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International Journal of Solids and Structures 44 (2007) 2852–2862
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Compressive buckling analysis of plates in unilateral contact

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Received 5 April 2006; received in revised form 31 July 2006
Available online 3 September 2006

Abstract

The unilateral contact buckling behaviour of delaminated plates in a composite member is studied in this paper, where the two-dimensional mechanical problem is simplified to a one-dimensional mathematical model following the assumption of a buckling mode function in terms of the lateral coordinates. After the governing differential equations for the plate parts in the contact and non-contact regions have been solved, the problem is reduced to just two nonlinear algebraic equations allowing the buckling coefficient to be obtained through an iteration method. A simplified method is also deduced based on an assumed zero length of contact. The numerical results of the buckling coefficients based on both methods show good agreement.

Keywords: Compressive buckling; Unilateral contacting plates; Composite panel; Simply supported rectangular plate; Clamped rectangular plate

1. Introduction

Composite members composed of several layers (especially where the layers vary significantly in thickness and material properties), may potentially exhibit compressive buckling under mechanical or thermal loadings, during which sections of the different layers become separated (delaminated), and then buckle away from each other. Most commonly, a thin layer or 'skin' will buckle away from a thicker layer, or 'core', typically of lower modulus. When the bonding action between the layers is ignorable, this type of buckling problem may be modelled by treating the distinct layers as elastic plates in a state of unilateral contact (Fig. 1). Buckling problems of this type are difficult to analyse due to the inherent nonlinearities and the complexity resulting from the unilateral contact effects.

Experiments conducted as part of a research project aimed at the development of composite components for the construction industry in New Zealand, CSA (2004) have shown the pivotal importance of initial skin buckling in the behaviour of composite wall panels, and also the failure initiation role played by post-buckling

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2006.08.029

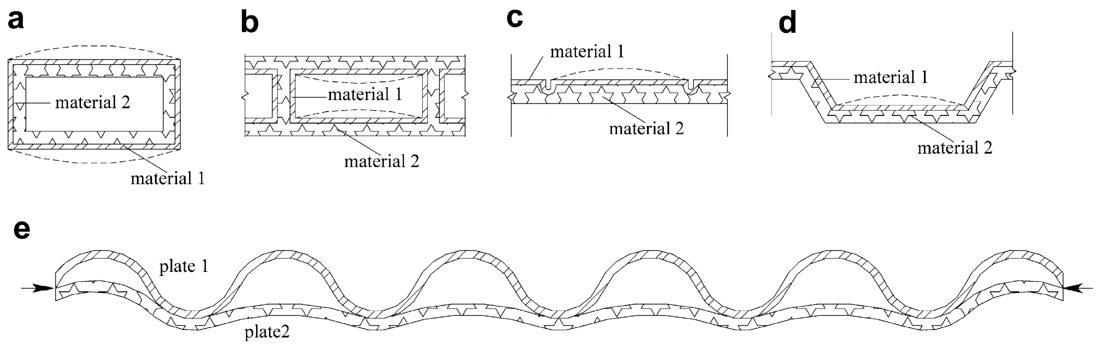

Fig. 1. Local buckling in composite members. (a) Cross-section of a thin-walled composite column, (b) cross-section of a hollow composite panel, (c) cross-section of a flat composite panel, (d) cross-section of a corrugated composite panel, (e) compressive buckling mode of contacting plates – longitudinal section.

behaviour. Fig. 1 shows the cross-section of a number of composite members in which delamination-type plate buckling could play a significant role. Confidentiality conditions currently prevent the publication of specific details of certain specimens being developed, but do not constrain the dissemination of the governing mechanics.

Seide (1958) and Co (1977) were among the earliest researchers of the contact buckling problem, which was extended recently to a compressive plate on a tensionless rigid foundation by Shahwan and Waas (1994, 1998) and Smith et al. (1999a). As a practical application, Wright (1995), Uy and Bradford (1996) and Smith et al. (1999b,c) studied the local buckling problem of composite steel-concrete members. Chai (2001) conducted analytical and experimental studies of post-local buckling response of unilaterally constrained thin plates. In general, the contact buckling problem of a thin plate resting on a tensionless rigid medium may be considered as a simple non-contact plate model with only one half-wave. In a new approach to this type of problem, de Holanda and Goncalves (2003) and Shen and Li (2004) constructed a numerical elastic contact post-buckling model for simply supported plates on tensionless deformable foundations using perturbation and iteration techniques. Most of the existing literature is confined to the case of a plate on a rigid or elastic spring constraint.

In this paper, the unilateral contact buckling problem between two infinite plates with immovable ends is investigated. Assuming a buckling mode function in terms of lateral coordinates allows the formulation of a one-dimensional mathematical model, taking into account the coupling effect between the two plates. After solving the governing buckling equations of the plate parts in both the contact and non-contact regions, the problem is reduced to two nonlinear algebraic equations allowing the buckling coefficient to be obtained by means of an iteration method.

2. Mechanical model of two plates in contact

Consider a single wavelength part of the two interacting plates (Fig. 2).

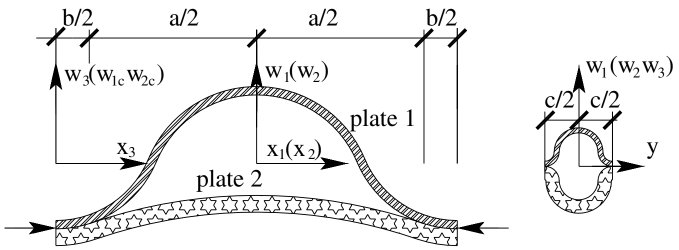

Fig. 2. Mechanical model for buckling of contacting plates.

