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# Using $\pi_2(1670) \to b_1(1235)\pi$ to constrain hadronic models

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**Abstract**

We show that current analyses of experimental data indicate that the strong decay mode $\pi_2 \to b_1\pi$ is anomalously small. Non-relativistic quark models with spin-1 quark pair creation, such as $^3P_0$, $^3S_1$ and $^3D_1$ models, as well as instanton and lowest order one-boson (in this case $\pi$) emission models, can accommodate the analyses of experimental data, because of a quark-spin selection rule. Models and effects that violate this selection rule, such as higher order one-boson emission models, as well as mixing with other Fock states, may be constrained by the small $\pi_2 \to b_1\pi$ decay. This can provide a viability check on newly proposed decay mechanisms. We show that for mesons made up of a heavy quark and anti-quark, the selection rule is exact to all orders of quantum chromodynamics (QCD) perturbation theory.



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## 1. Analyses of experimental data on $\pi_2(1670) \to b_1(1235)\pi$

Recently, the VES Collaboration published for the first time an upper bound of 0.0019 on the branching fraction for Br[$\pi_2 \to b_1\pi$], at the 97.7% confidence level. This branching fraction is measured in 37 GeV $\pi^-$ collisions on a nucleus, in the reaction $\pi^- A \to \omega\pi^-\pi^0 A^*$ [1]. This small branching fraction is consistent with a preliminary analysis performed by the E852 Collaboration [2] of data on the reaction $\pi^- p \to \omega\pi^-\pi^0 p$, in collisions of an 18 GeV $\pi^-$ beam with a proton target.

The decay $\pi_2 \to b_1\pi$ is allowed by conservation of parity, angular momentum, isospin and G-parity, and so its strength should be comparable with that of other decays which are allowed by the same quantum numbers, which are conserved to an extraordinary degree by the strong interactions. In order to show that the branching ratio is small for dynamical reasons, independent of any model, factors due to phase space and flavor should be removed.

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Table 1
Branching fractions, and ratios $R(X) = |\mathcal{M}(X)|^2/|\mathcal{M}(f_2\pi)|^2$ and $\widetilde{R}(X) = |\widetilde{\mathcal{M}}(X)|^2/|\widetilde{\mathcal{M}}(f_2\pi)|^2$ of partial widths with phase space and flavor factors removed to those of the dominant decay mode. $\mathcal{M}$ and $\widetilde{\mathcal{M}}$ are defined in the text. The decay is assumed to proceed via the bold-faced $L$ wave, since in all modes (except for $f_2\pi$, where [3] the $D$ wave is $(0.18 \pm 0.06)^2 = (3.2 \pm 2.2)\%$ of the $S$ wave) the contributions from the different partial waves are not known. Although the branching fractions do not add to unity, since Ref. [3] constrained a subset of these modes by unitarity, those outside of this subset were defined relative to the dominant $f_2\pi$ mode, and so this does not affect the ratios $R(X)$ and $\widetilde{R}(X)$. The constraint for the $\rho(1450)\pi$ mode is incorrectly quoted [18] in Refs. [1,3] and should read $\mathrm{Br}[\pi_2(1670) \to \rho(1450)\pi]\,\mathrm{Br}[\rho(1450) \to \omega\pi] < 0.36\%$. Since $\mathrm{Br}[\rho(1450) \to \omega\pi]$ is poorly known, estimates for a branching ratio of a third are provided

| Mode $X$ | $p$ (GeV) | $L$ | $f^2$ | $\mathrm{Br}(\pi_2 \to X)$ (%) [3] | $R(X)$ | $\widetilde{R}(X)$ |
|---|---|---|---|---|---|---|
| $f_2\pi$ | 0.326 | **S**, D, G | 2 | $56.2 \pm 3.2$ | 1.00 | 1.00 |
| $\sigma\pi$ | 0.634 | **D** | 2 | $13 \pm 6$ | 0.73 | 1.00 |
| $\omega\rho$ | 0.308 | **P**, F | 2 | $2.7 \pm 1.1$ | 0.53 | 0.53 |
| $\rho(1450)\pi$ | 0.143 | **P**, F | 4 | $< 0.36 \times 3$ | $< 0.36 \times 3$ | $< 0.33 \times 3$ |
| $\rho\pi$ | 0.649 | **P**, F | 4 | $31 \pm 4$ | 0.33 | 0.46 |
| $K\overline{K}^*$ | 0.450 | **P**, F | 2 | $4.2 \pm 1.4$ | 0.27 | 0.30 |
| $b_1\pi$ | 0.363 | **D** | 4 | $< 0.19$ | $< 0.09$ | $< 0.09$ |

