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Physics Letters B 556 (2003) 80–86
PHYSICS LETTERS B
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A note on the dual of $\mathcal{N}=1$ super-Yang–Mills theory

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Received 5 December 2002; received in revised form 15 January 2003; accepted 20 January 2003

Editor: M. Cvetič

Abstract

We refine the dictionary of the gauge/gravity correspondence realizing $\mathcal{N}=1$ super-Yang–Mills by means of D5-branes wrapped on a resolved Calabi–Yau space. This is done by fixing an ambiguity on the correct interpretation of the holographic dual of the running gauge coupling and amounts to identify a specific 2-cycle in the dual ten-dimensional supergravity background. In doing so, we also discuss the role played in this context by gauge transformations in the relevant seven-dimensional gauged supergravity. While all nice properties of the duality are maintained, this modification of the dictionary has some interesting physical consequences and solves a puzzle recently raised in the literature. In this refined framework, it is also straightforward to see how the correspondence naturally realizes a geometric transition.

1. Introduction and summary of the results

One of the goals recently pursued in the context of the AdS/CFT correspondence has been to look for gravity duals of $\mathcal{N}=2$ and $\mathcal{N}=1$ super-Yang–Mills (SYM) theory, both with and without matter.

An important step toward this goal was done by Maldacena and Nuñez (MN) in Ref. [1] where a supergravity dual of pure $\mathcal{N}=1$ SYM was proposed. As it is the case for gravity duals of confining gauge theories, one cannot obtain an exact duality since extra degrees of freedom, not belonging to the gauge theory, cannot be decoupled within the supergravity regime.[1] Nevertheless, many interesting properties of the gauge theory are encoded in the dual supergravity background and can be described in detail.

The MN model is constructed engineering a $\mathcal{N}=1$ SYM theory by wrapping N D5-branes on a non-trivial 2-cycle of a resolved Calabi–Yau (CY) space. The unwrapped part of the brane world-volume remains flat and supports a four-dimensional gauge theory. By implementing the proper topological twist so to preserve 4 supercharges [3], some of the world-volume fields become massive and decouple and one ends up, in the IR, with four-dimensional pure $\mathcal{N}=1$ SYM theory. This is obtained by considering the world-volume theory of the D5-branes at energies where both the higher string modes as well as the KK excitations on the 2-cycle decouple. The back-reaction

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[1] Similar considerations hold, for instance, for another notable example of a gravity dual of $\mathcal{N}=1$ SYM, the Klebanov–Strassler solution [2], which, although displaying a different UV completion, is equivalent to the MN solution in the IR.

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doi:10.1016/S0370-2693(03)00100-X

of the D-branes deforms the original background. The topology of the resulting space is in general very different from the starting CY space. In this case, as discussed by Vafa in Ref. [4], one expects the resulting space to be a deformed CY space, where the 2-cycle has shrunk but a 3-sphere has blown-up, rendering a ten-dimensional non-singular solution. The question is whether one can extract information on the gauge theory, possibly at non-perturbative level, from the dual supergravity background.

This question was recently addressed in a rather detailed way by Di Vecchia, Lerda and Merlatti (DLM) in Ref. [5] (see Refs. [6,7] for previous works discussing these issues) and a number of informations on the gauge theory were shown to be predicted by the dual supergravity background in a precise and quantitative way. In particular, the expected running of the gauge coupling with the corresponding β-function, the chiral symmetry anomaly, the phenomenon of gaugino condensation with the corresponding breaking of the chiral symmetry to $\mathbb{Z}_2$ in the IR as well as the instanton action contribution, were all derived from the supergravity solution.

The gauge/gravity dictionary can be derived from two basic equations [5] expressing the gauge coupling constant and the gaugino condensate $\langle\lambda^2\rangle$, which is a protected operator of the gauge theory, in terms of supergravity degrees of freedom. These two equations read

$$\frac{1}{g^2_{\mathrm{YM}}} = F(\rho) \sim \mathrm{Vol}\big(S^2\big), \tag{1}$$

$$\big\langle\lambda^2\big\rangle \sim \left(\frac{\Lambda}{\mu}\right)^3 = G(\rho), \tag{2}$$

where Λ is the dynamically generated scale, μ is the subtraction energy at which the gauge theory is defined and $F(\rho)$ and $G(\rho)$ are two given functions of the radial coordinate ρ of the ten-dimensional supergravity background. In particular, $F(\rho)$ is proportional to the volume of the 2-cycle the D5-branes wrap as seen in the deformed geometry. The identification in Eq. (2) (which is written in units of the energy scale) gives instead the radius/energy relation in the correspondence.

