu_1(\eta) + M^2 u_2(\eta) + \cdots \)

For small values of \( M \), the substitutions of Eqs. (27)–(30) into Eqs. (23)–(25) and the comparison of the coefficients of the like powers of \( M \), give the following differential equations:

\[
\begin{align*}
\psi''(\eta) + 2Sc(\eta + a)\psi'(\eta) - 4mSc\psi(\eta) &= 0 \tag{31} \\
f''(\eta) + 2c(\eta + a)f'(\eta) - 4mf(\eta) &= 0 \tag{32} \\
u''_c(\eta) + 2(\eta + a)u'_c(\eta) - 4(m + 1)u_0(\eta) &= -4f(\eta) - 4\lambda \psi(\eta) \tag{33} \\
u''_c(\eta) + 2(\eta + a)u''_c(\eta) - 4(m + 2)u_1(\eta) &= -4u_0(\eta) \tag{34} \\
u''_c(\eta) + 2(\eta + a)u''_c(\eta) - 4(m + 3)u_2(\eta) &= 4u_0(\eta) \tag{35}
\end{align*}
\]

where a dash denotes differentiation with respect to \( \eta \), and \( c = \frac{3\lambda^{1/4}}{M} \).

The corresponding boundary conditions (26) reduce to

\[
\begin{align*}
\psi(0) &= 1, \quad \psi(\infty) \rightarrow 0 \\
f(0) &= 1, \quad f(\infty) \rightarrow 0 \\
u_0(0) &= 0, \quad u_0(\infty) \rightarrow 0 \\
u_1(0) &= 0, \quad u_1(\infty) \rightarrow 0 \\
u_2(0) &= 0, \quad u_2(\infty) \rightarrow 0
\end{align*} \tag{36}
\]

3. Method of solutions

To solve Eqs. (31)–(35) subject to the conditions (36), we consider the following transformations:

\[
\zeta = \eta + a, \quad \zeta = \sqrt{2Sc\zeta}, \quad \tilde{\rho} = \sqrt{2\zeta}, \quad \zeta = \sqrt{2\zeta}
\]

In the aid of the transformations (37), Eqs. (31)–(35) in new system are

\[
\begin{align*}
\frac{d^2 \psi}{d\zeta^2} + \frac{\zeta d\psi}{d\zeta} - 2mp\psi &= 0 \tag{38} \\
\frac{d^2 f}{dp^2} + \frac{d f}{dp} &= 0 \tag{39} \\
\frac{d^2 u_0}{d\zeta^2} + \epsilon d u_0 + 2(m + 1)u_0 &= -2f(\eta) - 2\lambda \psi(\eta) \tag{40} \\
\frac{d^2 u_1}{d\zeta^2} + \epsilon d u_1 + 2(m + 2)u_1 &= 2u_0 \tag{41} \\
\frac{d^2 u_2}{d\zeta^2} + \epsilon d u_2 - 2(m + 3)u_1 &= 2u_1 \tag{42}
\end{align*}
\]

The transformed boundary conditions become

\[
\begin{align*}
\psi(\sqrt{2Sc\zeta}) &= 1, \quad \psi(\infty) \rightarrow 0 \\
f(\sqrt{2Sc\zeta}) &= 1, \quad f(\infty) \rightarrow 0 \\
u_0(\sqrt{2\zeta}) &= 0, \quad u_0(\infty) \rightarrow 0 \\
u_1(\sqrt{2\zeta}) &= 0, \quad u_1(\infty) \rightarrow 0 \\
u_2(\sqrt{2\zeta}) &= 0, \quad u_2(\infty) \rightarrow 0
\end{align*} \tag{43}
\]

