HISTORIA MATHEMATICA 3 (1976), 417-439

THE ANTECEDENTS OF OLD BABYLONIAN PLACE NOTATION AND THE EARLY HISTORY OF BABYLONIAN MATHEMATICS

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SUMMARIES

This article is devoted to the elucidation of a little known phenomenon which profoundly affected the development of ancient mathematics, in spite of the fact that we are now unable to document those effects in detail. I try to show that Babylonian place notation, far from being a creation of the Old Babylonian period (ca. 2000--1600 BC), actually has roots deep in the third millennium and was, in fact, invented before the end of the Third Dynasty of Ur (ca. 2112--2004 BC). In the latter part of the paper, I try to demonstrate that, contrary to prevailing opinion, the origins of Babylonian mathematics can now be traced back to the middle of the third millennium BC. In doing so, I wish to call to the attention of historians of mathematics the importance of the Sumero-Akkadian background to Babylonian mathematics and to underscore the importance for historical research of the basic principle that nature and her children *make no leaps.*

Dieser Aufsatz behandelt ein wenig bekanntes Phänomen, das die Entwicklung der Mathematik tief beeinflusst hat, obwohl wir nun die einzelnen Wirkungen dieses Einflusses nicht mehr dokumentieren können. Ich versuche zu beweisen, dass das babylonische Positionssystem, statt eine Erfindung der alt-babylonischen Zeit (ca. 2000--1600 v.Chr.) zu sein, seine Wurzel weit zurück im dritten Jahrtausend hat, und, in der Tat, schon vor dem Ende der dritten Dynastie von Ur (ca. 2112--2004 v.Chr.) erfunden wurde. Im abschliessenden Teil des Aufsatzes versuche ich zu beweisen, dass man jetzt entgegen der herkömmlichen Meinung die Entstehung der babylonischen Mathematik bis zur Mitte des dritten Jahrtausends zurückverfolgen kann. Ich möchte damit die Historiker der Mathematik auf die Wichtigkeit

des sumerisch-akkadischen Hintergrunds zur babylonischen Mathematik aufmerksam machen. Ferner wird durch diesen Aufsatz das für die historische Forschung wichtige Prinzip hervorgehoben, dass die Natur und ihre Kinder *keine Sprünge machen.*

MODERN CONCEPTIONS OF SEXAGESIMAL PLACE NOTATION

Conceptions about the nature and origin of Babylonian place notation have always been closely linked or inextricably entangled with notions about the sexagesimal system of counting. The two phenomena are not, however, identical. Babylonian place notation presupposes the existence of sexagesimal counting, but the latter neither presupposes nor predestines the existence of the former. I have discussed this distinction elsewhere [1972a, 1972b] and, therefore, do not propose to go into it here, but the fact that Sumerian sexagesimal counting is an ethno-linguistic phenomenon and *not* a mathematical creation should be borne in mind.

We know that the Sumerians, who lived in southern Iraq throughout the third millennium BC, spoke a language which employed sexagesimal number words. The reasons why their language used this counting structure are inadequately known and cannot be reconstructed with certainty [1]. The origins of the system of notation, on the other hand, is another matter, since *notation,* by definition, implies writing, and with the invention of writing around 3000 BC we begin to have evidence for a history of numerical notation. The volume of evidence increases as the third millenium passes, and by the end of the millennium it is enormous, but it is only the last four centuries of this era (ca. 2400--2000 BC) which provide us with documentation sufficient to really study the system of notation and to observe its evolution in some detail. It is with this latter period that this paper is concerned

That Greek mathematicians occasionally used sexagesimal numbers has long been known [Thureau-Dangin 1939, esp. 95 ff. & 137 ff.], but all that was known in the early nineteenth century about the Babylonian sexagesimal system was what could be gleaned from the remnants of Berossos' history with regard to the *sōssos* (= 60 year period), *nēros* (600 years), and *saros* (3600 years) [Montucla 1799, 57]. It was not until the middle of the nineteenth century with the decipherment of cuneiform that European scholars began to become aware of the true nature of Babylonian sexagesimal notation. The earliest reference to this type of notation by a European scholar that I have been able to discover is in a notice by Edward Hincks [1854; 1855]. In the following year, Henry Rawlinson, the foremost decipherer of the cuneiform script, published the first example of a table of squares in sexagesimal notation [Rawlinson 1855, 217 ff., n. 4].

It took some time for this knowledge to percolate through the rather limited circle of Assyriologists and Orientalists into the scholarly community at large. This process seems to have been accomplished primarily by means of debates on ancient metrology conducted in the meetings of the Berlin Academy of Sciences during the 1870's [Lepsius 1877a; 1877b; Oppert 1877], for it was not until the first edition of Moritz Cantor's *Vorlesungen* that a comprehensive history of ancient mathematics tried to deal with Babylonian mathematics and the sexagesimal place system [Cantor 1880 vol. 1, Ch. 3, 67-94]. Even so, it is only since the pioneering work of the philologist François Thureau-Dangin and the more mathematically systematic work of Otto Neugebauer, which came to fruition in the 1930's, that Babylonian mathematics and its role in the development of ancient mathematics has come to be fully appreciated. Chiefly through the careful work of these two scholars, the Babylonians have now taken their proper place in the history of mathematics as the earliest inventors of a type of numerical logic, akin to, but at the same time, distinct from, today's algebra [2], and the importance of their system of place notation in achieving this is generally recognized.

As is well known, the majority of all Babylonian mathematical texts date to the Old Babylonian period (ca. 2000--1600 BC) [Neugebauer & Sachs [1945, 1]. None have ever been discovered which could be dated with certainty earlier than this [3]. Neugebauer [1962, 29] has stated categorically: "For the Old-Babylonian texts no prehistory can be given. We know nothing about an earlier, presumably Sumerian, development." And Neugebauer and Sachs [1945, 3] remark: "No conclusion can be drawn from the use of the Sumerian language as to the time or place of origin of these texts." Even today we cannot speak of a specifically Sumerian background to Old Babylonian mathematics, because the symbiosis of this people with the Semitic Akkadians makes it almost impossible to sort out what is specifically Sumerian from what is Akkadian in the culture of southern Iraq during the latter third millennium BC. Nevertheless, it is now possible to substantiate Thureau-Dangin's belief that the Sumerians did indeed play a role in the creation of both the sexagesimal place system and in the foundation of Babylonian mathematics [4], as recently published documents once again remind us how much we have yet to learn about Mesopotamian civilization in the third millennium BC.