An important point to notice is that the gravity quantities to be compared with gauge theory operators should all be computed in the ten-dimensional framework, this being the natural one from a string theory point of view. This was done only partially in Ref. [1]. The 2-cycle entering Eq. (1) was identified within the seven-dimensional gauged supergravity geometry, while all other quantities, as the chiral anomaly and the gauge theory instanton contribution were obtained considering the ten-dimensional geometry. The observation above overcomes this hybrid interpretation and leads to a very homogeneous picture of the entire duality, as it was drawn in Ref [5]. However, as pointed out recently in Ref. [8], this posed a new problem since it seemed that in doing so a singular transformation in the gauge coupling was needed in order to get the NSVZ β-function [9] from the β-function obtained from the corresponding gravitational dual.

In this Letter we reconsider this issue, and clarify what is the correct 2-cycle in the ten-dimensional deformed geometry to be considered, related to the 2-cycle of the original resolved CY space used to engineer the $\mathcal{N}=1$ SYM theory. It turns out that the 2-cycle considered in the literature is not the correct one. As we are going to show, this observation solves the problem raised in Ref. [8], without spoiling, on the other hand, all nice results obtained in Ref. [5]. In particular, we will get their same result for the β-function, but determining now unambiguously the two-loop coefficient. It turns out that supergravity, through the holographic relations (1) and (2), gives a β-function which is in the same scheme as that obtained by NSVZ, the Pauli–Villars scheme. Redefinitions of the holographic relation (2) by means of analytic functions of the gauge coupling respecting the symmetry of $\langle\lambda^2\rangle$, correspond to a change of regularization scheme. This modifies the β-function beyond two-loops only, showing that supergravity naturally respects the expected universality of the 2-loop coefficient.[2] As a further check for the validity of our analysis, in this refined framework it is easy to see that the MN model realizes a geometric transition, as predicted for these kind of gauge/gravity dualities by the general picture discussed by Vafa in Ref. [4].

[2] We thank Wolfang Mueck for sharing with us his recent findings on related topics.

2. The geometry revisited

Let us start by summarizing the explicit form of the MN solution. This solution is obtained from a non-singular domain wall solution of seven-dimensional gauged supergravity [10], parameterized by coordinates $(x_0, \dots, x_3, \rho, \theta_1, \phi_1)$, uplifting to ten dimensions along a 3-sphere [11,12], parameterized by coordinates (ψ, θ_2, ϕ_2). The relevant fields (the metric, the dilaton and the RR 3-form the D5-branes magnetically couple to) are

$$ds^2 = e^{\Phi}\, dx_{1,3}^2 + e^{\Phi}\alpha' g_s N\left[e^{2h}\left(d\theta_1^2 + \sin^2\theta_1\, d\phi_1^2\right) + d\rho^2 + \sum_{a=1}^{3}\left(\sigma^a - A^a\right)^2\right], \tag{3}$$

$$e^{2\Phi} = \frac{\sinh 2\rho}{2e^h}, \tag{4}$$

$$F^{(3)} = 2\alpha' g_s N \prod_{a=1}^{3}\left(\sigma^a - A^a\right) - \alpha' g_s N \sum_{a=1}^{3} F^a \wedge \sigma^a, \tag{5}$$

where

$$A^1 = -\frac{1}{2}a(\rho)\, d\theta_1,$$
$$A^2 = \frac{1}{2}a(\rho)\sin\theta_1\, d\phi_1,$$
$$A^3 = -\frac{1}{2}\cos\theta_1\, d\phi_1, \tag{6}$$

$$e^{2h} = \rho\coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4},$$
$$a(\rho) = \frac{2\rho}{\sinh 2\rho}, \tag{7}$$

A^a being the three $SU(2)_L$ gauge fields of the relevant seven-dimensional gauged supergravity. The σ^a are the left-invariant one-forms parameterizing the 3-sphere

$$\sigma^1 = \frac{1}{2}(\cos\psi\, d\theta_2 + \sin\psi\sin\theta_2\, d\phi_2),$$
$$\sigma^2 = -\frac{1}{2}(\sin\psi\, d\theta_2 - \cos\psi\sin\theta_2\, d\phi_2),$$
$$\sigma^3 = \frac{1}{2}(d\psi + \cos\theta_2\, d\phi_2). \tag{8}$$