The solutions of Eqs. (38)–(42) under the conditions (43) are

\[
\begin{align*}
\psi &= A_1 H_{2m} \left( \sqrt{2Sc\zeta} \right) \tag{44} \\
f &= A_2 H_{2m} \left( \sqrt{2Sc\zeta} \right) \tag{45}
\end{align*}
\]

\[
u_0 = \begin{cases}
  A_1 H_{2m+2} \left( \sqrt{2\zeta} \right) + B_2 H_{2m+2} \left( \sqrt{2\zeta} \right) + C_3 H_{2m+2} \left( \sqrt{2Sc\zeta} \right); & c \neq 1, Sc \neq 1 \\
  A_1 H_{2m+2} \left( \sqrt{2\zeta} \right) + B_4 H_{2m+2} \left( \sqrt{2Sc\zeta} \right) + C_3 H_{2m+2} \left( \sqrt{2Sc\zeta} \right); & c = 1, Sc \neq 1 \\
  A_1 H_{2m+2} \left( \sqrt{2\zeta} \right) + B_5 H_{2m+2} \left( \sqrt{2Sc\zeta} \right) + C_4 H_{2m+2} \left( \sqrt{2Sc\zeta} \right); & c \neq 1, Sc = 1 \\
  B_5 \left[ H_{2m} \left( \sqrt{2\zeta} \right) - \frac{H_{2m} \left( \sqrt{2\zeta} \right)}{H_{2m} \left( \sqrt{2\zeta} \right)} \right] H_{2m+2} \left( \sqrt{2\zeta} \right); & c = 1, Sc = 1
\end{cases} \tag{46}
\]
where the function ‘$H_{h_n}$’ is defined as follows (Jeffrey and Jeffrey [15]):

$$H_{h_n}(x) = \begin{cases} 
\int_{-\infty}^{x} \frac{(-1)^{n+1} e^{-\nu t}}{\nu^n} dt; & n \in \mathbb{Z}, n \geq 0 \\
(-1)^{n+1} \left( \frac{x}{\nu} \right)^n e^{-\nu x}; & n \in \mathbb{Z}, n < 0
\end{cases}$$

And the constants involved in the above solutions are defined by

$$A_1 = \frac{1}{H_{h_2}(\sqrt{2}c)}, \quad A_2 = \frac{1}{H_{h_2}(\sqrt{2}c)}, \quad B_3 = \frac{2A_2}{1 - c},$$

$$C_3 = \frac{2A_1}{1 - c},$$

$$A_3 = \frac{B_1 H_{h_{n-2}}(\sqrt{2}a) + C_1 H_{h_{n-2}}(\sqrt{2}c)}{H_{h_{n-2}}(\sqrt{2}a)},$$

$$A_4 = \frac{B_1 H_{h_n}(\sqrt{2}a) + C_1 H_{h_{n-2}}(\sqrt{2}c)}{H_{h_{n-2}}(\sqrt{2}a)},$$

$$B_4 = \frac{1}{H_{h_2}(\sqrt{2}a)}, \quad C_4 = \lambda B_4, \quad B_5 = (1 + \lambda) B_4,$$

$$B_6 = \frac{2B_5}{c - 1}, \quad C_6 = \frac{2C_3}{c - 1},$$

$$A_5 = \frac{B_1 H_{h_{n-2}}(\sqrt{2}a) + C_1 H_{h_2}(\sqrt{2}a)}{H_{h_{n-2}}(\sqrt{2}a)},$$

$$A_6 = \frac{B_1 H_{h_2}(\sqrt{2}a)}{H_{h_{n-2}}(\sqrt{2}a)}.$$
\[ B_{13} = \frac{A_3}{2}, \quad C_{13} = \frac{2B_3}{c - 1}, \quad E_2 = -\frac{C_3}{3}, \quad F_1 = -\frac{F}{2}, \quad G_1 = -\frac{G}{3} \]

\[ A_{13} = \frac{1}{Hh_{2n+1}} (\sqrt{2a}) \left[ A_3 Hh_{2n+1} (\sqrt{2a}) - B_{13} Hh_{2n+2} (\sqrt{2a}) - C_{13} Hh_{2n+4} (\sqrt{2a}) \right] \]

\[ A_{14} = \frac{1}{Hh_{2n+1}(\sqrt{2a})} \left[ E H h_{2n+2} (\sqrt{2a}) - F_1 H h_{2n+2} (\sqrt{2a}) - G_1 H h_{2n} (\sqrt{2a}) \right] \]