Up to the present time, considerable confusion has prevailed concerning the origin of sexagesimal place notation. In a widely read and cited book, B. L. van der Waerden [1961, 42] states: "The most ancient Sumerian texts, from which the Sumerian number system was deduced, dating from the time of Shulgi (about 2000), were tables of inverses ($1/x$) and multiplication tables." This statement is both ambiguous and incorrect. It is ambiguous because it does not distinguish between a system of number words and a system of number symbols [Menninger 1958 or 1969]. From the failure to make this distinction, all sorts of ambiguities and errors follow.

THE USE OF SEXAGESIMAL PLACE NOTATION IN THE UR III PERIOD

The Sumerian system of number words, as well as a system of number symbols reflecting those number words, are attested as early as about 2800 BC [5]. The system of sexagesimal place notation, on the other hand, has never been definitely dated earlier than the Old Babylonian period, chiefly because the majority of all mathematical texts can only be loosely dated on the basis of script and language. However, as I have pointed out [1972a, 14 ff., n. 17], an Ur III text published over fifty years ago provides evidence that the "Old Babylonian" system of place notation was already in use before the end of the Third Dynasty of Ur, and, since the text can be dated on the basis of the year formula to the fifth year of Amar-Suena (i.e., ca. 2042 BC) or the second year of Ibbi-Sin (ca. 2027 BC), we have in this a *terminus ante quem* for the invention of this type of notation. The pertinent passage of this text [Keiser 1919, no. 293:1-6, museum no. YBC 1793] reads as follows:

Transliteration	*Translation*
14,54	0;14,54
29,56,50	0;29,56,50
17,43,40	0;17,43,40
30,53,20	0;30,53,20
(blank)	(blank)
šu+nigin$_2$ 1½ *ma-na* 3½ *gín* *lá* 7 *še kug-a*	Total: 1½ *mana*, 3½ *shekels*, minus 7 barleycorns in silver

The total entered in standard metrological notation in line 6 is, in effect, a statement of the sum of the sexagesimal numbers in lines 1-4, for these numbers add up to 1,33,27,50, i.e., 1;33,27,50. Since the following relations hold

	mana	*gin*	*še*
mana	1	1,0	3,0,0
gin	0;1	1	3,0
še	0;0,0,20	0;0,20	1

the amount stated in the total in standard notation is equal to 1;33,27,40 in sexagesimal notation. An absolutely accurate statement in standard notation would have been 1½ *mana* 3½ *gin* minus 6½ *še*. Here, according to common practice in documents dealing with relatively large weights, the half-barleycorn is dropped in the final summation. Thus, we have here unequivocal evidence that the "Old Babylonian" system of place notation was invented before the end of the third millennium BC.

But this document has a greater significance than is evident from the cuneiform copy. When I collated this text in the Yale Babylonian Collection (June 17, 1974), it turned out to be a kind of ancient "scratch pad." It has a form similar to a school text, being rather thick and having flat edges. The writing surface is extremely flat, and the back side, which was not used, is convex. The writing surface shows clear traces of having been previously used. The appearance of the tablet suggests that it was moistened and smoothed off after use.

Here we have at last an explanation for why so little trace of sexagesimal notation has survived from the Ur III period [6]. Calculations in sexagesimal notation were made on temporary tablets which were then moistened and erased for reuse after the calculation had been transferred to an archival document in standard notation. Moreover, another unusual characteristic of this text suggests how the Sumerian system of notation functioned without a sign for zero: the sexagesimal numerals are arranged in quite clear columns according to their proper power. Thus does this curious tablet buried away in a volume of Ur III admininstrative documents provide us with a number of useful suggestions for unraveling the riddles of "Babylonian" mathematics.

I have made the point elsewhere [1972a, 14 ff.] that the introduction of this place notation system must be ascribed to conscious invention on the part of some nameless Sumero-Akkadian mathematician, but it cannot be doubted that the milieu out of which this system of notation arose was that of the Sumerian system of number words interacting with the system of number symbols used for metrology. Thureau-Dangin [1939, 111; 1903, no. 306] has already called attention to Ur III expressions such as the following:

šár:7 *gešu*:4 *géš*:5 33 10 *gín*

= $(60^2)7 + (60 \cdot 10)4 + (60 \cdot 5) + 33 +$ "10 *shekels*"

= 7,45,33;10

Here the term "ten *shekels*" is used to express "one-sixth." The use of "shekel" here signifies "one-sixtieth" (0;1) and has nothing to do with shekels used in weighing per se, for the number in question refers to bundles of reeds. Other expressions of this type are also attested:

21,15,½ *gín geme*$_2$ *ud-1-šè*	"21,15;30 women workers for one day" [Schneider 1931, no. 250:1]
5,16,1/3 *gín geme*$_2$*-uš-bar*	"5,16;20 women weavers" [Boson 1941, 159 ff. i 2]

Here, in accordance with Sumerian accounting practice, the expressions "one-half/one-third worker" refer to the portion of the work day, not to the person per se. Also of importance in understanding the Sumerian system of fractional expressions is to note that the expressions ½ *gin* and 1/3 *gin* do *not* mean "one-half" or "one-third *shekel*" but "one-half" or "one-third *mana*," i.e., 30 and 20 shekels respectively, just as I have indicated in my translation. The *mana* is the prime unit, and *gin* is used to express fractions. I stress this distinction because many Assyriologists do not appear to be aware of it.