Let us now turn to the identification of the actual S^2 of the ten-dimensional geometry (3) entering the gauge/gravity relation (1). Naively one would say that this cycle is the cycle parameterized by the two coordinates (θ_1, ϕ_1). This is indeed the original cycle already present in the seven-dimensional solution one starts from to derive the ten-dimensional one. This was the choice made both in Ref. [1] and Ref. [5], within the seven and ten-dimensional geometry, respectively. In fact, the seven-dimensional solution is non-trivially embedded in ten dimensions, the non-triviality coming from the topological twist performed in seven dimensions. As a result of this, there is a non-trivial mixing between the three coordinates of the S^3 along which one uplifts the solution (θ_2, ϕ_2, ψ) and those of the S^2 along which the original seven-dimensional domain wall is wrapped (θ_1, ϕ_1). This mix can be seen explicitly by the appearance of the seven-dimensional gauge connection in the ten-dimensional metric (3). We could say that the seven-dimensional domain wall already knows about the ten-dimensional geometry via the twist, that from a seven-dimensional point of view actually mixes space–time degrees of freedom with internal ones (note that in ten dimensions all these degrees of freedom are relative to space–time). For this reason, it will turn out that the proper 2-cycle is different from that suggested by the naive intuition.

To identify the relevant 2-cycle (and the 3-cycle dual to it) we now focus on the five-dimensional angular part of the metric (3). Let us consider two particular limits, $\rho \to \infty$ and $\rho \to 0$. At large ρ, from the solution (3) we easily get

$$ds_5^2 \sim \rho\left(d\theta_1^2 + \sin^2\theta_1\, d\phi_1^2\right) + \frac{1}{4}\left(d\theta_2^2 + \sin^2\theta_2\, d\phi_2^2\right) + \frac{1}{4}(d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2)^2. \tag{9}$$

It is easy to see that this is precisely the metric of the $T^{1,1}$ manifold, that topologically is $S^2 \times S^3$. Even if it differs from the 'standard' $T^{1,1}$ (see for instance Refs. [13,14]) as now it is re-scaled in a way it is no longer an Einstein space, we can anyhow determine the non-trivial cycles. They are those of the standard $T^{1,1}$, since the only difference with the above manifold is just a metric difference.

In the above set of coordinates, see Refs. [15–18], the 2-cycle is not uniquely defined and it turns out there are two different, but physically equivalent, choices

$$S^2:\quad \theta_1=-\theta_2,\qquad \phi_1=-\phi_2,\qquad \psi=0, \tag{10}$$

$$S^2:\quad \theta_1=\theta_2,\qquad \phi_1=-\phi_2,\qquad \psi=\pi. \tag{11}$$

The value of ψ is fixed by the physical requirement that the cycle is that of minimal volume, this being proportional to the wrapped D5-brane tension. It is easy to show that with the first choice, $\theta_1=-\theta_2$, $\phi_1=-\phi_2$, the minimal volume in the geometry (3) is for $\psi=0$. Analogously, for $\theta_1=\theta_2$, $\phi_1=-\phi_2$ we have that $\psi=\pi$. Let us stress that the two 2-cycles (10) and (11) are physically equivalent. Indeed we can see from Eqs. (3) and (5) that the two corresponding volumes are equal, namely

$$\mathrm{Vol}\big(S^2\big)\sim\left[\mathrm{e}^{2h(\rho)}+\frac{1}{4}\big(a(\rho)-1\big)^2\right]\times\big(d\theta^2+\sin^2\theta\, d\phi^2\big) \tag{12}$$

and the projection of the RR field strength along both cycles vanishes at the origin, as it should be. Moreover, all the gauge theory implications we will discuss in the next section are the same for the two cycles.

As already discussed the 3-cycle is instead parameterized by

$$S^3:\quad \theta_1=\phi_1=0. \tag{13}$$

Let us now study the metric at the origin. It has the following form

$$\begin{aligned} ds_5^2\sim{}&\frac{1}{4}(\cos\psi\sin\theta_2\, d\phi_2-\sin\psi\, d\theta_2-\sin\theta_1\, d\phi_1)^2\\ &+\frac{1}{4}(\sin\psi\sin\theta_2\, d\phi_2+\cos\psi\, d\theta_2+d\theta_1)^2\\ &+\frac{1}{4}(d\psi+\cos\theta_1\, d\phi_1+\cos\theta_2\, d\phi_2)^2. \end{aligned}\tag{14}$$

This is precisely the metric of a deformed conifold at the apex, see Ref. [13].[3] The parameterization of the non-trivial 2 and 3-cycle is known for this metric, and is consistent with the ones found before. By implementing Eq. (10) (or equivalently Eq. (11)) and Eq. (13) in the above metric one finds a vanishing radius for the 2-sphere and a finite one for the 3-sphere, as expected for a deformed conifold. We will come back to this issue in the last section.