For thin plates in compression, the governing equations are

$$D_1\left(w_{1c,xxxx} + 2w_{1c,xxyy} + w_{1c,yyyy} + \frac{\pi^2 K_1}{c^2} w_{1c,xx}\right) = q_r, \quad q_r \geqslant 0, \tag{1a}$$

$$D_2\left(w_{2c,xxxx} + 2w_{2c,xxyy} + w_{2c,yyyy} + \frac{\pi^2 K_2}{c^2} w_{2c,xx}\right) = -q_r, \quad q_r \geqslant 0, \tag{1b}$$

$$D_i\left(w_{i,xxxx} + 2w_{i,xxyy} + w_{i,yyyy} + \frac{\pi^2 K_i}{c^2} w_{i,xx}\right) = 0, \quad i = 1, 2, \tag{1c}$$

$$D_i = \frac{E_i t_i^3}{12(1 - v_i^2)}, \quad i = 1, 2 \quad \text{and} \tag{2}$$

$$K_i = \frac{c^2 \sigma_{xi} t_i}{\pi^2 D_i}, \quad i = 1, 2, \tag{3}$$

where w_{ic}, w_i are the vertical displacements in the contact and non-contact regions respectively of plate i; D_i is the well-known plate rigidity and K_i is a dimensionless coefficient; q_r is the contact force between the two plates; E_i, v_i, t_i are the elastic modulus, Poisson's ratio and thickness of plate i and σ_{xi} is the normal stress in the x-direction. The subscripts ,x(,y) indicate partial differentiation, $\partial/\partial x(\partial/\partial y)$, etc.

Assuming the two plates have the same compressive strain,

$$\sigma_{x1}/E_1 = \sigma_{x2}/E_2, \tag{4}$$

(3) may be rewritten as

$$K_2 = \frac{t_1^2(1 - v_2^2)}{t_2^2(1 - v_1^2)} K_1. \tag{5}$$

The inequality $q_r \geqslant 0$ in (1), which means that plate 1 is supported by plate 2, requires

$$K_2 < K_1, \quad \text{or} \quad t_1^2(1 - v_2^2) < t_2^2(1 - v_1^2).$$

Considering that $w_{1c} = w_{2c} = w_3$ in the contact area, adding (1a) to (1b), one obtains

$$w_{3,xxxx} + 2w_{3,xxyy} + w_{3,yyyy} + \frac{\pi^2}{c^2} K_3 w_{3,xx} = 0, \tag{6}$$

$$\text{where } K_3 = \frac{D_1 K_1 + D_2 K_2}{D_1 + D_2}. \tag{7}$$

Thus the mechanical model is reduced to three Eqs. (1c) and (6), governing buckling.

Assuming $w_i(x_i, y) = f_i(x_i)g(y)$, Eqs. (1c) and (6) may be rewritten as

$$f_i'''' g + 2f_i'' g'' + f_i g'''' + \frac{\pi^2}{c^2} K_i f_i'' g = 0, \quad i = 1, 2, 3. \tag{8}$$

Assuming $q_r(x_3, y) = q_x(x_3)g(y)$, (1a) may be rewritten as

$$D_1\left(f_3'''' g + 2f_3'' g'' + f_3 g'''' + \frac{\pi^2}{c^2} K_1 f_3'' g\right) = q_x g, \tag{9}$$

where primes denote differentiation with respect to x or y.

The boundary conditions of the problem are

$$g(\pm c/2) = g''(\pm c/2) = 0 \quad \text{for simply supported plates} \tag{10a}$$

$$\text{and } g(\pm c/2) = g'(\pm c/2) = 0 \quad \text{for clamped plates.} \tag{10b}$$

The continuity condition between f_1, f_2, f_3 is defined by

$$f_i(-a/2) = f_3(b/2), \quad i = 1,2, \tag{11a}$$

$$f_i'(-a/2) = f_3'(b/2), \quad i = 1,2 \text{ and} \tag{11b}$$

$$f_i''(-a/2) = f_3''(b/2), \quad i = 1,2. \tag{11c}$$

To satisfy the boundary condition of (10), the lateral (buckling) mode function $g(y)$ may be assumed as

$$g(y) = \cos\frac{\pi y}{c} \quad \text{for a simply supported plate} \tag{12a}$$

$$\text{and } g(y) = [1/4 - (y/c)^2]^2 \quad \text{for a clamped plate,} \tag{12b}$$

where (12a) is the exact buckling mode function; while (12b) is approximate, but of good precision (Jones and Milne, 1976; Smith et al., 1999a).