### 4. Coefficient of skin friction

The viscous drag at the plate per unit area in the direction of the plate velocity is quantified by the Newton’s law of viscosity in the form:

\[ \tau = \frac{\mu \partial u}{\partial y} \bigg|_{y=0} = \frac{L \rho \beta \sqrt{2a}}{2} \frac{\partial u}{\partial y} \bigg|_{y=0} \]  \hspace{1cm} (49)

The coefficient of the skin friction at the plate is as follows:

\[ \tau = \frac{2 \tau}{L \beta \rho \sqrt{2a}} = \frac{\partial u}{\partial y} \bigg|_{y=0} \]

\[ = u_0(0) + M u_0(0) + M^2 u_0(0) + \cdots \]

\[ = \tau_0 + M^2 \tau_1 + M^2 \tau_2 + \cdots \]  \hspace{1cm} (50)

where

\[ \tau_0 = \begin{cases} 
-\sqrt{2} [A_3 H h_{2n+1} (\sqrt{2a}) + \sqrt{c} B_{13} H h_{2n+2} (\sqrt{2a}) + \sqrt{Sc} C_{13} H h_{2n+4} (\sqrt{2a})] 
&; c \neq 1, Sc \neq 1 \\
-\sqrt{2} [A_3 H h_{2n+1} (\sqrt{2a}) + B_{13} H h_{2n+2} (\sqrt{2a}) + \sqrt{Sc} C_{13} H h_{2n+4} (\sqrt{2a})] 
&; c = 1, Sc \neq 1 \\
-\sqrt{2} [A_3 H h_{2n+1} (\sqrt{2a}) + \sqrt{Sc} C_{13} H h_{2n+4} (\sqrt{2a})] 
&; c \neq 1, Sc = 1 \\
\end{cases} \]

\[ \tau_1 = \begin{cases} 
-\sqrt{2} \left[ A_3 H h_{2n+1} (\sqrt{2a}) - \sqrt{c} A_3 H h_{2n+1} (\sqrt{2a}) + \sqrt{Sc} C_{13} H h_{2n+4} (\sqrt{2a}) \right] 
&; c \neq 1, Sc \neq 1 \\
-\sqrt{2} \left[ A_3 H h_{2n+1} (\sqrt{2a}) - A_3 H h_{2n+1} (\sqrt{2a}) + \sqrt{Sc} C_{13} H h_{2n+4} (\sqrt{2a}) \right] 
&; c = 1, Sc \neq 1 \\
-\sqrt{2} \left[ A_3 H h_{2n+1} (\sqrt{2a}) - A_3 H h_{2n+1} (\sqrt{2a}) + \sqrt{Sc} C_{13} H h_{2n+4} (\sqrt{2a}) \right] 
&; c \neq 1, Sc = 1 \\
-\sqrt{2} [E H H h_{2n+1} (\sqrt{2a}) + F H H h_{2n+1} (\sqrt{2a}) + G H h_{2n+1} (\sqrt{2a})] 
&; c = 1, Sc = 1 \\
\end{cases} \]

### 5. Coefficient of rate of heat transfer

The heat flux \( q' \) from the plate to the fluid is quantified by the Fourier law of conduction in the form

\[ q' = -\kappa \frac{\partial T}{\partial y} \bigg|_{y=0} = -\kappa \sqrt{16 a^2 + \frac{L}{2}} \frac{f'(0)}{2} \]  \hspace{1cm} (54)

The coefficient of the rate of heat transfer from the plate to the fluid in terms of the Nusselt number is given by

\[ Nu = \frac{q'}{2 \sqrt{a}} = -f'(0) = \sqrt{2} A_2 H h_{2n+2} (\sqrt{2a}) \]  \hspace{1cm} (55)
6. Coefficient of mass transfer

The mass flux $M_w$ at the plate is determined by the Fick’s law of mass diffusion

$$M_w = -D \frac{\partial C}{\partial y} \bigg|_{y=0} = -\frac{DL \beta}{2\sqrt{\pi \tau}} \psi'(0)$$  \hspace{1cm} (56)