Both the invention of a new system of sexagesimal number symbols, of which the distinctive feature is place notation, and the intimate relationship of this new system to the Sumero-Akkadian system of metrological notation is evidenced by the examples cited above. I have suggested elsewhere [1972a, 17] that the impetus for creation of this place notation system is to be sought in the circumstances brought into being by the Ur III empire. The conquests of Ur-Nammu and Shulgi resulted in the formation of a state embracing all of southern Iraq and at times including territories lying far beyond this. The Ur III government not only controlled the area politically, but large sections of the economy were also under direct control of the state. This resulted in the creation of a large bureaucracy of scribes and other civil servants to manage and keep account of the voluminous flow of goods passing in and out of state warehouses. It appears to have been in the context of this situation that some Sumero-Akkadian scribe accustomed to working with very large and very small numbers invented the place notation system to replace the older and more cumbrous system of standard notation and, with this act, created the facile instrument upon which the success of Old Babylonian mathematics is predicated.

Linking the invention of place notation to the creation of the Ur III empire, and all that this political system implied, is simply a hypothesis invented by me to explain the known facts. This hypothesis has, however, recently received some indirect support in the form of newly published texts of mathematical character which likewise come from a period following the creation of a large empire.

SEXAGESIMAL CALCULATIONS AND MATHEMATICS IN THE PRE-UR III PERIOD

Around the middle of the twenty-fourth century BC, a Semite from northern Babylonia, whom we know as Sargon of Akkad, united southern Mesopotamia into a more or less centrally governed state. The texts which I now discuss cannot be dated precisely, but they probably come from the latter years of this empire, perhaps around 2200 BC [7]. These texts are significant for the following reasons: (1) they provide further and older evidence for an intimate linkage between the system of weight metrology and the expression of sexagesimal fraction; (2) they are the oldest well-defined group of cuneiform documents showing an unequivocal interest in playing with numbers; and (3) as problem texts, they are 500 years older than their Old Babylonian counterparts.

With the foundation of the Akkad empire, a number of metrological innovations also appear. Among these, the most striking is the appearance of a large unit of capacity called *gur* with the structure:

(a) 1 *gur* = 5 *bariga* = 30 *ban* = 5,0 (= 300) *sila* [8].

Prior to this time, the most commonly used *gur* seems to have been one having the structure:

(b) 1 *gur* = 4 *bariga* = 24 *ban* = 4,0 (= 240) *sila*;

but this was not the only *gur* in use, for one important state, Lagash, used a *gur* with the structure:

(c) 1 *gur* = 4 *bariga* = 24 *ban* = 2,24 (= 144) *sila*.

The reasons underlying the change from a *gur* with 4 *bariga* to one with 5 *bariga* cannot be reconstructed with certainty, but it seems to have been motivated by pratical considerations relating to the facilitation of cost accounting and calculations involving sexagesimal ratios. It is not difficult to understand why the *gur* of 144 *sila* went out of existence, for it is a cumbersome unit with which to calculate. For this purpose, the *gur* of 240 *sila* is much better suited, but it too had its drawbacks, which will be clear if we examine the advantages of using the *gur* with 300 *sila*.

As is well known, from about 2400 BC, documents from southern Babylonia record the issue of large quantities of barley to dependents of the state. The fact that the state was responsible for the maintenance of these dependents meant that elaborate records had to be kept, and the larger the state became the larger the volume of goods handled by state officials also became. Along with the increase in volume of goods came an increase in the number and magnitude of calculations necessary to keep the accounts balanced. It is precisely in connection with calculations of large magnitude that a *gur* with the structure of *a* becomes really useful.

The pivotal unit used by Sumero-Akkadian scribes in dealing with capacity calculations seems to have been the *bariga* of 60 *sila*. With a *gur* containing 5 of these units, the scribe has only to know the number of *gur* expended monthly in order to know instantly, without further calculation, how many *bariga* will be expended in a year's time, for 1 *gur* = 5 *bariga*, which, multiplied by 12 (months) = 1,0 *bariga*. Moreover, this relationship has another very practical application. The Mesopotamians seem to have reckoned one *sila* of barley per day as the amount of food necessary for the minimal subsistence of one adult human. With a *gur* of 300 *sila*, if the scribe knew the number of *gur* to be expended monthly, he would know immediately the number of *sila* required for the entire year, for 1 *gur* = 5 *bariga*, and 5 times 12 = 1,0 *bariga* = 1,0,0 *sila*. Naturally, if the scribe knew how many *gur* were available for the entire year, it would be a very simple matter, using a *gur* with the structure described above, to find the number of *gur* available each month by multiplying the number of *gur* by 5, i.e., 1,0 *gur* per year, times 0;5 = 5 *gur* per month.

I am not trying to suggest here that the Sumerians and Akkadians were already using place notation, but what these metrological relations do suggest is that they already fully understand the reciprocal relationship of 5 and 12 [9]. This would not be surprising in view of the fact that the fractions 1/3, 1/4, and 1/6 are already attested in documents from ca. 2375-2350 in the form *igi-3-g̃al, igi-4-g̃al,* and *igi-6-g̃al*. This is a Sumerian mode of expressing numbers which we call "fractions," but I doubt that they conceived of these numbers as fractions. In any case, the *igi-x-g̃al* construct is the key element in the whole conceptual underpinning of the tables of reciprocals, which in turn constitute the *sine qua non* of Babylonian mathematics [10].

Metrology tends to be extremely conservative. For this reason, no amount of scholarly speculation about sexagesimal ratios is likely to have occasioned a metrological change such as that I have described above. This *gur* is referred to in the documents as the "*gur* of Akkad." This fact permits two hypotheses to explain its origin: (1) it was a local north Babylonian *gur* introduced into the south after the Akkadian conquest, or (2) it was a conscious creation of scholars under the patronage of an early Akkadian king [11]. I am inclined to favor the latter hypothesis, for there is another metrological innovation of the Akkad period which suggests a concern with sexagesimal ratios. This is the length unit known as the *kuš-numun* or "seed cubit."