[3] We thank E. Gimon for a useful comment on this point.

Let us anticipate that with the above identification of the 2-cycle, which has of course non-trivial consequences on the explicit form of the function entering in Eq. (1) (see the explicit expression in Eq. (12)), the main results about the gauge theory obtained in Ref. [5] do not change drastically. On the other hand, as already noticed, the problem related to the determination of the proper β-function by means of the gravitational dual will be solved.

Before studying the gauge theory implications of what we have been discussing so far, we want to illustrate another way one can get the same result. In doing so, we also clarify the meaning of the seven-dimensional gauge transformations. Indeed in the non-singular seven-dimensional solution we are free to make $SU(2)$ gauge transformations

$$A\to g^{-1}Ag+\mathrm{i}g^{-1}\,dg,$$

where g is an element of the $SU(2)$ group and A is the $SU(2)$ gauge connection.

The ten-dimensional solution then is not completely determined, even if all the possible solutions should be equivalent. Indeed different seven-dimensional gauge choices correspond to different parameterizations of the relevant ten-dimensional geometry. We show this with one concrete example. Consider then the following gauge transformation

$$g=\mathrm{e}^{-\frac{\mathrm{i}}{2}\theta_1\sigma_1}\mathrm{e}^{-\frac{\mathrm{i}}{2}\phi_1\sigma_3} \tag{15}$$

on the A^a's in Eq. (6). The new gauge connection is

$$\begin{aligned} A'^1={}&\frac{1}{2}\big(a(\rho)-1\big)\\ &\times(-\cos\phi_1\, d\theta_1+\cos\theta_1\sin\theta_1\sin\phi_1\, d\phi_1),\\ A'^2={}&\frac{1}{2}\big(a(\rho)-1\big)\\ &\times(\sin\phi_1\, d\theta_1+\cos\theta_1\sin\theta_1\cos\phi_1\, d\phi_1),\\ A'^3={}&-\frac{1}{2}\big(a(\rho)-1\big)\sin^2\theta_1\, d\phi_1, \end{aligned}\tag{16}$$

where the function $a(\rho)$ is again given by Eq. (7). Now we have that $A'\to 0$ for $\rho\to 0$. As already discussed at the beginning of this section, the seven-dimensional gauge connection is the field responsible for the non-trivial mixing between the seven-dimensional coordinates and the ten-dimensional ones. Moreover, the seven-dimensional solution represents a domain wall located precisely at $\rho=0$ (corresponding to the wrapped D5-branes). Hence if the gauge potential vanishes at $\rho = 0$, the 2-cycle no longer mixes with the S^3 used to uplift the solution to ten dimensions. Then, also in the ten-dimensional solution, the 2-cycle will be simply parameterized by θ_1 and ϕ_1, while, as usual, the 3-cycle by θ_2, ϕ_2 and ψ.

Once the cycles are properly identified, it is completely equivalent to study the gauge theory by means of this solution (that in terms of A', Eq. (16)) or of the other one (that in terms of A, Eq. (6)). All the physical results we are going to describe in the next section will not change. Note that also for this cycle the ρ-dependent volume is precisely given by Eq. (12) and the projection of the RR field-strength on it vanishes at the origin.

3. The gauge/gravity dictionary revisited

Let us now investigate what are the consequences of the above discussion on the gauge/gravity dictionary. As recalled in the introduction, the two crucial equations in relating gauge and gravity quantities are those expressing the gauge coupling and the energy scale of the gauge theory as functions of gravity fields. Using the solution (3)–(7) they read in our case

$$\frac{1}{g_{\mathrm{YM}}^2} = \frac{1}{2(2\pi)^3\alpha' g_s}\int_{S^2} \mathrm{e}^{-\phi}\sqrt{\det G} = \frac{N}{16\pi^2}Y(\rho), \tag{17}$$

$$\left(\frac{\Lambda}{\mu}\right)^3 = a(\rho), \tag{18}$$

where

$$Y(\rho) = 4\mathrm{e}^{2h(\rho)} + \big(a(\rho)-1\big)^2 = 4\rho\tanh\rho,$$
$$a(\rho) = \frac{2\rho}{\sinh 2\rho}, \tag{19}$$

Eq. (17) is obtained identifying the Yang–Mills coupling constant from the DBI action of the D5-branes while the energy/radius relation (18) was obtained in Ref. [5] from the identification of the gaugino condensate in terms of the supergravity field $a(\rho)$ [7].