Taking the integral of (8) and (9) after multiplying both sides by function $g(y)$, and introducing $f_i(x_i) = \tilde{f}_i(\xi_i)$, where $\xi_{1,2} = x_{1,2}/a$, $\phi_{1,2} = a/c$, $\xi_3 = x_3/b$, $\phi_3 = b/c$, we get

$$\tilde{f}_i''''(\xi_i) - \phi_i^2(\lambda_1 - \pi^2 K_i)\tilde{f}_i''(\xi_i) + \lambda_2\phi_i^4\tilde{f}_i(\xi_i) = 0, \quad i = 1,2,3 \tag{13}$$

$$\text{and } D_1\lfloor\tilde{f}_i''''(\xi_3) - \phi_3^2(\lambda_1 - \pi^2 K_1)\tilde{f}_3''(\xi_3) + \lambda_2\phi_3^4\tilde{f}_3(\xi_3)\rfloor/b^4 = q_x, \tag{14}$$

where $\lambda_1 = 2\pi^2$, $\lambda_2 = \pi^4$ for a simply supported plate; or

$\lambda_1 = 24$, $\lambda_2 = 504$ for a clamped plate.

The symmetric solution of (13) may be written as

$$\tilde{f}_i(\xi_i) = A_{1i}f_{1i} + A_{2i}f_{2i}, \tag{15}$$

where the functions f_{1i} and f_{2i} depend on the value of the parameter $\Delta_i = \left(\frac{\pi^2 K_i - \lambda_1}{2}\right)^2 - \lambda_2$ as follows:

Case 1: $\Delta_i > 0$:

$$f_{1i}(\xi_i) = \cos(\alpha_i\xi_i), \tag{16a}$$

$$f_{2i}(\xi_i) = \cos(\beta_i\xi_i), \tag{16b}$$

$$\alpha_i, \beta_i = \phi_i\left[\frac{\pi^2 K_i - \lambda_1}{2} \pm \sqrt{\Delta_i}\right]^{1/2}. \tag{16c}$$

Case 2: $\Delta_i = 0$:

$$f_{1i}(\xi_i) = \cos(\alpha_i\xi_i), \tag{17a}$$

$$f_{2i}(\xi_i) = \xi_i\sin(\alpha_i\xi_i), \tag{17b}$$

$$\alpha_i = \phi_i\left[\frac{\pi^2 K_i - \lambda_1}{2}\right]^{1/2}. \tag{17c}$$

Case 3: $\Delta_i < 0$:

$$f_{1i}(\xi_i) = (e^{\alpha_i\xi_i} + e^{-\alpha_i\xi_i})\cos(\beta_i\xi_i), \tag{18a}$$

$$f_{2i}(\xi_i) = (e^{\alpha_i\xi_i} - e^{-\alpha_i\xi_i})\sin(\beta_i\xi_i), \tag{18b}$$

$$\alpha_i, \beta_i = \phi_i\left[\sqrt{\left(\frac{\pi^2 K_i - \lambda_1}{2}\right)^2 - \Delta_i} \mp \left(\frac{\pi^2 K_i - \lambda_1}{2}\right)\right]^{1/2}. \tag{18c}$$

The continuity condition, (11), may be rewritten as

$$\tilde{f}_i(-1/2) = \tilde{f}_3(1/2), \quad i = 1,2, \tag{19a}$$

$$\tilde{f}'_i(-1/2)/a = \tilde{f}'_3(1/2)/b, \quad i = 1,2 \quad \text{and} \tag{19b}$$
$$\tilde{f}''_i(-1/2)/a^2 = \tilde{f}''_3(1/2)/b^2, \quad i = 1,2. \tag{19c}$$

(19) may be expressed in matrix form as

$$[\Lambda][A] = 0, \tag{20}$$

where

$$\Lambda = \begin{bmatrix} f_{11}(-1/2) & f_{21}(-1/2) & -f_{12}(-1/2) & -f_{22}(-1/2) & 0 & 0 \\ f'_{11}(-1/2)/a & f'_{21}(-1/2)/a & -f'_{12}(-1/2)/a & -f'_{22}(-1/2)/a & 0 & 0 \\ f''_{11}(-1/2)/a^2 & f''_{21}(-1/2)/a^2 & -f''_{12}(-1/2)/a^2 & -f''_{22}(-1/2)/a^2 & 0 & 0 \\ f_{11}(-1/2) & f_{21}(-1/2) & 0 & 0 & -f_{13}(1/2) & -f_{23}(1/2) \\ f'_{11}(-1/2)/a & f'_{21}(-1/2)/a & 0 & 0 & -f'_{13}(1/2)/b & -f'_{23}(1/2)/b \\ f''_{11}(-1/2)/a^2 & f''_{21}(-1/2)/a^2 & 0 & 0 & -f''_{13}(1/2)/b^2 & -f''_{23}(1/2)/b^2 \end{bmatrix}$$

$$\text{and}[A] = [A_{11} \quad A_{21} \quad A_{12} \quad A_{22} \quad A_{13} \quad A_{23}]^{\mathrm{T}}.$$

For a non-trivial solution of (20) we require that the determinant of the coefficient matrix should vanish,

$$|\Lambda| = 0. \tag{21}$$

Observing that the shearing force equilibrium equation on the borderline between the contact and non-contact areas is

$$\begin{aligned} &D_1\lfloor w_{3,xxx} + (2-\nu_1)w_{3,xyy}\rfloor_{x=b/2} + D_2\lfloor w_{3,xxx} + (2-\nu_2)w_{3,xyy}\rfloor_{x=b/2} \\ &\quad = D_1[w_{1,xxx} + (2-\nu_1)w_{1,xyy}]_{x=-a/2} + D_2[w_{2,xxx} + (2-\nu_2)w_{2,xyy}]_{x=-a/2} \end{aligned} \tag{22}$$

and recalling that $w_i(x_i, y) = f_i(x_i)g(y)$, and using the relationships of (11b) and (15), (22) may be rewritten as

$$\begin{aligned} &D_1[A_{13}f''''_{13}(1/2) + A_{23}f''''_{23}(1/2)]/b^3 - D_1[A_{11}f''''_{11}(-1/2) + A_{21}f''''_{21}(-1/2)]/a^3 \\ &\quad = D_2[A_{12}f''''_{12}(-1/2) + A_{22}f''''_{22}(-1/2)]/a^3 - D_2[A_{13}f''''_{13}(1/2) + A_{23}f''''_{23}(1/2)]/b^3. \end{aligned} \tag{23}$$

