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Advances in Mathematics 225 (2010) 3576–3578

ADVANCES IN Mathematics

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Corrigendum

Corrigendum to “Homology of perfect complexes” [Adv. Math. 223 (5) (2010) 1731–1781]

Luchezar L. Avramov [a], Ragnar-Olaf Buchweitz [b], Srikanth B. Iyengar [a,*], Claudia Miller [c]

[a] *Department of Mathematics, University of Nebraska, Lincoln, NE 68588, USA*
[b] *Department of Computer and Mathematical Sciences, University of Toronto Scarborough, Toronto, ON M1A 1C4, Canada*
[c] *Mathematics Department, Syracuse University, Syracuse, NY 13244, USA*

Received 12 May 2010; accepted 16 May 2010

Communicated by Karen Smith

In the statement of Theorem 7.4 the functor $\mathsf{t}\colon \mathsf{D}(S) \to \mathsf{D}(\Lambda)$ is not the right one. Theorem 7.4, its proof, and Remark 7.6.3 should be replaced by the text given below. These changes do not affect any other results or proofs in the paper.

Theorem 7.4. *Let X be the DG module described above.*

The functor $\mathrm{Hom}_\Lambda(X^*, -)$ *induces an exact functor* $\mathsf{h}\colon \mathsf{D}(\Lambda) \to \mathsf{D}(S)$.

The functor $\mathsf{t} = X^* \otimes^{\mathbf{L}}_S - \colon \mathsf{D}(S) \to \mathsf{D}(\Lambda)$ *is left adjoint to* h.

This pair of functors restricts to inverse equivalences of triangulated categories:

$$\begin{array}{ccc} \mathsf{D}(\Lambda) & \underset{\mathsf{h}}{\overset{\mathsf{t}}{\leftrightarrows}} & \mathsf{D}(S) \\ \cup| & & \cup| \\ \mathsf{thick}_\Lambda(k) & \xleftrightarrow{\equiv} & \mathsf{thick}_S(S) \\ \cup| & & \cup| \\ \mathsf{thick}_\Lambda(\Lambda) & \xleftrightarrow{\equiv} & \mathsf{thick}_S(k) \end{array} \qquad (7.4.1)$$

DOI of original article: 10.1016/j.aim.2009.10.009.

* Corresponding author.

E-mail addresses: avramov@math.unl.edu (L.L. Avramov), ragnar@utsc.utoronto.ca (R.-O. Buchweitz), iyengar@math.unl.edu (S.B. Iyengar), clamille@syr.edu (C. Miller).

0001-8708/$ – see front matter
doi:10.1016/j.aim.2010.05.017

For $d = |\xi_1| + \cdots + |\xi_c|$ *there are isomorphisms in* $\mathsf{D}(S)$ *and* $\mathsf{D}(\Lambda)$*, respectively*:

$$\begin{array}{rcl} \mathsf{h}(k) \simeq S, & & \mathsf{t}(S) \simeq k, \\ & \textit{and} & \\ \mathsf{h}(\Lambda) \simeq \Sigma^d k & & \mathsf{t}(k) \simeq \Sigma^{-d} \Lambda. \end{array} \tag{7.4.2}$$

The DG module structures of $\operatorname{Hom}_\Lambda(X^*, N)$ over S and of $X^* \otimes^{\mathbf{L}}_S M$ over Λ are those induced by X^*, as explained in 7.6.3.

7.6.3. For every DG Λ-module N the complex $\operatorname{Hom}_\Lambda(X^*, N)$ of k-vector spaces has a canonical structure of DG S-module, isomorphic to $\operatorname{Hom}_k(S^*, N)^\delta$.

Multiplication with δ annihilates $\operatorname{Hom}_\Lambda(\Lambda \otimes_k S^*, k)^\natural$. This observation shows that the following natural S-linear isomorphisms are morphisms of DG S-modules:

$$\operatorname{Hom}_\Lambda\big(X^*, k\big) \cong \operatorname{Hom}_\Lambda\big(\big(\Lambda \otimes_k S^*\big)^{-\delta}, k\big) = \operatorname{Hom}_\Lambda\big(\Lambda \otimes_k S^*, k\big) \cong S.$$

For each DG S-module M the complex $X^* \otimes_S M$ of k-vector spaces has a structure of DG Λ-module, isomorphic to $(\Lambda^* \otimes_k M)^\delta$. Thus, $X^* \otimes_S -$ is a functor from DG S-modules to DG Λ-modules; it induces a functor $X^* \otimes^{\mathbf{L}}_S - : \mathsf{D}(S) \to \mathsf{D}(\Lambda)$.

