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Sum rules and spin-dependent polarizabilities of the deuteron in effective field theory

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Abstract

We construct sum rules for the forward vector and tensor polarizabilities for any spin-S target and apply them to the spin-1 deuteron. We calculate these polarizabilities of the deuteron to the next-to-leading order in the pionless effective field theory.

Low-energy photon scattering on a composite system can be characterized by a host of electromagnetic polarizabilities, many of which depend on the polarization (spin) state of the system. Comparing experimental measurements of these polarizabilities with theoretical predictions allows one to learn about the underlying dynamics of the composite system. In this Letter, we are interested in the *spin-dependent* vector and tensor polarizabilities of the deuteron. The spin-independent electric polarizability α_{E0} of the nucleus has long been a subject of investigation in the literature, and it has been explored extensively in the potential models [1]. More recently, it has been calculated in effective field theories with and without explicit pion degrees of freedom [2,3]. Experimentally, α_{E0} has been measured through deuteron scattering off heavy atoms $(0.70 \pm 0.05\ \text{fm}^3)$ [4] and also extracted from the photo-production data through a sum rule $(0.69 \pm 0.04\ \text{fm}^3)$ [5].

We communicate two sets of results in this Letter. First is the sum rules for the spin-dependent vector and tensor polarizabilities. Because the deuteron binding-energy is 2.2 MeV, extracting the polarizabilities directly from Compton scattering is difficult experimentally. One would need high-intensity, polarized photon beams at energy of order 1 MeV or less, which are not available at the present time. (Compton scattering on the deuteron has been studied in the past and sum rules have been explored [6,7], and recently it has been investigated in effective field theories [8].) One could scatter a polarized deuteron beam off the Coulomb field of a heavy nucleus, as in [4], observing spin-dependent effects. However, an easier way might be to extract these polarizabilities from the spin-dependent photo-production cross sections through sum rules. The HIGS facility at Duke with photon energy from ~ 1 to 200 MeV could be used for this purpose [9].

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The second set of results is on the effective field theory (EFT) calculations of the forward spin polarizabilities. A systematic EFT approach to the deuteron structure and scattering processes has been developed in the last few years [10] and has been applied successfully to many experimental observables (see [11] for a review). Here we use the pionless version of the theory [3] to calculate the vector and tensor polarizabilities up to the next-to-leading order. Since all the counter terms to this order have been fixed from other processes, there are no free parameters in our prediction.

Before specializing to the spin-1 deuteron case, we consider the forward scattering of a circularly polarized photon of positive helicity on a nucleus of spin S and the magnetic quantum number m_S (we choose the direction of photon momentum as the quantization axis z). The total number of forward scattering amplitudes is easily found to be $2S + 1 + [S]$, where $[S]$ denotes the integer part. These amplitudes arise from the initial deuteron and photon states with the total angular momentum projection $S+1, S, (S-1)^2, \ldots, 1^2, 0^2$ for integer nuclear spin, and $S+1, S, (S-1)^2, \ldots, (3/2)^2, (1/2)^2$ for half integer nuclear spin, where the superscripts denote the multiplicity of the amplitudes. Here we are concerned with the forward amplitudes without helicity flip, for which there are exactly $2S+1$.

Let us denote the scattering amplitude for $\gamma(+1) + A(m_S) \to \gamma(+1) + A(m_S)$ by $f^{(m_S)}(\theta)$, where θ is the scattering angle, and the corresponding cross section by $\sigma^{(m_S)}$. Then the well-known optical theorem states

$$\mathrm{Im}\, f^{(m_S)}(0) = \frac{k}{4\pi}\sigma^{(m_S)}, \tag{1}$$

where k is the center-of-mass momentum. Since $f^{(m_S)} \sim \chi^*_{m_S}\chi_{m_S}$ where χ_{m_S} is the spin wave function of the target, we can couple the initial and final spin wave functions into tensors with definite total angular momentum,

$$\tilde{f}_J = \frac{1}{\sqrt{2S+1}} \times \sum_{m_S} (-1)^{S-m_S} \langle S - m_S D m_S | J0\rangle f^{(m_S)}, \tag{2}$$

and a similar relation can be used to define $\tilde{\sigma}_J$ such that the optical theorem exists between them: $\mathrm{Im}\, \tilde{f}_J(0) = \frac{k}{4\pi}\tilde{\sigma}_J$. Obviously for $J=0$, one has

$$\tilde{f}_0 = \frac{1}{2S+1}\sum_{m_S} f^{(m_S)}, \tag{3}$$

which is just the unpolarized scattering amplitude.