Considering the contact pressure between the two plates, (14) may be rewritten as the following inequality:

$$\begin{aligned} q_x = D_1\{&A_{13}\lfloor f''''_{13}(\xi_3) - \phi_3^2(\lambda_1 - \pi^2K_1)f''_{13}(\xi_3) + \lambda_2\phi_3^4 f_{13}(\xi_3)\rfloor + A_{23}[f''''_{23}(\xi_3) - \phi_3^2(\lambda_1 - \pi^2K_1)f''_{23}(\xi_3) \\ &+ \lambda_2\phi_3^4 f_{23}(\xi_3)]\}/b^4 \geqslant 0, \quad |\xi_3| \leqslant 1/2. \end{aligned} \tag{24}$$

Enforcing the tensionless condition on the borderline contact force requires

$$Q_x = D_1[A_{11}f''''_{11}(-1/2) + A_{21}f''''_{21}(-1/2)]/a^3 - D_1[A_{13}f''''_{13}(1/2) + A_{23}f''''_{23}(1/2)]/b^3 \geqslant 0. \tag{25}$$

In the non-contact area, the displacements of plate 1 cannot be less than those of plate 2, thus $w_1 \geqslant w_2$, which means

$$A_{11}\tilde{f}_{11}(\xi_1) + A_{21}\tilde{f}_{21}(\xi_1) - [A_{12}\tilde{f}_{12}(\xi_1) + A_{22}\tilde{f}_{22}(\xi_1)] \geqslant 0, \quad |\xi_1| \leqslant 1/2. \tag{26}$$

Considering Eqs. (5) and (7), and inequalities (24)–(26), leads to solutions of Eqs. (21) and (23), for values of K_1 and ϕ_2 for varying ϕ_1. The true value of ϕ_1 is that which minimizes K_1. The minimum K_1 defines the buckling coefficient $K_{1\mathrm{cr}}$.

3. Simplified model of plates in contact

If the contact length is assumed negligible, $b \to 0$, and the boundary condition for (19) becomes

$$\tilde{f}_1(1/2) = \tilde{f}_2(1/2) \quad \text{and} \tag{27a}$$
$$\tilde{f}'_1(1/2) = \tilde{f}'_2(1/2) = 0. \tag{27b}$$

The shearing force equilibrium equation at the boundary may be written as

$$D_1\tilde{f}'''_1(1/2) + D_2\tilde{f}'''_2(1/2) = 0 \tag{27c}$$

$$\text{and thus} \quad [\widetilde{A}][\widetilde{A}] = 0, \tag{28}$$

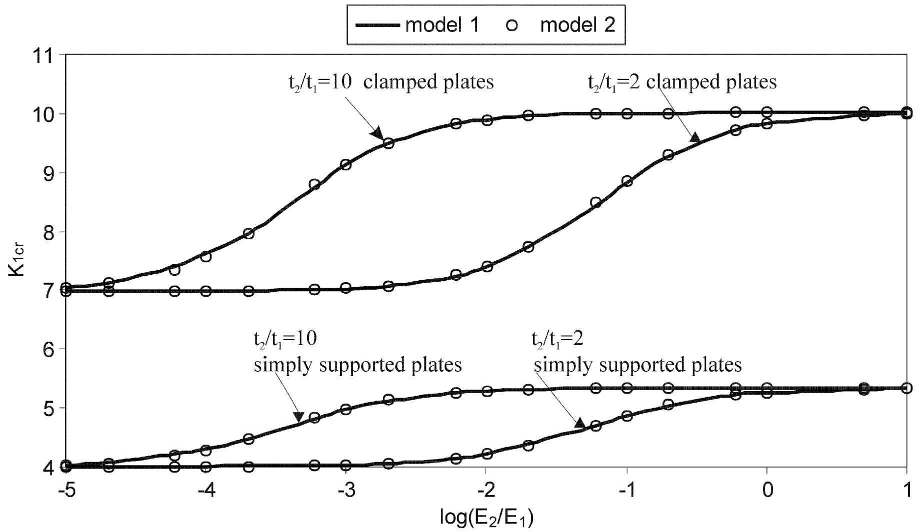

Fig. 3. Relationship between buckling coefficient and elastic modulus ratio.