The existence of this metrological unit has been known for many years [Deimal 1930, no. 318, 56], but only in 1973 has a text been discovered and published which enables one to determine its relation to the known measures of length and to define its function. This same text also constitutes, in a much simpler form, one of the oldest examples yet discovered of a problem text like those from the Old Babylonian period, and, along with a few others of similar character, permits us our first glimpse into the third millennium background to Babylonian mathematics. The text [Limet 1973, no. 38; Powell 1975, 184] reads as follows:

Transliteration	*Translation*
2,40 *ús ḫ̮i*	2,40 *nindan* (is the length of) both sides;
sag 1 *(iku)* $GANA_2$	front (= width, is such that it encloses an area of) 1 *iku*;
sag-bi 3 *kùš-numun* 1 *GIŠ.BAD* 1 *zipaḫ̮*	its front (is) 3 seed cubits, 1 *GIŠ.BAD*, 1 *zipaḫ̮*

This is a very simple problem which involves finding the short side of a rectangle when the long sides and the area are known. We would call this an equation of the first degree and solve the problem by dividing the area by the length. It is, however, virtually certain that the scribes of the third millennium solved this problem in the same way that it would have been done in the Old Babylonian period, namely by multiplying the area by the reciprocal of the length as follows:

us (length) = 2,40 *nindan*	*sag* = *ašag* $\cdot \frac{1}{us}$
ašag (area) = 1,240 square *nindan*	*sag* = 1,40 $\cdot$ 22,30
sag (width) = to be found	*sag* = 37,30 = 0;37,30

This is precisely the answer found by the scribe--actually a student or an apprentice scribe--for, from other sources, we are able to determine that

1 *zipaḫ* = ½ regular *cubit* = 0;2,30 *nindan*

1 *nindan* = 12 regular *cubits*

1 *iku* = 100 square *nindan* = 1,40 sq. *nindan*

and, by a knowledge of the structure of the metrological system combined with a process of deduction, we arrive at the following:

1 *GIŠ.BAD* = 1 regular *cubit* = 0;5 *nindan*

1 *kuš-numun* = 2 regular *cubits* = 0;10 *nindan*,

therefore, 3 *kuš-numun* + 1 *GIŠ.BAD* + 1 *zipaḫ* = 0;37,30 *nindan* [12].

It seems peculiar that one should devise a "seed cubit" exactly twice the size of the normal cubit, until one realizes that this is explicable in terms of sexagesimal ratios, especially in terms of the unique 6 and 10 system of counting that arises from the Sumerian system of number words and plays such a central role in the system of numerical notation. Thus, when the scribe wrote 1, 2, 3, 4, or 5 *kuš-numun*, he automatically knew that this was to be interpreted as 0;10, 0;20, 0;30, 0;40 or 0;50 *nindan*. Thus, we have in this problem text additional evidence for conscious manipulation of the metrological system to facilitate the use of sexagesimal ratios and to facilitate the expression of these ratios by the system of notation [13].

More important perhaps than the concern with sexagesimal ratios is the interest in numerical relationships of a purely abstract nature that this document reveals. This is not immediately apparent until one realizes that the rectangle described is 256 times as long as it is wide, i.e., about 960 by 3.75 meters [14]. In other words, the practical function of teaching the young scribe how to deal with field mensuration is clearly subordinate to the abstract interest in numerical relationships. The same conclusion follows from another text in the same collection. This text [Limet 1973, no. 39] reads:

Transliteration	*Translation*
4,3 *ús*	4,3 *nindan* (is the) side;
sag 1 *(iku)* $GANA_2$	front (is such that it encloses an area of) 1 *iku*;
sag-bi	its front
pà-dè-dam	is to be found.

This is an example of what might have confronted us had we attended school around 2200 BC in southern Mesopotamia. The problem is written out in this form and given to the student to solve. The student then copies the problem, finds the solution, and produces a tablet like Limet's no. 38 translated above. Another interesting fact emerges, if we solve the problem, for

$$sag = a\check{s}ag \cdot \frac{1}{us} = 1{,}40 \cdot \frac{1}{4{,}3}$$

and, as given, $sag = 1{,}40 \cdot 14{,}48{,}53{,}20$

therefore, $sag = 0;24{,}41{,}28{,}53{,}20$ *nindan*.

Thus, the problem has a finite sexagesimal solution which can be expressed as the ratio of regular numbers, but the startling fact is that there is no way of expressing this solution in the metrological terminology of the third--or of the second--millennium. The closest one can come to expressing this solution is by the following combination:

(Standard metrological notation)	*(Expressed sexagesimally in nindan)*
2 *kuš-numun* (seed cubit)	= 0;20
+ 2 *šudua* (2/3 normal cubit)	= 0; 3,20
+ 8 *šusi* (fingers)	= 0; 1,20
+ ½ *še* (barleycorn)	= 0; 0, 0,50
	= 0;24,40,50

This still leaves a remainder of 0;0,0,38,53,20 unaccounted for. This poses for us the interesting question of how the third millennium mathematicians dealt with the problem of remainders, or, more specifically, how they stated the solution to problems such as this.

At the present time we cannot provide a definitive answer to this question, but another recently published exercise text provides some very suggestive information along these lines. This text [Limet 1973, no. 36] involves calculation of the area of a square and reads as follows:

Transliteration	*Translation*
11 *NÍG.DU* 1 *kùš-numun* 1 *GIŠ.BAD* 1 *zipaḫ*	11 *nindan,* 1 seed *cubit,* 1 *GIŠ.BAD,* 1 *zipaḫ* (is the side of the square);
a-šà-bi 1¼ *(iku)* $GANA_2$ 2½(*!*) *šar* 6 *gín* 15 *gín-tur* [15]	its area: 1¼ *iku,* 2½(*!*) *šar* 6 *shekels,* 15 little *shekels*
ba-pa	is found.