Based on the above relation (2), a class of sum rules for scattering amplitude can be established [7]. In this Letter, we want to establish the sum rules for forward polarizabilities. Therefore, we need to find the general structures for the forward Compton scattering amplitudes and define the forward polarizabilities as coefficients of them. We choose to write down the scattering amplitude in terms of the following tensor structures for a general spin target,

$$f = f_0 \hat{\epsilon}^* \cdot \hat{\epsilon} + f_1 i \hat{\epsilon}^* \times \hat{\epsilon} \cdot \vec{S} + f_2 (\hat{k} \otimes \hat{k})^{(2)} \cdot (\vec{S} \otimes \vec{S})^{(2)} \hat{\epsilon}^* \cdot \hat{\epsilon} + \cdots, \tag{4}$$

where $\hat{\epsilon}$ is the photon polarization, $\vec{S}$ is the angular momentum operator of the target, and $\otimes$ indicates a tensor coupling. (For example, $(\hat{k}\otimes\hat{k})^{(2)} \cdot (\vec{S}\otimes\vec{S})^{(2)} = (\hat{k}\cdot S)(\hat{k}\cdot S) - S^2/3$.) A general odd-$J$ term has the structure $i(\vec{S}\otimes\vec{S}\otimes\cdots\otimes\vec{S})^{(J)} \cdot ((\hat{\epsilon}^*\times\hat{\epsilon})\otimes\hat{k}\cdots\otimes\hat{k})^{(J)}$; an even-$J$ term has the structure $(\vec{S}\otimes\vec{S}\otimes\cdots\otimes\vec{S})^{(J)} \cdot (\hat{k}\otimes\hat{k}\otimes\cdots\otimes\hat{k})^{(J)}\hat{\epsilon}^*\cdot\hat{\epsilon}$. (If one considers the spin-flip forward amplitudes as well, one has one more structure $(\vec{S}\otimes\vec{S}\otimes\cdots\otimes\vec{S})^{(J)} \cdot (\hat{\epsilon}^*\otimes\hat{\epsilon}\otimes\hat{k}\otimes\cdots\otimes\hat{k})^{(J)}$ for every even-J.) The structures are chosen in the above way so that the coefficient f_i's have one to one correspondence to the $\tilde{f}_i$'s in relation (2). With the proportional coefficient fixed, we can write down the following relations:

$$f_0 = \tilde{f}_0,$$
$$f_1 = -\sqrt{\frac{3}{S(S+1)}}\tilde{f}_1 = -\frac{3}{S(S+1)}\frac{1}{2S+1}\sum_{m_S} m_S f^{(m_S)}, \tag{5}$$

and

$$f_2 = \sqrt{\frac{5}{S(S+1)}}\frac{3}{\sqrt{(2S-1)(2S+3)}}\tilde{f}_2 = \frac{3\cdot 5}{S(S+1)(2S-1)(2S+3)}\frac{1}{2S+1} \times \sum_{m_S}\left(3m_S^2 - S(S+1)\right) f^{(m_S)}, \tag{6}$$

and so on.

Because of the crossing symmetry under exchanging of photon four-momentum $k^\mu \leftrightarrow -k^\mu$ and $\epsilon'^* \leftrightarrow \epsilon$, all even-$J$ amplitudes are even functions of the photon energy ω, and all odd-J amplitudes are odd functions of ω. Using the analyticity of the f_J, we write down once-subtracted dispersion relations for even-J,

$$f_J(\omega) = f_J(0) + \frac{2}{\pi}\omega^2 \int_0^\infty \frac{d\omega'}{\omega'} \frac{\mathrm{Im}\, f_J(\omega')}{\omega'^2 - \omega^2}. \tag{7}$$

Using the optical theorem, one has [6,7],

$$f_J(\omega) = f_J(0) + \frac{\omega^2}{2\pi^2} \int_0^\infty d\omega' \frac{\sigma_J(\omega')}{\omega'^2 - \omega^2}, \tag{8}$$

which is the basis for various sum rules.