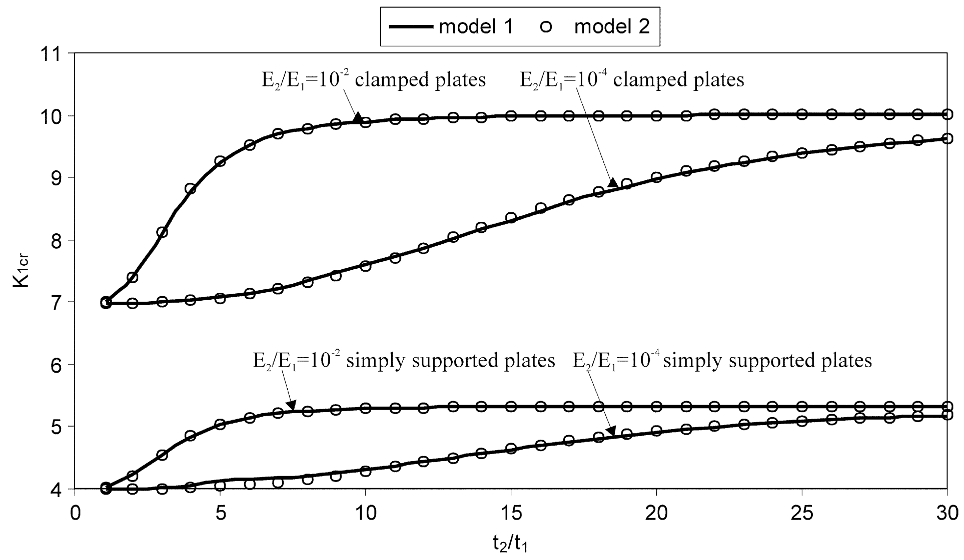

Fig. 4. Relationship between buckling coefficient and thickness ratio.

where

$$\widetilde{\Lambda} = \begin{bmatrix} f_{11}(1/2) & f_{21}(1/2) & -f_{12}(1/2) & -f_{22}(1/2) \\ f'_{11}(1/2) & f'_{21}(1/2) & 0 & 0 \\ 0 & 0 & f'_{12}(1/2) & f'_{22}(1/2) \\ D_1 f'''_{11}(1/2) & D_1 f'''_{21}(1/2) & D_2 f'''_{12}(1/2) & D_2 f'''_{22}(1/2) \end{bmatrix} \quad \text{and}$$

$$[\widetilde{A}] = [A_{11} \quad A_{21} \quad A_{12} \quad A_{22}]^{\mathrm{T}}.$$

For a non-trivial solution of (28) we require that the determinant of the coefficient matrix vanish,

$$|\widetilde{\Lambda}| = 0. \tag{29}$$

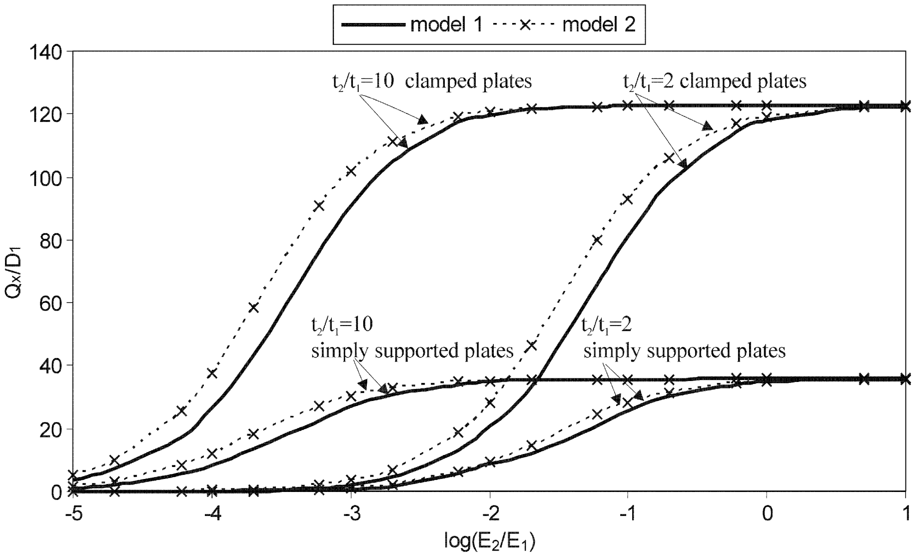

Fig. 5. Relationship between contact force and modular ratio (assumes $A_{11} + A_{21} = 1$).

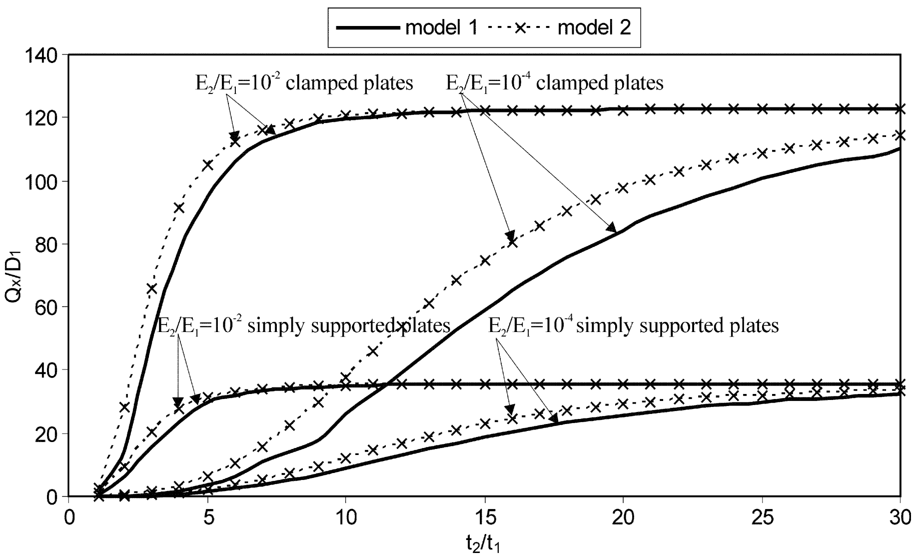

Fig. 6. Relationship between contact force and thickness ratio (assumes $A_{11} + A_{21} = 1$).