Eq. (17) differs from the analogous equation of Ref. [5], Eq. (4.7), the difference being in the precise ρ-dependence of the function $Y(\rho)$ (that is essentially the volume of the S^2). In particular, now $Y(\rho)$ goes to zero at small ρ, and we get

$$\frac{1}{g_{\mathrm{YM}}^2} \simeq \frac{N\rho}{4\pi^2} \quad \text{for } \rho \to \infty \text{ which means } \mu \gg \Lambda, \tag{20}$$

$$\frac{1}{g_{\mathrm{YM}}^2} \simeq 0 \quad \text{for } \rho \to 0 \text{ which means } \mu \sim \Lambda. \tag{21}$$

The large ρ behavior is the same as in DLM, while at $\rho = 0$ we get a Landau pole. We will comment more on this point later.

From the above equations one can get the complete perturbative $\mathcal{N} = 1$ β-function. We can write

$$\beta(g_{\mathrm{YM}}) = \frac{\partial g_{\mathrm{YM}}}{\partial \ln(\mu/\Lambda)} = \frac{\partial g_{\mathrm{YM}}}{\partial\rho}\frac{\partial\rho}{\partial \ln(\mu/\Lambda)} \tag{22}$$

and compute the two derivative contributions from Eqs. (17) and (18), respectively. In doing so, let us first disregard the exponential corrections, which are sub-leading at large ρ and which give rise to non-perturbative contributions. In this case the expansion in Eq. (20) is exact. We easily get

$$\frac{\partial g_{\mathrm{YM}}}{\partial\rho} = -\frac{N g_{\mathrm{YM}}^3}{8\pi^2},$$
$$\frac{\partial\rho}{\partial\ln(\mu/\Lambda)} = \frac{3}{2}\left(1-\frac{1}{2\rho}\right)^{-1} = \frac{3}{2}\left(1-\frac{N g_{\mathrm{YM}}^2}{8\pi^2}\right)^{-1}, \tag{23}$$

where in the last step of the second equation we have used again Eq. (20). The final result is then

$$\beta(g_{\mathrm{YM}}) = -3\frac{N g_{\mathrm{YM}}^3}{16\pi^2}\left(1-\frac{N g_{\mathrm{YM}}^2}{8\pi^2}\right)^{-1} \tag{24}$$

which is the NSVZ β-function [9]. Note how this differs from the result of DLM. Besides exponentially suppressed corrections, in their case the expression (20) received also sub-leading corrections as power series in $1/\rho$ and $\log\rho$. These corrections should be taken into account when deriving the perturbative β-function. The contributions in the $1/\rho$ change the result beyond two-loop only, hence respecting the universality of the two-loop coefficient of the β-function. The contributions proportional to $\log\rho$, instead, spoil this universality. This gives, as a result, a β-function not belonging to the same universality class of the NSVZ β-function. As discussed in Ref. [8], in order to get rid of the unwanted logarithmic corrections and get a β-function respecting the universality of the two-loop coefficient, a singular transformation in the gauge coupling is needed. We have shown here that the correct identification of the relevant 2-cycle in the geometry gives instead directly the result (24) and the complications discussed in Ref. [8] are not present. Let us stress that this is not an option: once the correct 2-cycle is identified, the result (24) naturally follows.

Note also how the correct gauge/gravity dictionary naturally respects the universality of the two-loop coefficient of the β-function. Indeed the geometric considerations leading to the identification of the gaugino condensate with the function $a(\rho)$ [5,7] are insensible to a redefinition of the holographic relation (18) by means of an analytic function of the gauge coupling [8]. If doing so, one can easily see that the result we have obtained, Eq. (24), changes beyond two-loops only.

As anticipated there are also some non-perturbative contributions to the β-function that supergravity suggests should be present. These are included by considering the full expression for $Y(\rho)$ and $a(\rho)$ in Eqs. (17) and (18). The analysis performed in Ref. [5] is essentially unchanged in this case and we do not repeat it here. It would be nice to check this (unexpected) prediction by doing some computations in the field theory.