Consider the example of $J = 0$, the low-energy expansion for the amplitude is

$$f_0 = \frac{e^2 Z^2}{4\pi M} + (\alpha_{E0} + \beta_{M0})\omega^2 + \cdots. \tag{9}$$

Substituting the above into the dispersion relation, we recover the well-known Baldin sum rule for the averaged cross section,

$$\alpha_{E0} + \beta_{M0} = \frac{1}{2\pi^2} \int_0^\infty d\omega' \frac{\sigma_0(\omega')}{\omega'^2}. \tag{10}$$

For the special case of the spin-1 deuteron, the sum rule becomes

$$\alpha_{E0} + \beta_{M0} = \frac{1}{6\pi^2} \int_0^\infty d\omega' \frac{\sigma^{(1)} + \sigma^{(0)} + \sigma^{(-1)}}{\omega'^2}, \tag{11}$$

where we remind the reader that $\sigma^{(m)}$ denotes the cross section for the deuteron in an m-state.

Let us review the calculations of α_{E0} and β_{M0} in the pionless effective field theory [2,3,12]. We remind the reader that in the pionless theory, the leading order effective Lagrangian is

$$\begin{aligned}\mathcal{L} = {} & N^\dagger\left(iD_0 + \frac{\vec{D}^2}{2M_N}\right)N \\ & - C_0^{(^3S_1)}(\mu)\left(N^T P_i N\right)^\dagger\left(N^T P_i N\right) \\ & - C_0^{(^1S_0)}(\mu)\left(N^T \bar{P}_i N\right)^\dagger\left(N^T \bar{P}_i N\right) \\ & + \frac{e}{2M_N} N^\dagger\left(\mu^{(0)} + \mu^{(1)}\tau_3\right)\vec{\sigma}\cdot\vec{B}N \\ & + N^\dagger i\left[\left(2\mu^{(0)} - \frac{1}{2}\right) + \left(2\mu^{(1)} - \frac{1}{2}\right)\tau_3\right] \\ & \times \frac{e}{8M_N^2}\vec{\sigma}\cdot(\vec{D}\times\vec{E} - \vec{E}\times\vec{D})N,\end{aligned} \tag{12}$$

where N is the nucleon field, $P_i = \tau_2\sigma_2\sigma_i/\sqrt{8}$ and $\bar{P}_i = \sigma_2\tau_2\tau_i/\sqrt{8}$ are the triplet S_1 and singlet S_0 two-nucleon projection operators, respectively. The covariant derivative is $\vec{D} = \vec{\partial} + ieQ\vec{A}$ with $Q = (1+\tau^3)/2$ as the charge operator and $\vec{A}$ the photon vector potential. $\mu^{(0)} = (\mu_p + \mu_n)/2$ and $\mu^{(1)} = (\mu_p - \mu_n)/2$ are the isoscalar and isovector nucleon magnetic moments in nuclear magnetons. The two-body coupling constants are

$$\begin{aligned} C_0^{(^1S_0)}(\mu) &= -\frac{4\pi}{M_N}\frac{1}{(\mu - 1/a^{(^1S_0)})}, \\ C_0^{(^3S_1)}(\mu) &= -\frac{4\pi}{M_N}\frac{1}{(\mu - \gamma)},\end{aligned} \tag{13}$$

where μ is a renormalization scale, $a^{(^1S_0)} = -23.714$ fm is the scattering length in the two-nucleon singlet S_0 channel and $\gamma = \sqrt{M_N B} = 45.703$ MeV with $B = 2.225$ MeV the deuteron binding energy.

To the $\mathrm{N^3LO}$ order, the scalar electric polarizability is [3,12]

$$\alpha_{E0} = \frac{\alpha_{\mathrm{em}} M_N}{32\gamma^4} Z_d\left[1 + \frac{2\gamma^2}{3M_N^2} + \frac{M_N\gamma^3}{3\pi}D_P\right], \tag{14}$$

where $Z_d = 1/(1 - \gamma\rho_d) = 1.69$ is the deuteron wave function renormalization; $\rho_d = 1.764$ fm; $D_p = -1.51\ \mathrm{fm}^3$. Numerically $\alpha_{E0} = 0.6339\ \mathrm{fm}^3$. The magnetic polarizability β_{M0} is suppressed by two orders of Q-counting relative to α_{E0} because of the non-relativistic dynamics of the deuteron [2],

$$\beta_{M0} = -\frac{\alpha_{\mathrm{em}}}{32\gamma^2 M_N}\left[1 - \frac{16(\mu^{(1)})^2}{3} - \frac{8\gamma M_N(\mu^{(1)})^2}{3\pi}\mathcal{A}_{-1}^{(^1S_0)}(-B)\right], \tag{15}$$

where $\mathcal{A}_{-1}^{(^1S_0)}(-B) = -(4\pi/M_N)(1/a^{(^1S_0)} - \gamma)^{-1}$ is the leading-order singlet S_0 scattering amplitude at energy $E = -B$. (Note that the above result differs from [2] by a term proportional to $(\mu^{(0)})^2$, which is canceled by the regular part of an omitted diagram