The inequality for tensionless contact force on the borderline is

$$Q_x = -D_1[A_{11}f'''_{11}(1/2) + A_{21}f'''_{21}(1/2)]/a^3 \geqslant 0. \tag{30}$$

Considering (5) and (30), and solving (29) yields values of K_1 for varying ϕ_1. The true value of ϕ_1 is that which minimizes K_1, with the minimum K_1 defining the buckling coefficient, K_{1cr}.

4. Numerical results and verification

For two long, finite-width plates in contact, with Poison's ratio $v_1 = v_2$, the calculated buckling coefficients for plate 1 (K_{1cr}) for varying ratios of elastic modulus and varying thickness ratios are shown in Figs. 3 and 4.

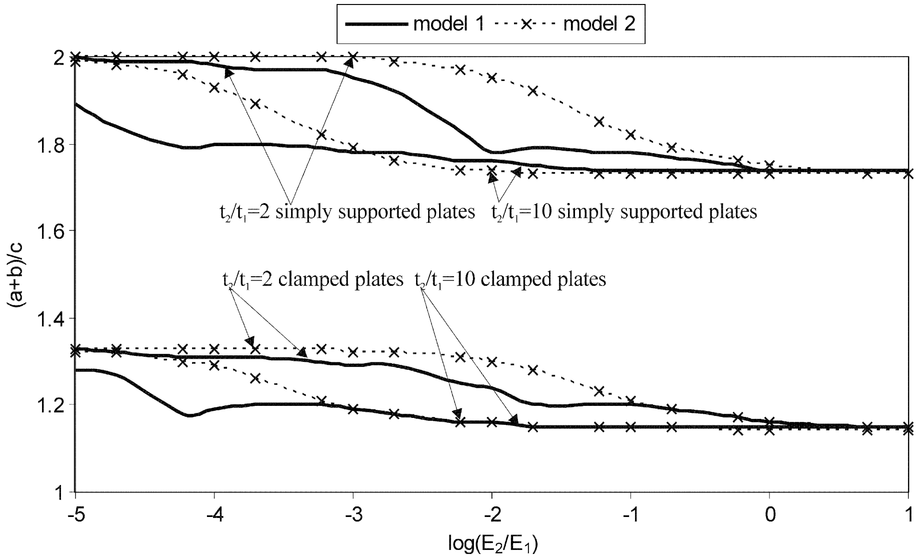

Fig. 7. Relationship between wavelength and modular ratio.

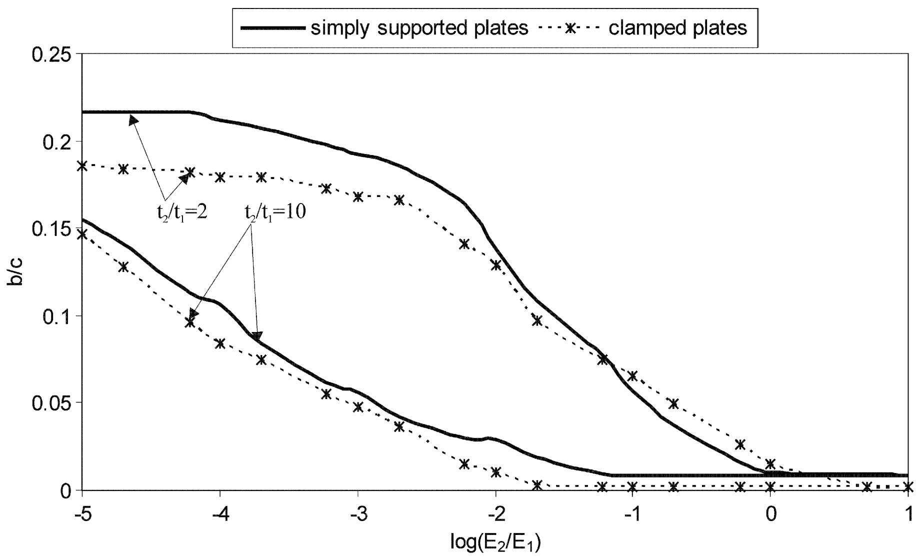

Fig. 8. Relationship between contact length and modular ratio.

The calculated contact forces between the two plates, Q_x are shown in Figs. 5 and 6, where model 1 refers to the precise method of (21) and (23), and model 2 denotes the simplified method of (29). The results show that the buckling coefficients and the contact forces increase when plate 2 has a higher elastic modulus or a greater thickness, tending to a constant value when the relative rigidity of plate 2 is sufficiently large to simulate a rigid constraint. The results also show that the simplified method compares well with model 1 for accuracy of calculated buckling coefficients, despite inevitable errors in wavelength due to the assumption of zero contact length. The wavelength of $\phi_1 + \phi_2$ (or $(a + b)/c$) and contact length of ϕ_2 (or b/c) are shown in Figs. 7 and 8. The buckling mode displacements of plates with lateral edges simply supported are given in Figs. 9 and 10, which show that the assumption of zero contact length in the simplified method leads to displacement