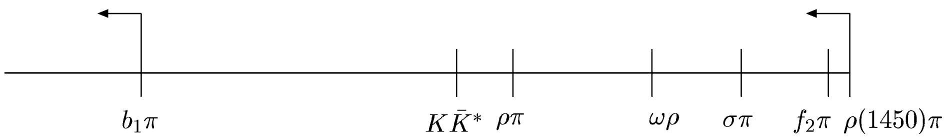


Fig. 1. Ratios ($|\mathcal{M}(X)|^2/|\mathcal{M}(f_2\pi)|^2$) plotted logarithmically.

The standard expression for the partial width is [3]

$$\Gamma = \frac{p}{8\pi(2J_{\pi_2} + 1)m_{\pi_2}^2}\left|p^L f \mathcal{M}\right|^2, \tag{1}$$

where $m_{\pi_2}$ and $J_{\pi_2}$ are the mass and total angular momentum of the decaying $\pi_2$, the decay momentum $p$ is measured in the rest frame of the $\pi_2$, the relative orbital angular momentum of the decay products is $L$, and $p^L f \mathcal{M}$ is the decay amplitude. The amplitude with the phase space ($p^L$) and flavor ($f$) factors removed is $\mathcal{M}$. In Table 1 we show the ratios of $|\mathcal{M}|^2$ for the observed decay modes of the $\pi_2$ to that of the dominant decay mode ($f_2\pi$). A further refinement is to remove the dependence on the kinematics of the decays from the form factors of the initial and final mesons. With universal Gaussian wave functions for the mesons, this can be accomplished by defining $\mathcal{M} = \exp(-p^2/[12\beta^2])\widetilde{\mathcal{M}}$, where $\beta = 0.4$ GeV [4].

The ratios of the squares of these amplitudes with the flavor, phase space, and kinematic factors removed is also shown in Table 1. It is evident that the $b_1\pi$ decay is a factor of between 3 and 11 weaker than the other decay modes for dynamical reasons, making it anomalously small. This is emphasized by Fig. 1, which shows the $|\mathcal{M}|^2$ ratios plotted logarithmically. Since there is only an experimental upper bound on the $b_1\pi$ mode, this suppression factor could be even larger. There is also evidence from recent analyses of E852 data [5] of a $\pi_2(1670)$ signal in the $f_1\pi$ and $a_2\eta$ final states. The discovery of additional final states will have the effect of further reducing the $b_1\pi$ branching fraction. We urge future experiments to put more restrictive bounds on the $\pi_2 \to b_1\pi$ decay mode.

## 2. Models that can accommodate $\pi_2(1670) \to b_1(1235)\pi$

The decay $\pi_2 \to b_1\pi$ is particularly clean in the sense that it is only sensitive to OZI allowed decays. This is because OZI-forbidden decay processes, which allow the creation of either the isovector $\pi_2$, $b_1$ or $\pi$ out of isoscalar

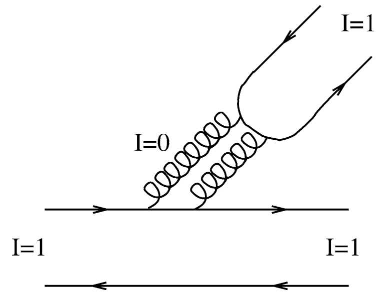


Fig. 2. OZI forbidden decays of an isovector meson to a pair of isovector mesons.

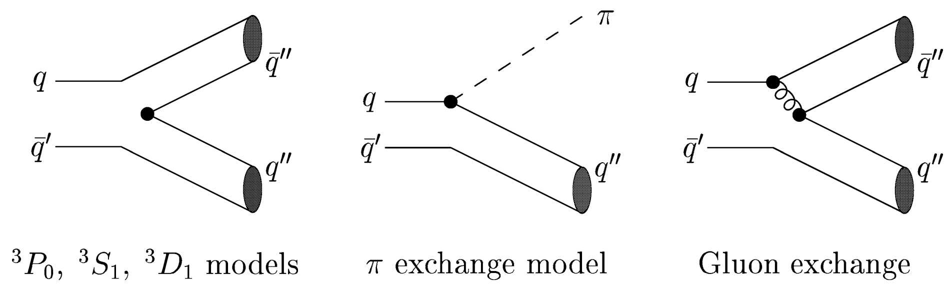


Fig. 3. The OZI allowed decay of an initial meson to two final mesons in various models.

gluons, are forbidden by isospin symmetry (see Fig. 2). The suppression of isospin symmetry breaking amplitudes is much greater than that of OZI forbidden amplitudes, the latter being about a factor of 10.