The coefficient of mass transfer at the plate in terms of Sherwood number is given by (see Fig. 1)

$$Sh = \frac{2\sqrt{\pi} M_w \beta}{DL \beta \rho m} = -\psi'(0) = \sqrt{2Sc} \frac{A}{H_t} H_{2m-1} \left(\sqrt{2Sc} \right)$$

$$= \frac{\partial \psi}{\partial y} \bigg|_{y=0}$$  \hspace{1cm} (57)

7. Results and discussion

In order to get clear insight of the physical problem, numerical computations from the analytical solutions for representative velocity field, temperature field, concentration field, and the coefficient of skin friction, the coefficient of the rate of heat transfer in terms of Nusselt number and the rate of mass transfer in terms of Sherwood number at the plate have been carried out by assigning some arbitrarily chosen specific values to the similarity parameters such as magnetic parameter $M$ (square of the Hartmann number), radiation parameter $N$, suction parameter $a$, the constant $m$, the constant ratio $\lambda$ and the normal coordinate $\eta$. Throughout our investigation, the value of the Prandtl number $Pr$ has been chosen to be 0.71 which corresponds to air as the numerical computations are concerned. That is to say that there is a substantial drop in concentration for high mass diffusivity. This observation is consistent with the physical fact that when mass (solute) diffuses in the solvent at a high rate, the concentration level of the medium of diffusion gets enhanced. Further the three figures as expected show that $\psi(\eta)$ decreases asymptotically as the normal coordinate $\eta$ increases.

The velocity profiles under the influence of the parameters $m$, $\lambda$, $N$, $Sc$, $M$ and $a$, the suction parameter are exhibited in Figs. 5–10. It is observed from Fig. 6 that the fluid flow is accelerated due to the effect of the parameter $\lambda$, while an increase in each of the values of the parameters $m$ and $Sc$ causes the flow to decelerate considerably as seen in Figs. 5 and 8. Fig. 7 establishes the fact that the thermal radiation has also some contributions in controlling the growth of the thickness of the velocity boundary layer to some extent. It is inferred from Fig. 9 that an increase in the magnetic parameter $M$ has an inhibiting effect on the fluid velocity to some extent. The fluid velocity $u$ gets continuously reduced with increasing $M$. In other words, the imposition of the transverse magnetic field causes the flow to retard slowly and steadily. This phenomenon has an excellent agreement with the physical fact that the Lorentz force that appears due to interaction of the transverse magnetic field with the fluid velocity acting as a resistive force to the fluid flow which serves to decelerate the flow. The variation in fluid velocity under the effect of suction is presented in Fig. 10. It is observed in this figure that like magnetic field, the effect of suction also results in a substantial decrease in the fluid velocity. As such the imposition of the magnetic field as well as the suction is an effective regulatory mechanism for the flow regime. All Figs. 5–10 uniquely establish the fact that the fluid velocity first increases in a thin layer adjacent to the plate and thereafter it decreases asymptotically indicating the fact that the buoyancy force has a significant effect on the flow near the plate and its effect gets nullified far away from the plate.

Figs. 11–15 correspond to the coefficient of skin friction $\tau$ at the plate against the suction parameter $a$ under the influence of $m$, $\lambda$, $N$, $Sc$, $M$. A trend of decay in the coefficient of skin friction $\tau$ is clearly marked in Figs. 11 and 15 due to increasing $m$ and $Sc$, thereby reducing the viscous drag at the plate. On the other hand, the frictional resistance of the fluid at the plate is seen to be enhanced under the effect radiation, imposition of

![Figure 1](image1.png)

**Figure 1** Flow configuration.

![Figure 2](image2.png)

**Figure 2** Concentration versus $\eta$ for $m = 1, a = 0.5$. 
Figure 3  Concentration versus $\eta$ for $Sc = 0.60$, $a = 0.5$.

Figure 4  Concentration versus $\eta$ for $Sc = 0.60$, $m = 1$.