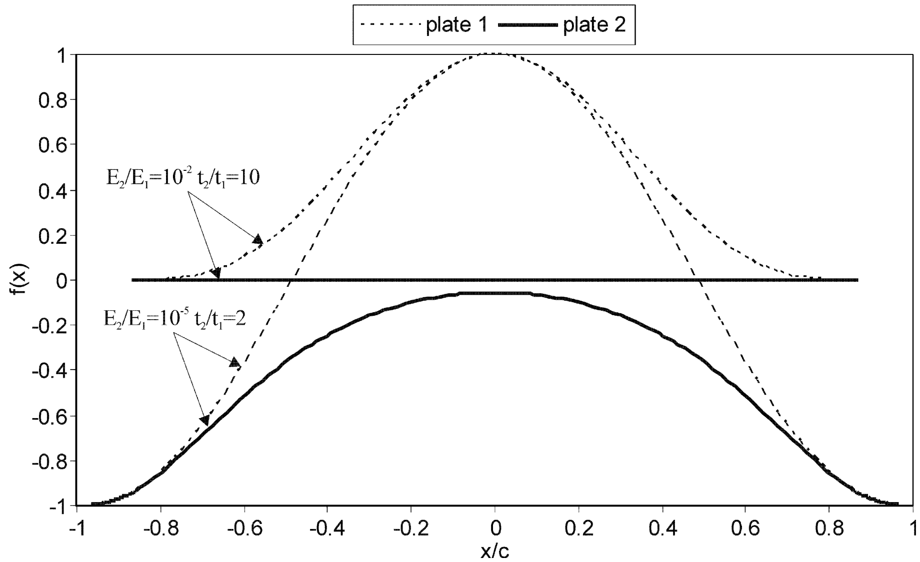

Fig. 9. Buckling mode of plates with lateral edges simply supported – Model 1.

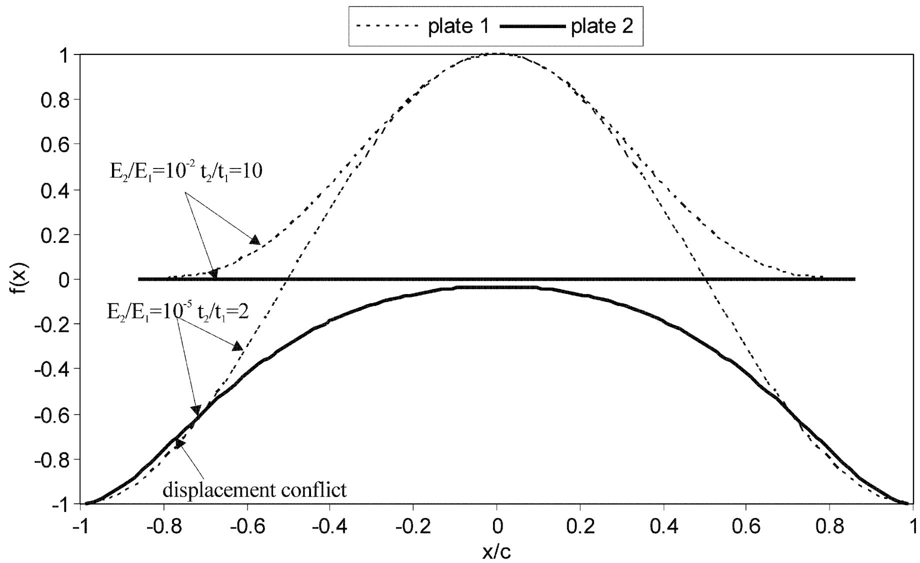

Fig. 10. Buckling mode of plates with lateral edges simply supported – Model 2 (simplified).

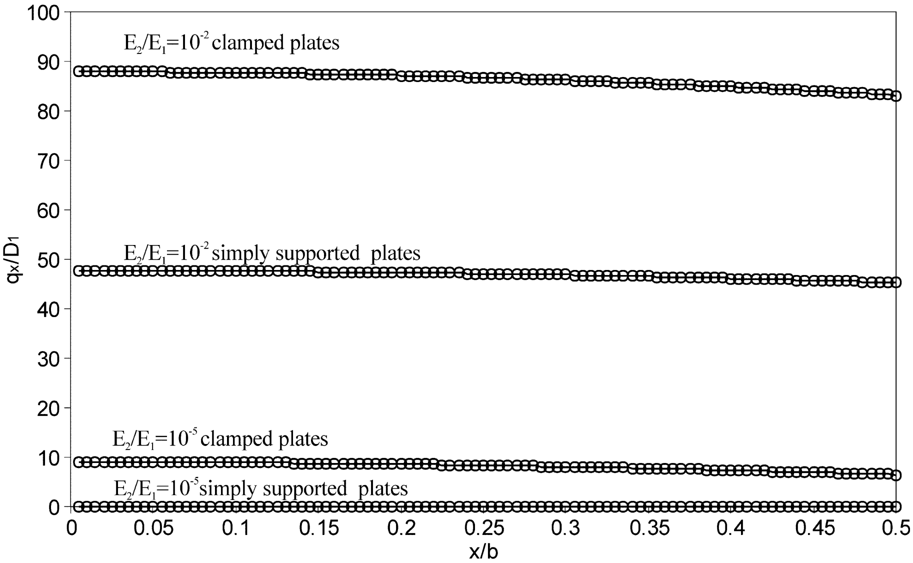

Fig. 11. Pressure distribution in the contact area (assumes $A_{11} + A_{21} = 1$, $t_2/t_1 = 2$).