In non-relativistic quark-pair-creation models, where OZI-allowed meson decay processes are modeled by an initial $q\bar{q}'$ pair decaying to the two pairs $q\bar{q}''$ and $q''\bar{q}'$ (see Fig. 3), a simple selection rule arises when all the mesons have quark-spin $S = 0$. If the $q''\bar{q}''$ pair is created with quark-spin $S_{\text{pair}} = 1$, then conservation of quark-spin implies that the amplitude is zero [6,7]. In the quark model, conventional mesons with $S = 0$ have $J^{PC} = 0^{-+}, 1^{+-}, 2^{-+}, 3^{+-}, 4^{-+}, 5^{+-}, \ldots$, of which only states corresponding to the first three $J^{PC}$ have been established experimentally [3]. The isovector resonances with these three $J^{PC}$ and in their radial ground states are $\pi$, $b_1$ and $\pi_2$, respectively. The only kinematically allowed decay involving these three $S = 0$ resonances is $\pi_2 \to b_1\pi$. Moreover, all other kinematically allowed decays involving $\pi$, $b_1$, $\pi_2$, and their isoscalar partners, are forbidden by the quantum numbers conserved by the strong interaction. The first explicit mention of the quark-spin selection rule or its application to $\pi_2 \to b_1\pi$ was in Ref. [6], although it is implicit in Ref. [8].

No other strong decay involving conventional mesons composed of quarks other than $u, d$ quarks currently appears to be able to test the selection rule. Decays $q\bar{q} \to q\bar{q} + q\bar{q}$ with $q \in \{s, c, b\}$, where each meson is in its radial ground state with the $S = 0$ quantum numbers $J^{PC} = 0^{-+}$, $1^{+-}$ or $2^{-+}$, are forbidden for the same reasons as decays between the isoscalar resonances above. With the exception of the pseudoscalars, quark-model mesons with the open flavor structure $K$, $D$, $D_s$, $B$, $B_s$ or $B_c$, and lying on the un-natural parity sequence $J^P = 0^-, 1^+, 2^-, 3^+, 4^-, 5^+, \ldots$ are mixtures of $S = 0$ and $S = 1$ states, since $S = 1$ components are no longer excluded by charge conjugation symmetry. In QCD, if one of the initial or final mesons in the decay has this open flavor structure, a second meson must also. This implies that the selection rule can only be tested in decays involving open flavor mesons if there are two open flavor pseudoscalar mesons involved. Since two pseudoscalars with an arbitrary relative angular momentum couple to the natural-parity sequence $J^P = 0^+, 1^-, 2^+, 3^-, 4^+, 5^-, \ldots$, the $S = 0$ selection rule cannot be tested with decays involving open flavor mesons. It is, therefore, evident how central and unique the decay mode $\pi_2 \to b_1\pi$ is for testing this selection rule.

The selection rule obtains when the $\pi_2$, $b_1$ and $\pi$ are treated non-relativistically as $S = 0$ mesons in the quark model. Remarkably, relativistic interactions cannot introduce $S = 1$ components in the $\pi_2$, $b_1$ and $\pi$ wave functions, so that the selection rule remains valid after relativistic interaction corrections to the quark model. This is because the $q\bar{q}'$ Fock state wave function of the $\pi_2$ can only have ${}^1D_2$ quantum numbers before relativistic interactions, and the interactions cannot change that. The analogous argument holds for $b_1$ and $\pi$. Even in the fully relativistic equal-time Bethe–Salpeter equation the selection rule is exact [9]. It remains an open question whether a selection rule would be found in field theoretic calculations of $\pi_2 \to b_1\pi$, e.g., in the lattice QCD, QCD sum rule, and Dyson–Schwinger equation approaches.