This text is the work of a student, like Limet's no. 38 above. Without specifically stating the figure to be a square, the side is given in standard notation which adds up to 11;17,30 *nindan*. The correct area is 2,7;30,6,15 *šar* (or square *nindan*), but the total arrived at by the student is 2,7;36;15. Comparison of these two numbers suggests that the pupil has made a calculation using a mental construct analogous to Old Babylonian place notation, but, during the course of computation, he apparently lost track of the correct "sexagesimal place" and has interpreted 0;0,6,15 *šar* as 6 *gin* + 15 *gintur,* whereas, in actuality, it is 6 *gintur* + 15/60 *gintur*. Just as in the preceding problem, we know of no way to express 15/60 *gintur* in standard metrological notation [16].

Astonishing as it may seem, the conclusion forced upon us by these texts is that calculations involving the conceptual framework implicit in Old Babylonian place notation were already being performed in the Sargonic period, some two to four centuries earlier than the earliest Old Babylonian mathematical texts. This conclusion may seem unbelievable to the historian of mathematics unused to working with cuneiform documents, but, in this respect, it is well to keep in mind the wise admonitions of O. Neugebauer [1934, 179; 1962, 30], who has repeatedly stressed that the written record constitutes an insufficient witness to the real nature of Babylonian mathematical thought. Obviously it would be imprudent to draw far-reaching conclusions on the basis of the few, simple mathematical texts presently known from the third millennium, but it is also equally obvious that a great deal of Sumero-Akkadian thought was either never committed to writing or that it still awaits the excavator. Indeed, probably both of these alternatives are true.

Moreover, there are probably other unpublished texts of this type already in museum collections containing Sargonic documents, and there may be a few others in the published material that I have missed in this survey. One text of this type was published in 1935, but its significance was not recognized, and I discovered it only after the basic manuscript of this paper was completed. It belongs to the same type as Limet's no. 38 above, but omits the phrase *sag-bi* and, like Gelb's no. 112 below, seems to have the name of the pupil on the reverse [Pohl 1935, 65; transliterated with collations as Westenholz 1975, 65]:

Transliteration	*Translation*
1,7½ *ús NÍG.DU*	1,7 + ½ *nindan* (is the) side
sag 1 *(iku)* $GANA_2$	the front (is such that it encloses an area of) 1 *iku*
1 *NÍG.DU* 5 *kùš* 2 *šu-dù-a* 3 *šu-si* 1/3 *šu-si*	(the front is) 1 *nindan* + 5 *cubits* + 2 double hands + 3 fingers + 1/3 finger
(blank)	

On the reverse:

il	*Il* (personal name?)
NI.LAGAR.DU.NI	(reading and meaning unclear)

The same metrological relations as in Limet's no. 38 hold: 1 sq. *nindan* = 1/1,40 *iku*; 12 *kuš* = 1/12 *nindan*; 1 *šudua* = 1/3 *kuš*; 1 *šusi* = 1/30 *kuš*. Thus, since the area is 1,40 and the length 67;30, the width is obtained by multiplying the area by the reciprocal of the length: 1,40 · 1/0;0,53,20 = 1;28,53,20. This is precisely the result obtained in the text, stated in standard metrological terms: 1;0 + 0;25 + 0;3,20 + 0;0,30 + 0;0,3,20 *nindan*.

I shall conclude this discussion of texts from the late third millennium with the treatment of a text which vividly illustrates the abstract concern for numbers and numerical relationships that characterizes these school exercises no less than the mathematical texts of the Old Babylonian period. This text involves finding the area of a rectangle and is simple enough as far as the structure of the problem itself, but the interpretation of the text is complicated for us by the fact that the symbols used to express $60^2 \cdot 10$ and 60^3 units of the area measure *bur* appear here for the first time. This text [Gelb 1970, no. 112] reads as follows: [17]

Transliteration	*Translation*
šár-gal 4,0 *NÍG* 4 *kùš-numun sá*	1,0,0,0 + 4,0 *nindan* + 4 seed cubits, equal
(blank)	(blank)
šár 1,32 *NÍG* 1 *kùš-numun sá*	1,0,0 + 1,32 *nindan* + 1 seed cubit, equal

ur-^{d}ištaran	*Ur-Ištaran*
ŠÁR:7+LÍL ŠAR'U:4+GAL	7,0,0,0 *(bur)* + 40,0,0 *(bur)* +
*ŠÁR:7 bur'u:*1	7,0 *(bur)* + 10 *(bur)* +
*bur:*7 *eše:*1	7 *(bur)* + 1 *(eše)* +
*iku:*3 *ubu:*1 *GANA$_2$* 10 *šar*	3*(iku)* + 1 *(ubu)* + 10 *šar* +
16 *gín* 2/3.*ŠA*	16 *gin* + 2/3 *(gin)*

A few words of explication will assist the reader in interpreting my translation, which I have purposely kept as "literal" as possible. The length of the rectangle is given in line 1, the width in line 3. Each of these is qualified by the adjective *sa*, which I translate "equal" and which is commonly used in mensuration texts to express "opposite sides of equal length." In line 3, at the very bottom of the obverse of the tablet is a personal name, Ur-Ištaran, which I take to be the name of the student who made the calculation and wrote the tablet. If my assumption is correct, Ur-Ištaran was doubtless an unhappy little Sumerian when he recited the result of his computation which we find on the reverse of the tablet, for, the correct solution is 1,1,36,16,49,41;26,40 *šar* (or square *nindan*), but the number computed by the student apparently equals 3,50,0,38,46,0;16,40 *šar*. However, even apart from the solution obtained by the student, the abstract nature of the problem becomes apparent when one realizes that the length of this rectangle is 1297.444 kilometers.