The correct supergravity prediction for the chiral anomaly and chiral symmetry breaking discussed in Ref. [5] is also unchanged. The gauge theory θ-angle is related to the flux of the RR 2-form $C^{(2)}$ through the 2-cycle and the N vacua of the gauge theory are parameterized by shifts in the angular variable ψ. Finally, the gauge theory instantons are described by euclidean D1-branes wrapped on the 2-cycle (10) (or equivalently (11)), and computing their corresponding action in the background (3)–(7) one easily finds the expected gauge theory instanton action, as in Ref. [5].

Summarizing, once the proper identification of the S^2 related to the gauge coupling is made, all the nice properties of the correspondence discussed in Ref. [5] still hold while the complications addressed in Ref. [8] turn out not to be present. The only property which is lost is soft confinement one had signs of, in the DLM picture, when taking ρ all the way to zero. We find a Landau pole, instead. However, this is not really an issue. The curvature of the MN background goes like $\alpha'\mathcal{R} \sim 1/g_s N$ so the regime in which the supergravity approximation is reliable is for large N. In this regime a Landau pole can indeed be present even if the gauge coupling remains finite at the scale Λ, since in Eq. (21) it is really $g^2_{\mathrm{YM}} N$ which is going to infinity and not the gauge coupling itself. To discuss the duality in the deep IR at finite N, one has to go beyond the supergravity approximation.

4. The duality as a geometric transition

As anticipated, a by-product of our analysis is that now it is easy to show that the MN solution is indeed an explicit example realizing the general picture proposed by Vafa in Ref. [4] (see Refs. [19, 20] for further clarifications).

The general idea discussed in Ref. [4], applied to the case at hand, is to engineer a supersymmetric gauge theory by means of D5-branes wrapped on a supersymmetric 2-cycle of a resolved CY manifold. The dual supergravity solution is conjectured to correspond to a deformed CY geometry, where the D-branes are absent and the manifold has undergone a geometric transition: on the deformed CY the S^2 is shrunk and an S^3 has blown-up. The D-branes are replaced by H_{NSNS} flux through a non-compact 3-cycle and H_{RR} flux through the S^3.

From the discussion in Section 2, it is clear that the MN duality indeed realizes a geometric transition. Starting from the ten-dimensional metric, Eq. (3), and taking the limit $\rho \to 0$ we get

$$ds^2 \sim dx^2_{1,3} + \alpha' g_s N \left[d\rho^2 + \rho^2 \left(d\theta_1^2 + \sin^2\theta_1 \, d\phi_1^2\right) + \sum_{a=1}^{3} \left(\sigma^a - A^a\right)^2 \right]. \qquad (25)$$

By using Eq. (13), combined with one parameterization of the 2-cycle (equivalently Eqs. (10) or (11) one immediately sees that the topology of the space at $\rho = 0$ is that of an S^3, which is blown-up: while the 2-sphere is shrunk ($R^2_{S^2} \sim \rho^2$), the radius of the 3-sphere remains finite, $R^2_{S^3} = \alpha' g_s N$. Hence the original resolved CY space used to engineer the $\mathcal{N} = 1$ SYM by means of D-branes wrapped on a non-vanishing 2-cycle has undergone a geometric transition to a deformed CY, where the S^2 has shrunk and an S^3 has blown-up, as predicted by Vafa duality.[4]

Let us end noticing an aspect where the MN correspondence is apparently different from Vafa general picture. In the MN supergravity solution there is just one 3-form, H_{RR}, switched-on while the NSNS one is not. As we have been extensively discussed, in the MN solution the gauge coupling is related to the volume of the S^2, rather than to the B_{NS} flux along the 2-cycle, as it is instead the case for Vafa duality. In fact, the MN configuration is related by T-dualities to fractional D3-branes on $\mathcal{N}=1$ orbifolds (having D4-branes suspended between non-parallel NS5 branes as an intermediate step). There, the volume of S^2 translates indeed into the B_2 flux along the S^2. So, in a sense, this difference amounts just to a U-duality gauge.

Acknowledgements

We are grateful to P. Di Vecchia and A. Lerda for some very useful remarks on a preliminary version of this Letter. We thank D. Berenstein, C. Nuñez and E. Imeroni for discussions and/or correspondence. M.B. is supported by an EC Marie Curie Postdoc Fellowship under contract number HPMF-CT-2000-00847. Work partially supported by the European Community's Human Potential Program under contract number HPRN-CT-2000-00131.

[4] The same conclusion could be reached considering the background defined by the seven-dimensional gauge connection (16).

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