It has been pointed out that a success of the non-relativistic ${}^3P_0$ pair-creation model (Fig. 3), where $S_{\text{pair}} = 1$, is the fact that the decay $\pi_2 \to b_1\pi$ is predicted to vanish [7]. Other decay models where $S_{\text{pair}} = 1$, such as the non-relativistic chromo-electric string-breaking model where the pair has ${}^3S_1$ or ${}^3D_1$ quantum numbers [10] (Fig. 3), will also have this suppression. Both the ${}^3P_0$ and ${}^3S_1$ models involve a decay operator proportional to $\boldsymbol{\sigma} \cdot \mathbf{p}$, where the $\boldsymbol{\sigma}$ is the spin of the created quark anti-quark pair, and $\mathbf{p}$ is a momentum operator. It is not surprising that the ${}^3P_0$, ${}^3S_1$ and ${}^3D_1$ models obey the selection rule, since these all treat the quarks non-relativistically, as though they are heavy. This is a special case of a result that is shown in Appendix A: when each of the mesons participating in the decay is composed of a very heavy quark and anti-quark, the selection rule is exact to all orders of QCD perturbation theory.

Since 't Hooft's instanton-induced six-quark vertices only affects strong decays where all participating mesons have $J = 0$, and their singlet flavor structure requires the presence of a strange quark (and anti-quark), decay models based on these vertices also predict vanishing $\pi_2 \to b_1\pi$ decay [11].

## 3. One-boson emission models

The one-boson exchange (OBE) model describes the coarse features of the baryon spectrum as being due to confinement and the exchange of pseudoscalar [12] and scalar and vector [13] bosons between the quarks. For light-quark baryons an important pseudoscalar exchange potential comes from pion exchange. This model is not applied to meson spectroscopy. Two reasons are often given for this. The first is that if the light pseudoscalar bosons are the pseudo-Goldstone bosons of spontaneously-broken chiral symmetry, then it is inconsistent to also treat them as quark–anti-quark bound states and allow OBE to act between the quark and anti-quark. This argument would not appear to be applicable to heavier quark–anti-quark bound states such as the $\pi_2(1670)$ and $b_1(1235)$.

A second reason for not applying this model to the meson spectrum is that if one-boson-exchange in baryons is viewed microscopically, with the pion treated as a $q\bar{q}$ pair, an exchange of quarks in the process $qq' \to q'q$ can be viewed in one time ordering as an exchange of $q\bar{q}'$, which can be identified with a meson. In a meson the exchange of quarks occurs in the process $q\bar{q}' \to \bar{q}'q$, which in one time ordering is the exchange of a di-quark $qq'$ and not a meson. Exchange of mesons like the pion between quarks is, therefore, not expected to be important to the structure of mesons, even if it is important for baryons.

Once one admits a quark-pseudoscalar meson vertex as employed in baryon spectroscopy, this vertex naturally leads baryons to decay to a baryon plus a pseudoscalar meson, and mesons to decay to a meson and a pseudoscalar meson. For this reason the OBE model of baryon spectroscopy *implies* a one-boson emission decay model in baryons and in spatially excited mesons. This model should, therefore, be confronted with $\pi_2 \to b_1\pi$.

In the ${}^3P_0$, ${}^3S_1$ and ${}^3D_1$ models, pionic decay of mesons proceeds via $q\bar{q}'$ pair decaying to the two final meson pairs $q\bar{q}''$ and $q''\bar{q}'$, one of which is identified with the pseudoscalar boson. As shown in Fig. 3, the one-pion emission model has either $q \to q''\pi$, or $\bar{q}' \to \bar{q}''\pi$. The lowest order one-pion coupling to the quark or anti-quark is given by the Lagrangian density [14–16]

$$\mathcal{L}_\pi = i\frac{g_A^q}{2f_\pi}\bar{\psi}(x)\gamma_5\gamma_\mu\partial^\mu\vec{\pi}(x)\cdot\vec{\tau}\psi(x) + \text{h.c.} \tag{2}$$

An expansion of this axial current gives a decay operator of the form $\boldsymbol{\sigma}_q \cdot \mathbf{k}$ (Eqs. (2) and (28) of Ref. [15]), where $\boldsymbol{\sigma}_q$ is the spin of the quark emitting the pion, and $\mathbf{k}$ is the pion momentum. This means that the operator creating the boson is a vector operator in the space of the spin of the decaying meson, and so cannot link an initial $S = 0$ meson to a final $S = 0$ meson, so the selection rule is also valid for lowest order one-boson emission.

We conclude that the phenomenologically successful pair-creation model for light–light mesons (the ${}^3P_0$ model) [7], the chromo-electric string-breaking model (${}^3S_1$ or ${}^3D_1$ model), instantons [11], and the lowest order one-boson emission model, which has successfully been applied to the decay of heavy-light mesons [15,16], are consistent with the experimental decay width of $\pi_2 \to b_1\pi$.