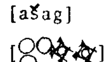
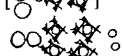
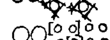
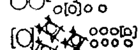
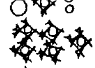
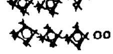
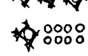
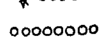
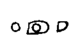
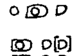
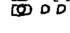
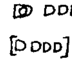
EVIDENCE FOR MATHEMATICAL INSTRUCTION CIRCA 2500 BC

As I have indicated above, some scholars have been inclined to regard the Sumerian logograms (word symbols) in Old Babylonian mathematical texts as pseudo-Sumerian, but no good reason for doubting the genuine Sumerian origin of these terms has ever been advanced. There is no evidence in the writing system as a whole for a tendency to create new word symbols of this type in the Old Babylonian period. Moreover, a review of Old Babylonian mathematical terminology reveals that all the basic mathematical procedures used in the texts--adding, subtracting, multiplying, raising to powers, finding roots, using reciprocals and coefficients--have Sumerian origins. In addition to these procedure terms, there is a large group of nouns and adjectives describing the basic elements of geometry and a few miscellaneous interrogatives, pronouns, and adverbs, all of Sumerian origin. Even when one excludes metrological and analogous terms, the list still amounts to at least fifty that have every probability of being originally used by the Sumerians themselves.

The problem in the past has been to find texts in the Sumerian period to which the phenomena of Old Babylonian mathematical texts relate. A. A. Vaiman of the Hermitage in Leningrad is presently engaged in a study of Ur III and earlier administrative texts which, I believe, will contribute substantially to an understanding of the Old Babylonian use of coefficients. However, over and above the administrative texts and the school texts that I have discussed above, there is even older evidence for the mathematical tradition which culminated in the Old Babylonian period.

In 1902-3, a German excavation at Fara (ancient Shuruppak) discovered about 1000 tablets dating to the period around 2500 BC [18]. Deimel [1923, no. 82] published from among these what is still the oldest example of a Sumerian multiplication table. It is actually a table of squares, and, although it has sometimes been conceived of as a metrological table, real metrological tables are organized according to the "list" principle and have an entirely different format. The standard metrological notation obscures the arithmetic principle, as I try to illustrate by the schematic transcription in Figure 1.

FIGURE 1: A Sumerian Table of Squares from ca. 2500 BC (Deimel, Fara II no. 82)

Obverse	Schematic Transcription				Sexagesimal Equivalent	
sag	[us]	[sa]	=	[ašag]	= in bur	= in šar
10,0 nindan	10,0 (nindan)	sa	=		= 3,20	1,40, 0, 0
9,0	9,0	sa	=		= 2,42	1,21, 0, 0
8,0	8,0	sa	=		= 2, 8	1, 4, 0, 0
7,0	7,0	sa	=		= 1,38	49, 0, 0
6,0	6,0	[sa]	=		= 1,12	36, 0, 0
5,0	5,0	sa	=		= 50	25, 0, 0
4,0	4,0	sa	=		= 32	16, 0, 0
3,0	3,0	sa	=		= 18	9, 0, 0
2,0	2,0	sa	=		= 8	4, 0, 0
1,0	1,0	sa	=		= 2	1, 0, 0
Reverse						
50	50	sa	=		= 1;23,20	41,40
40	40	sa	=		= 0;53,20	26,40
30	30	sa	=		= 0;30	15, 0
20	20	sa	=		= 0;13,20	6,40
10	[10]	[sa]	=		= 0; 3,20	1,40
5	[5]	[sa]	=		= 0; 0,50	25

The division into obverse and reverse is not arbitrary or accidental. It will be observed that the number of *bur* on the obverse can be arrived at easily by multiplying each product (stated in *šar*) by the constant factor 2. This is because, as I have stressed elsewhere [1972b, 175 ff., 219], the notation of the area measure *bur* is basically sexagesimal. I would add, *consciously* sexagesimal. No such simple relation governed the smaller surface units contained on the reverse, and the standard notation, together with the number of *šar* in each, seems to have been memorized by the student, as later metrological texts indicate [Hilprecht 1906, nos. 39 & 40]. Another text similar to the Fara table of squares is known from Bismaya (ancient Adab). It probably dates to about 2400 BC and consists of a table of small units of length and their squares stated in standard metrological notation [Edzard 1969].

More important, however, than these table texts are a number of others from Fara [19] which tell us something about the nature of mathematical instruction. One of these [Jestin 1937, no. 77] is a geometrical exercise preserved on a fragmentary school tablet of the typical lenticular type used for exercises. This tablet and its geometric design is reproduced schematically in Figure 2.

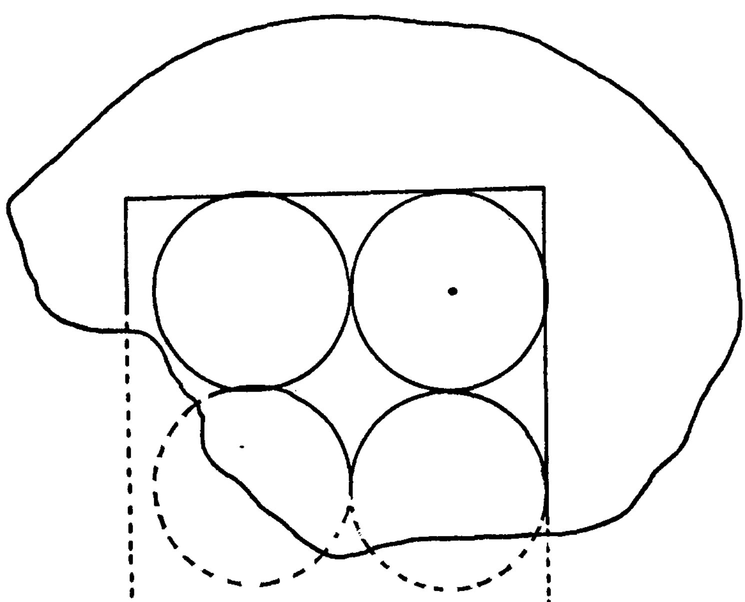


FIGURE 2: A School Tablet with a Geometrical Exercise from ca. 2500 BC [Jestin 1937, no. 77]. The identical diagram appears in an Old Babylonian text [Saggs 1960, 133], but the cuneiform text describing the figure is broken out [Saggs 1960, text N].

It may have something to do with finding the area of a circle, but any interpretation must remain hypothetical since, according to R. Jestin, only this side is legible. The other two texts which call for comment here represent two versions of the same exercise, and they are significant, not only because of the abstract interest in numerical relations indicated by the enormous numbers involved, but also because they concern the problem of irregular numbers.