Proof of Theorem 7.4. The functor $\operatorname{Hom}_\Lambda(X^*, -)$ preserves quasi-isomorphisms, by 7.6.5, so it induces an exact functor $\mathsf{h} : \mathsf{D}(\Lambda) \to \mathsf{D}(S)$.

It is clear that the functor $\mathsf{t} = X^* \otimes^{\mathbf{L}}_S -$ is a left adjoint to h. Let

$$\sigma : \mathsf{Id}^{\mathsf{D}(S)} \to \mathsf{ht} \quad \text{and} \quad \lambda : \mathsf{th} \to \mathsf{Id}^{\mathsf{D}(\Lambda)},$$

respectively, be the unit and the counit of the adjunction. Thus, for each $M \in \mathsf{D}(S)$ and $N \in \mathsf{D}(\Lambda)$ there are morphisms of DG modules over S and Λ, respectively:

$$M \xrightarrow{\sigma(M)} \operatorname{Hom}_\Lambda\big(X^*, X^* \otimes^{\mathbf{L}}_S M\big) \quad \text{and} \quad X^* \otimes^{\mathbf{L}}_S \operatorname{Hom}_\Lambda\big(X^*, N\big) \xrightarrow{\lambda(N)} N.$$

To prove that h and t restrict to inverse equivalences between $\mathsf{thick}_\Lambda(k)$ and $\mathsf{thick}_S(S)$ it suffices to show $\lambda(k)$ and $\sigma(S)$ are quasi-isomorphisms.

Consider the chain of morphisms of DG Λ-modules

$$X^* \otimes^{\mathbf{L}}_S \operatorname{Hom}_\Lambda\big(X^*, k\big) \cong X^* \otimes^{\mathbf{L}}_S S \cong X^* \xrightarrow{\rho} k,$$

where the first isomorphism comes from 7.6.3. By the same token, the DG S-module $\operatorname{Hom}_\Lambda(X^*, k)$ is semi-free, so in the formula above one may replace $\otimes^{\mathbf{L}}_S$ with $\otimes_S$. Now a direct verification shows that the composite map equals $\lambda(k)$. Since ρ is a quasi-isomorphism, see 7.6.2, so is $\lambda(k)$.

In the following chain of morphisms of DG S-modules the last one is from 7.6.3:

$$S \xrightarrow{\sigma(S)} \operatorname{Hom}_\Lambda\big(X^*, X^*\big) \xrightarrow{\operatorname{Hom}_\Lambda(X^*, \rho)} \operatorname{Hom}_\Lambda\big(X^*, k\big) \cong S.$$

A direct computation shows that the composite map is id^S. Since ρ is a quasi-isomorphism so is $\operatorname{Hom}_\Lambda(X^*, \rho)$, by 7.6.5; hence $\sigma(S)$ is a quasi-isomorphism.

We have proved that h and t restrict to inverse equivalences between $\mathsf{thick}_\Lambda(k)$ and $\mathsf{thick}_S(S)$. We now verify the isomorphisms in (7.4.2).

We obtain an isomorphism $\mathsf{h}(k) \simeq S$ from 7.6.3. Using the quasi-isomorphism ρ of DG Λ-modules from 7.6.2 and the exactness of the functor $\operatorname{Hom}_\Lambda(-, \Lambda)$, see 7.6.6, one gets quasi-isomorphisms

$$\mathsf{h}(\Lambda) = \operatorname{Hom}_\Lambda\bigl(X^*, \Lambda\bigr) \simeq \operatorname{Hom}_\Lambda(k, \Lambda) \cong \Sigma^d k$$

of complexes of vector spaces. It yields $\mathsf{h}(\Lambda) \simeq \Sigma^d k$ in $\mathsf{D}(S)$, see Proposition 3.12(1).

Since $\lambda : \mathsf{th} \to \mathrm{id}^{\mathsf{D}(\Lambda)}$ is an isomorphism on $\mathsf{thick}_\Lambda(k)$, it follows from the calculations above that there are quasi-isomorphisms of DG Λ-modules

$$\mathsf{t}(S) \simeq \mathsf{th}(k) \simeq k \quad \text{and} \quad \mathsf{t}(k) \simeq \Sigma^{-d}\mathsf{th}(\Lambda) \simeq \Sigma^{-d}\Lambda.$$

Finally, as h restricts to an equivalence $\mathsf{thick}_\Lambda(k) \equiv \mathsf{thick}_S(S)$ and $\mathsf{h}(\Lambda) \simeq \Sigma^d k$ holds, further restriction yields an equivalence $\mathsf{thick}_\Lambda(\Lambda) \equiv \mathsf{thick}_S(k)$. $\square$