## 4. Models possibly constrained by $\pi_2(1670) \to b_1(1235)\pi$

Higher order contributions in one-boson emission models contain terms that are not of the form $\boldsymbol{\sigma}_q \cdot \mathbf{p}$, which violate the selection rule. An example is interactions where *both* a pseudoscalar boson is emitted, *and* a particle is exchanged between the quark and anti-quark in the initial meson (Eqs. (13), (38) and (39) of Ref. [15]). The amplitudes corresponding to the higher order contributions can be similar in size to those corresponding to the lowest order contribution.[1] This suggests that consistency with the small decay branch for $\pi_2 \to b_1\pi$ can constrain models which do not obey the selection rule, such as the higher order contributions introduced in one-pseudoscalar-boson emission models [15] to cure problems with the lowest order contribution [15,16]. It can also provide a viability check on proposed decay mechanisms. An example, depicted in Fig. 3, is where there is a single gluon exchanged between a quark in the decaying hadron and the vertex at which the quark pair is created. Although this one-gluon exchange quark pair creation decay mechanism violates the selection rule,[2] it is found to be sub-dominant relative to the ${}^3P_0$ model [17], so that it is not expected to be constrained by $\pi_2 \to b_1\pi$. If appreciable strength for $\pi_2 \to b_1\pi$, inconsistent with experiment, is predicted by either higher order terms present in the one-boson emission decay mechanism, or by the one-gluon exchange pair creation decay mechanism, one of these decay models could be ruled out. This could distinguish between the OBE and one-gluon exchange models of the coarse features of the light baryon spectrum.

Even though the main models commonly applied to strong decays have been discussed, a comprehensive discussion of all proposed decay mechanisms has not been given. Such mechanisms should be confronted with the experimental data on $\pi_2 \to b_1\pi$.

## 5. Further constraints due to $\pi_2(1670) \to b_1(1235)\pi$

In addition to aspects of the decay models discussed in the previous section, further breaking of the selection rule can arise from mixing with other Fock states. The mixing of mesons participating in the decay with non-$q\bar{q}'$ Fock states is constrained by the experimentally measured $\pi_2 \to b_1\pi$ width. Examples of such mixing are mixing between the $S = 0$ meson $\pi_2$ and the $S = 1$ hybrid $\pi_2$ meson expected nearby in mass, and non-mesonic Fock states in the pseudo-Goldstone boson $\pi$.

[1] See Table 4 of Ref. [15]. Note that the size of the part of the higher-order interaction that is not of the form $\boldsymbol{\sigma} \cdot \mathbf{p}$ is not evaluated in Ref. [15].

[2] One-gluon exchange involves both Coulomb and transverse interactions. The former has a simple $\boldsymbol{\sigma} \cdot \mathbf{p}$ pair creation operator, but the latter involves *both* spin vector pair creation, and an additional term at the vertex where the quark or anti-quark emits a gluon (See Eqs. (B5)–(B7) of Ref. [17]). This additional term includes a $\boldsymbol{\sigma} \cdot \mathbf{p}/m$ contribution [17], so that the overall transverse gluon interaction has spin vector operators at *both* interaction vertices of the gluon, giving rise to a violation of the selection rule.

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## Appendix A. The quark-spin selection rule is exact for heavy quarks

The quark–gluon interaction in the QCD Lagrangian density (suppressing flavor and color) is

$$\mathcal{L} = g\bar{\psi}(x)\gamma_\mu A^\mu(x)\psi(x) + \text{h.c.} \tag{3}$$

Second quantize the free quark fields in the usual way,

$$\psi(x) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E(p)}} \sum_\nu \left[a^\nu(p)u^\nu(p)e^{ip\cdot x} + b^{\nu\dagger}(p)v^\nu(p)e^{-ip\cdot x}\right], \tag{4}$$

where $a^\nu(p)$ and $b^\nu(p)$ are the quark and anti-quark annihilation operators. Substituting Eq. (4) into Eq. (3) yields