Figure 5  Velocity versus $\eta$ for $N = 3$, $Pr = 0.71$, $a = 0.5$, $Sc = 0.60$, $\lambda = 0$, $M = 0.01$.

Figure 6  Velocity versus $\eta$ for $N = 3$, $Pr = 0.71$, $a = 0.5$, $Sc = 0.60$, $m = 1$, $M = 0.01$.

Figure 7  Velocity versus $\eta$ for $m = 1$, $Pr = 0.71$, $a = 0.5$, $Sc = 0.60$, $\lambda = 3$, $M = 0.01$.

Figure 8  Velocity versus $\eta$ for $N = 3$, $Pr = 0.71$, $a = 0.5$, $m = 1$, $\lambda = 3$, $M = 0.01$. 
Figure 9  Velocity versus $\eta$ for $N = 3$, $Pr = 0.71$, $a = 0.5$, $Sc = 0.60$, $\lambda = 0$, $m = 1$.

Figure 10  Velocity versus $\eta$ for $N = 3$, $Pr = 0.71$, $m = 1$, $Sc = 0.60$, $\lambda = 3$, $M = 0.01$.

Figure 11  Skin friction versus $a$ for $N = 3$, $Pr = 0.71$, $Sc = 0.60$, $\lambda = 1$, $M = 0.01$.

Figure 12  Skin friction versus $a$ for $N = 3$, $Pr = 0.71$, $Sc = 0.60$, $m = 1$, $M = 0.01$.

Figure 13  Skin friction versus $a$ for $m = 1$, $Pr = 0.71$, $Sc = 0.60$, $\lambda = 3$, $M = 0.01$.

Figure 14  Skin friction versus $a$ for $N = 3$, $Pr = 0.71$, $Sc = 0.60$, $\lambda = 3$, $m = 1$. 
Figure 15  Skin friction versus $a$ for $N = 3$, $Pr = 0.71$, $M = 0.01$, $\lambda = 3$, $m = 1$.

Figure 16  Nusselt number versus $a$ for $N = 3$, $Pr = 0.71$.

Figure 17  Nusselt number versus $a$ for $m = 1$, $Pr = 0.71$.

Figure 18  Sherwood number versus $a$ for $m = 1$.

Figure 19  Sherwood number versus $a$ for $Sc = 0.60$.

Figure 20  Velocity versus $\eta$ (Figure No. 6 of Perdikis and Rapti [14]).
the transverse magnetic field and for increasing values of \( \lambda \) as observed from Figs. 12–14. An interesting behavior of the skin friction \( \tau \) at the plate against the suction parameter \( a \) is observed in Fig. 14. This figure indicates that in the absence of the magnetic field, the drag force due to viscosity gets reduced under the effect of suction whereas in the presence of magnetic field, this behavior takes a reverse turn.

The effects of the parameters \( m \), \( a \) and \( \lambda \) on the Nusselt number \( Nu \) are displayed in Figs. 16 and 17. These figures simulate that an increase in each of the values of the above parameters leads the rate of heat transfer from the plate to the fluid to increase substantially. Figs. 18 and 19 present how the rate mass transfer at the plate in terms of Sherwood number \( Sh \) is affected by the Schmidt number \( Sc \), the suction parameter \( a \) and parameter \( m \). Like Nusselt number, the Sherwood number is also enhanced comprehensively under the influence of the aforesaid parameters. Further, it is noticed that the effect of radiation on the rate of heat transfer, and the effect of mass diffusion on the Sherwood number are more pronounced for large suction.

8. Comparison of results

In order to highlight the accuracy of the numerical computations from the analytical solutions in the present work, one of the results of the present study for a special case has been compared with those of Perdikis and Rapti [14]. Comparing Figs. 5 and 20 (Fig. 6 of the work done by Perdikis and Rapti [14]), we see that the two figures are almost identical as the behavior of the velocity field against the normal coordinate \( \eta \) under the influence of the parameter \( m \) is concerned. There is an excellent agreement between the results of the present work and those of Perdikis and Rapti [14].

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References