One of these texts [Jestin 1937, no. 50] was treated by Geneviève Guitel [1963], (mistakenly, I believe) as a problem in division, and she posits a method of solution "absolutely analogous to modern practice." It is, however, precisely this close correspondence to modern practice that makes the solution suspect. If modern long division had been used in the Fara period, it is virtually certain that it would appear somewhere in Old Babylonian mathematical texts, which is not the case. Moreover, the most significant text pertaining to the problem of how the calculation was performed is not Jestin's no. 50 at all. It is rather a text [Jestin 1937, no. 671] written by a bungler who did not know the front from the back of his tablet, did not know the difference between standard numerical notation and area notation, and succeeded in making half a dozen writing errors in as many lines, but nevertheless was not without a modicum of ability and probably finished school with a low passing grade, took a post with the government and became a bureaucrat. The writer of no. 50 no doubt became a scholar and died penniless. However probable these postulated eventualities may be, the modern scholar may well be more grateful to our third millennium bungler than to his competent classmate. The reason for this will, I believe, be apparent if we compare the two texts.

Jestin no. 50	*Jestin no. 671*	*Translation of no. 50*
še $guru_7$:1	(rev) *še* $sila_3$ 7[!] $guru_7$	The grain (is) 1 silo.
$sila_3$ 7		7 sila (seven liters)
lú:1 *šu ba-ti*	*lú*:1 *šu ba-ti*	each man received.
lú-bi	(obv) *guruš*	Its men:
45,42,51	45,36,0 (written on three lines)	45,42,51.
še $sila_3$:3 $šu^?$-tag_4?		3 sila of grain (remaining.)

As one can see, the two problems are identical in type and form. No. 671, in addition to the handwriting errors, which I have not shown, also has *guruš* (man = Latin *vir*) instead of *lu* (man = *homo*) and omits the verb form at the end, because, as we shall see shortly, his solution did not require a remainder. A silo *(guru)* in this period contained 40,0 *gur*, each of which contained 8,0 *sila*. Thus, the number being "divided" by 7 is 5,20,0,0. Seven is the only integer between 1 and 10 that will not produce an even result, therefore, given this fact and the fact that two exercises dealing with the same problem have survived, the choice of 7 can hardly be coincidental. Moreover, the choice of 7 has no material explanation, because the seven-day week played no role in Sumerian accounting procedures, and, having read thousands of Sumerian and Akkadian texts from the third millennium, I cannot recall a single case where 7 functions as a divisor. Thus, the choice of 7 can hardly be motivated by any other cause than that it is an irregular number. Moreover, the two different answers suggest that the object is an exercise in using the reciprocal of an irregular number.

This deduction flows from the following considerations: (1) multiplication of the "dividend" by the reciprocal of the "divisor" is the only means of "dividing" attested in the Babylonian mathematical tradition, except when the "divisor" is 2 (halving); (2) the two answers obtained to the problem are explicable by a single hypothesis, but only if one assumes multiplication by the reciprocal. The correct answer seems to have been obtained by the following process:

(1) $5,20,0,0 \cdot 0;8,34,17,8 = 45,42,51;22,40$

(2) $45,42,51 \cdot 7 = 5,19,59,57$

(3) $5,20,0,0 - 5,19,59,57 = 3$

In (1), the number of *sila* is multiplied by the reciprocal of 7 calculated to the fourth place (a three-place reciprocal will not work, unless one assumes the use of rounding). In (2), the fractional number of men is discarded and the whole number multiplied by 7 to obtain the number of *sila* passed out on a seven-each basis. In (3), the product of (2) is subtracted from the original number of *sila*, giving the remainder 3. In the text containing the wrong answer (no. 671), the pupil has apparently used 0;8,33 as the reciprocal of 7, for $5,20,0,0 \cdot 0;8,33 = 45,36,0$ which is the answer contained in the text. How the pupil arrived at the choice of 0;8,33 for the reciprocal of 7, I have no idea, but perhaps someone else will see the solution where I have not.

It may seem rather startling to suggest that the Sumerians were working problems involving the use of reciprocals calculated to the fourth place in the middle of the third millennium, but I must confess that I find it difficult to believe that the relationships exhibited in the two problems are merely the result of coincidence. Also, although up to the present time there has been no definite evidence for this sort of thing, the Sumerians had been using a type of sexagesimal notation and dealing with very large numbers for several hundred years [5]. A context out of which the use of reciprocals could have emerged was, therefore, clearly in existence by the middle of the third millennium.

CONCLUSION

The origins of Babylonian mathematics go back much further than anyone has heretofore realized. In the first place, the sexagesimal place notation system was in existence during the Third Dynasty of Ur, by ca. 2050 BC. Secondly, mathematical instruction can be documented from ca. 2500 to 2200 BC. Thirdly, two problem texts from ca. 2500 and one from ca. 2200 seem to indicate the use of a mental construct analogous to place notation and the use of sexagesimal reciprocals. The texts treated are elementary mathematical exercises, but they reveal the same abstract interest in numerical relationships that is found in Old Babylonian mathematical texts. We have recently learned, in the case of Sumerian literature, that many of the texts so diligently copied in Old Babylonian schools were already in existence in the middle of the third millennium [Biggs 1974]. In my opinion, we will find evidence sooner or later to substantiate in detail the hypothesis that the basic elements of Old Babylonian mathematics are really Sumerian in origin.

NOTES

1. For further bibliography and a possible explanation for the use of sixty as a base, see [Powell 1972a, 17 n. 8], where the proposed etymology, which has there been made unintelligible by a printing error, derives Sumerian *g̃eš* (sixty) from *nig̃* (something) plus *eš* (much). The theory of van den Brom [1969], which I previously overlooked, cannot be supported by the linguistic evidence.