$$\begin{aligned}\mathcal{L} = g \int & \frac{d^3p\, d^3p'}{(2\pi)^3\sqrt{(2E(p))(2E(p'))}} \\ & \times \sum_{\nu\nu'} \Big[u^{\nu\dagger}(p)\gamma_0\gamma_\mu u^{\nu'}(p')A^\mu(x)e^{i(p'-p)\cdot x}a^{\nu\dagger}(p)a^{\nu'}(p') \\ & \quad + v^{\nu\dagger}(p)\gamma_0\gamma_\mu v^{\nu'}(p')A^\mu(x)e^{i(p-p')\cdot x}b^{\nu}(p)b^{\nu'\dagger}(p') \\ & \quad + u^{\nu\dagger}(p)\gamma_0\gamma_\mu v^{\nu'}(p')A^\mu(x)e^{-i(p+p')\cdot x}a^{\nu\dagger}(p)b^{\nu'\dagger}(p') \\ & \quad + v^{\nu\dagger}(p)\gamma_0\gamma_\mu u^{\nu'}(p')A^\mu(x)e^{i(p+p')\cdot x}b^{\nu}(p)a^{\nu'}(p')\Big] + \text{h.c.}\end{aligned} \tag{5}$$

The first and second terms describe the quark and anti-quark interactions with the gluon field, respectively, the third term describes creation of a quark–anti-quark pair, and the fourth term annihilation of a quark–anti-quark pair.

In the limit of very heavy quarks

$$u^\nu(p) = \sqrt{2m_Q}\begin{pmatrix}\chi^\nu \\ 0\end{pmatrix}, \qquad v^\nu(p) = \sqrt{2m_Q}\begin{pmatrix}0 \\ \chi^\nu\end{pmatrix}, \tag{6}$$

where the $\chi^\nu$ are the usual Pauli spinors. Then the first and second terms in Eq. (5) contain

$$u^{\nu\dagger}(p)\gamma_0\gamma_\mu u^{\nu'}(p') = v^{\nu\dagger}(p)\gamma_0\gamma_\mu v^{\nu'}(p') = 2m_Q\chi^{\nu\dagger}\chi^{\nu'}\delta_{\mu 0} = 2m_Q\delta_{\nu\nu'}\delta_{\mu 0}, \tag{7}$$

so quark–gluon and anti-quark–gluon interactions do not change the spin of heavy quarks or anti-quarks. The third and fourth terms in Eq. (5) contain

$$u^{\nu\dagger}(p)\gamma_0\gamma_\mu v^{\nu'}(p') = v^{\nu\dagger}(p)\gamma_0\gamma_\mu u^{\nu'}(p') = 2m_Q\chi^{\nu\dagger}\sigma_i\chi^{\nu'}\delta_{\mu i}, \tag{8}$$

where $i \in \{1, 2, 3\}$. Hence quark–anti-quark pair creation and annihilation involve a spin change described by the Pauli matrices $\sigma_i$.

The spin of a propagating heavy quark remains unchanged by quark–gluon interactions, according to the first and second terms of the interaction in Eq. (5), and Eq. (7). The exception to this is when the quark travels in a Z-graph, which corresponds to quark–anti-quark pair creation and then annihilation via the third and fourth terms of the interaction in Eq. (5). However, these Z-graphs are suppressed by powers of $1/m_Q$, so that for very heavy quarks they do not contribute. The spin of a propagating heavy quark remains unchanged to all orders in QCD perturbation theory.

This implies that the spin of a quark or anti-quark is changed only when a quark–anti-quark pair is created or annihilated, through an operator of the form $\boldsymbol{\sigma} \cdot \mathbf{A}$ (Eqs. (5) and (8)). When an initial heavy-quark meson $Q\overline{Q}'$ pair undergoes an OZI allowed decay to the two final heavy-quark meson pairs $Q\overline{Q}''$ and $Q''\overline{Q}'$, the spin is only changed when the $Q''\overline{Q}''$ pair is created.[3] Also, since the individual mesons are composed of very heavy quarks, moving non-relativistically, they have a specific quark-spin (assuming no accidental mixing with states nearby in mass). It follows that the spin selection rule is exact to all orders in QCD perturbation theory when the mesons participating in the decay are built from very heavy quarks and anti-quarks. Light quark loops do not change these conclusions. For very heavy quarks, $1/m_Q$ corrections are negligible compared to higher order corrections in $\alpha_s$, because $\alpha_s(m_Q)$ depends only logarithmically on $m_Q$.

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[3] According to Eqs. (5), (7) and (8), only the time-like component of the gluon field couples to the propagating quark or anti-quark, while only the space-like component couples in the case of quark–anti-quark pair creation or annihilation. The time-like component can change into the space-like component via gluon-loop diagrams in both covariant and Coulomb gauge, and also via ghost-loop diagrams in covariant gauge.