Table 1
Comparison of results

Boundary condition	Method in the paper			Existing theory		
	E_2/E_1	K_{1cr}	$\phi_1 + \phi_2$		K_{1cr}	$\phi_1 + \phi_2$
Simply supported	10^{-5} ($t_2/t_1 = 2$)	4.0	2.00 (Model 1) 2.00 (Model 2)	Plate a	4.0 (Bloom)	2.0 (Bloom)
	10 ($t_2/t_1 = 10$)	5.33	1.74 (Model 1) 1.73 (Model 2)	Plate b	5.33 (Shahwan)	1.73 (Shahwan)
Clamped	10^{-5} ($t_2/t_1 = 2$)	6.98	1.33 (Model 1) 1.32 (Model 2)	Plate a	6.97 (Bloom)	1.32 (Bloom)
	10 ($t_2/t_1 = 10$)	10.01	1.15 (Model 1) 1.14 (Model 2)	Plate b[a]	9.80 (Bloom) 10.30 (Uy)	1.20 (Bloom) 1.00 (Uy)

[a] The single wavelength part of an infinite plate with lateral edges clamped and resting on rigid constraints is similar to an unconstrained plate with four edges clamped.

incompatibility between the two plates. The pressure distributions between the two plates in the contact area are shown in Fig. 11.

To verify the results above we consider two extreme conditions. ‘Plate a’ represents the conditions when the flexural stiffness of plate 2 is very small, and the behaviour of the skin sheet approaches that of a long, unconstrained plate. ‘Plate b’ corresponds to the case when the flexural stiffness of plate 2 is very large, and the behaviour of the skin sheet is similar to that of a long plate resting on a tensionless, rigid constraint. The comparison between the results of the methods in this paper and existing theory (Bloom and Coffin, 2001; Shahwan and Waas, 1998; Uy and Bradford, 1996) are shown in table 1.

5. Conclusion

Two unilateral contact models have been presented for the problem of local buckling of delaminated plates in a composite panel. Good agreement with existing solutions has been demonstrated for two limiting cases. Numerical results show that the simplified model, in which the contact length is assumed to be zero, generally leads to satisfactory results for both the buckling coefficients and the interaction forces at the points of contact between the plates, although there are inevitable errors in the calculation of wavelength and buckling mode.

Acknowledgements

The authors are grateful for the financial support from The New Zealand Foundation for Research, Science and Technology.

References

Bloom, F., Coffin, D., 2001. Handbook of Thin Plate Buckling and Postbuckling. Chapman & Hall/CRC, New York.

Chai, H., 2001. Contact buckling and postbuckling of thin rectangular plates. Journal of Mechanics and Physics of Solids 49 (2), 209–230.

Co, C., 1977. Bifurcation theory for elastic plates subjected to unilateral conditions. Journal of Mathematical Analysis and Applications 60 (2), 435–448.

CSA, 2004. Composite Structural Assemblies Consortium. Research Consortium funded by the New Zealand Foundation for Research Science and Technology, pp. 2004–2010. Available from: <http://www.composite.net.nz>.

de Holanda, A.S., Goncalves, P.B., 2003. Postbuckling analysis of plates resting on a tensionless elastic foundation. Journal of Engineering Mechanics-ASCE 129 (4), 438–448.

Jones, R., Milne, B.J., 1976. Application of the extended Kantorovich method to the vibration of clamped rectangular plates. Journal of Sound and Vibration 45 (3), 309–316.

Seide, P., 1958. Compressive buckling of a long simply supported plate on an elastic foundation. Journal of the Aeronautical Sciences 25 (6), 382–384.

Shahwan, K.W., Waas, A.M., 1994. A mechanical model for the buckling of unilaterally constrained rectangular plates. International Journal of Solids and Structures 31 (1), 75–87.

Shahwan, K.W., Waas, A.M., 1998. Buckling on unilaterally constrained infinite plates. Journal of Engineering Mechanics-ASCE 124 (2), 127–136.

Shen, H., Li, Q.S., 2004. Postbuckling of shear deformable laminated plates resting on a tensionless elastic foundation subjected to mechanical or thermal loading. International Journal of Solids and Structures 41 (16–17), 4769–4785.

Smith, S.T., Bradford, M.A., Oehlers, D.J., 1999a. Numerical convergence of simple and orthogonal polynomials for the unilateral plate buckling problem using the Rayleigh–Ritz method. International Journal for Numerical Methods in Engineering 44 (11), 1685–1707.

Smith, S.T., Bradford, M.A., Oehlers, D.J., 1999b. Local buckling of side-plated reinforced-concrete beams. I: Theoretical study. Journal of Structural Engineering-ASCE 125 (6), 625–634.

Smith, S.T., Bradford, M.A., Oehlers, D.J., 1999c. Local buckling of side-plated reinforced-concrete beams II: Experimental study. Journal of Structural Engineering-ASCE 125 (6), 635–643.

Uy, B., Bradford, M.A., 1996. Elastic local buckling of steel plates in composite steel-concrete members. Engineering Structures 18 (3), 193–200.

Wright, H.D., 1995. Local stability of filled and encased steel section. Journal of Structural Engineering-ASCE 121 (17), 1382–1388.