2. Goetsch [1968] has reviewed the basic corpus of presently known evidence and appraised the achievements and limitations of Babylonian mathematical thought. In a review of the reprint of Neugebauer [1934], Mahoney [1971] has criticized Neugebauer's claim that the Babylonians used a form of algebra. It is probably true that Babylonian mathematical logic was basically numerical, but Mahoney's inference that *ipso facto* Babylonian mathematical thought can be characterized as "mythopoeic," as opposed to "rational" Greek thought is based on an incomplete view of both Greek and Babylonian culture, which is widely accepted but nonetheless false and misleading.

3. It is true that Delaporte [1911] published a multiplication table discovered by de Sarzec at Telloh (ancient Girsu), but he offered no evidence for dating the text to the Ur III period, although this is claimed in the title. In fact, there are no criteria other than the style of writing, and, although this does not exclude an Ur III date, it is by no means conclusive in favor of it. [Neugebauer 1935, 10]

4. To my knowledge, Thureau-Dangin never made a direct claim for a Sumerian origin to Babylonian mathematics, but he did regard the Sumerian terms in Babylonian mathematical texts as resting on genuine Sumerian tradition. "Les Sumériens, dont, en ce cas comme en beaucoup d'autres, les Accadiens ont conservé la terminologie ..." [Thureau-Dangin 1936, 55] Cf. also [Thureau-Dangin 1939, 101 and 135 ff.].

5. For the Sumerian character of the Jemdet Naṣr notation, see [Powell 1972b, 168-172].

6. Other published texts observed by me to contain notation similar to YBC 1793 are: 1. Nikol'ski [1915, nos. 402 and 403]. 2. There is an instructive example of mixed notation (sexagesimal + standard metrological notation) written on the edge of a tablet dated to the last year of Shulgi [King 1898, pl. 30 no. 19027]. 3. Myrhman [1911, no. 56], dated to the second year of Ibbi-Sin, contains in the last column a broken sexagesimal entry, which should probably be restored to 2,1[6;26] for 2,16 *gur* + 2 *bariga* + 1 *ban* of barley. 4. Two undated texts from this period which indicate the use of place notation in making calculations are Thureau-Dangin [1903, nos. 408 and 413]. 5. When I discussed YBC 1793 with W. W. Hallo, curator of the Yale Babylonian Collection, he told me that he remembered seeing sexagesimal notation rather frequently in balanced accounts from the Ur III period and referred me to YBC 4179, published by Ellis [1970].

7. Dated on the basis of script, format of contents, external appearance of the tablets, and their presence in collections which can be dated to this period on the basis of contents. [Limet 1973, 16 ff; Gelb 1970, xvi-xx]

8. A *sila* contained approximately 1 liter.

9. I do not wish to leave the impression that this is the only hypothesis permitted by the evidence, for some documents from this period, which use the same type of *gur*, calculate amounts for 10-month rather than 12-month periods, e.g. Hackman [1958, no. 182]. Also, the relationship suggested by me would apply only to the normal Babylonian year, which consisted of 12 months of 30 days each. A separate calculation would have to be made for intercalary months, but that would still not diminish the usefulness of the other relationship.

10. For documentation and discussion of this expression, see for the moment Powell [1971, 54-69, 221-224, 245]. I expect to treat the problem in an article in the near future.

11. One thinks naturally of Sargon or Naram-Sin. Van der Waerden [1968] has tried to make a case for Dareios the Great as a patron of science. The role of the state in the support of learning is very poorly documented for Babylonia, but the peculiar relationship of the Babylonian monarch to the intelligentsia and, in a very real sense, his dependence upon it makes the role of patron an inherently good hypothesis.

12. For further comment on the metrological units, see Powell [1975]. For my interpretation of the terms *us* (wr. *UŠ*) and *sag*, see note 14 below.

13. The seed cubit may have had even wider application in calculating the amount of seed necessary to sow a given amount of land, for, as I have shown in my essay on area measures [1972b, 182 ff.], the Sumerians probably calculated on sowing one shekel (= 0;1 *sila*) of seed per *nindan* of length.

14. The sexagesimal ratio is 2,40 : 0;37,30. As we know from numerous mensuration documents, no real field ever had this shape. The shape of fields was determined largely by the exigencies of irrigation, which also accounts for the term *sag*, "front," referring originally to the part of the field which "fronted" on the irrigation ditch, as opposed to *us*, "side," which referred to the longer sides adjoining other fields fronting on the same canal. The term *sag* is derived from the Sumerian word for "head," and *us* is derived from a verb meaning "to adjoin/be adjacent to."

15. The copy has 2 + 1 *šar*, but it should read 2 + 1/2 (or 2½) (collated by me in Liège 16 July 1974).

16. Since 1 *šar* (or sq. *nindan*) = 1,0 *gin* = 1,0,0 *gintur* = 3,0,0 *še*, 15/60 *gintur* would equal 3/4 *še*, but there is no known way to write 3/4 *še* in standard metrological notation.

17. For the system of transcription of area measures, see [Powell 1972b, 214-216].

18. The best discussion of what we presently know about the Fara period texts and the associated set of unresolved questions is [Biggs 1974, 19-27].

19. I am much indebted to A. A. Vaiman, The Hermitage, Leningrad, who read the next to last version of this paper and called my attention to the fact that a number of additional mathematical texts from the Fara period are published in Jestin [1937]. Having excluded doubtful cases, the following texts can be assigned to the category of elementary mathematical exercises: Jestin [1937], nos. 50, 51, 77 (on pl. LIX), 81, 91, 188, 242, 245, 251, 260, 554, 613, 619, 648, 649, 671, 725, 748, 758, 775, 780, 828, 930, and 969. Deimel [1924], nos. 93 and 125 may also belong to this category, and there are probably a few others in Jestin [1957], which was inaccessible to me when I made the final revisions of this paper.

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ACKNOWLEDGEMENTS

I wish to acknowledge the useful criticism of my teacher, Tom B. Jones, who read an early draft of this paper in 1971, and especially that of my colleague in the History of Science, Joe D. Burchfield, who read the paper in its more or less final form in 1974.