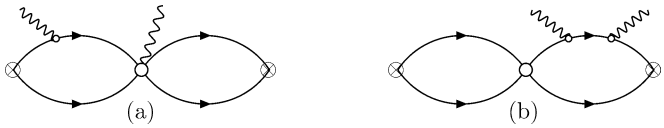

Fig. 1. Leading-order contribution to the electric tensor polarizability of the deuteron. The small circle denotes the electric current coupling. The large circle represents the S–D mixing interaction $C_0^{(sd)}$, which by gauge principle can couple to a photon as in (a). The crossing circles represent the deuteron interpolating fields. The photon cross diagrams are not shown.

with the triplet S_1 bubble chain between the two photon insertions.) Numerically, $\beta_{M0} = 0.067\ \mathrm{fm}^3$, about 10% of α_{E0}.

Turning to the case of $J = 2$, we have the low-energy expansion,

$$f_2(\omega) = (\alpha_{E2} + \beta_{M2})\omega^2 + \mathcal{O}(\omega^4), \tag{16}$$

which allows us to write a $J = 2$ sum rule,

$$\alpha_{E2} + \beta_{M2} = \frac{1}{2\pi^2}\int_0^\infty d\omega' \frac{\sigma_2}{\omega'^2}. \tag{17}$$

Specializing to the spin-1 deuteron, the sum rule becomes,

$$\alpha_{E2} + \beta_{M2} = \frac{1}{4\pi^2}\int_0^\infty d\omega' \frac{\sigma^{(1)} + \sigma^{(-1)} - 2\sigma^{(0)}}{\omega'^2}. \tag{18}$$

In the effective theory with pions, the leading contribution comes at the NLO in Q-counting from the potential pion exchange [2,8]. On the other hand, in the pionless theory, there is a leading contribution coming from the two-body operator that couples the triplet S_1 and triplet D_1 channels [13],

$$\mathcal{L} = -\mathcal{T}_{ij,xy}^{(sd)} C_0^{sd}\big(N^T P^i N\big)^\dagger \big(N^T \mathcal{O}_2^{xy,j} N\big), \tag{19}$$

where $\mathcal{T}_{ij,xy}^{(sd)} = \delta_{ix}\delta_{jy} - \delta_{ij}\delta_{xy}/(n-1)$ and $\mathcal{O}_2^{xy,j} = -(D^x D^y P^j + P^j D^x D^y - D^x P^j D^y - D^y P^j D^x)/4$. C_0^{sd} is related to the asymptotic D/S ratio η_{sd} of the deuteron (0.0254) through [3]

$$C_0^{(sd)} = -\eta_{sd}\frac{6\sqrt{2}\pi}{M_N\gamma^2(\mu - \gamma)}. \tag{20}$$

The leading-order Feynman diagrams are shown in Fig. 1, and a straightforward calculation yields,

$$\alpha_{E2}^{\mathrm{LO}} = -\frac{3\sqrt{2}\alpha_{\mathrm{em}}\eta_{sd}M_N}{32\gamma^4}. \tag{21}$$

At the NLO, there are contributions from Fig. 2 plus the correction for wave function renormalization. The result is that

$$\begin{aligned} \alpha_{E2}^{\mathrm{LO+NLO}} &= -\frac{3\sqrt{2}\alpha_{\mathrm{em}}\eta_{sd}M_N}{32\gamma^4}(1 + \gamma\rho_d) \\ &\approx -\frac{3\sqrt{2}\alpha_{\mathrm{em}}\eta_{sd}M_N}{32\gamma^4}Z_d, \end{aligned} \tag{22}$$

where in the second line we have introduced $Z_d = 1/(1 - \gamma\rho_d) = 1.69$ [12]. Numerically, we have $\alpha_{E2} = -0.068\ \mathrm{fm}^3$ at this order, which is very close to the result from the potential pion contribution [2]. For completeness, we quote the magnetic tensor polarizability β_{M2} which formally comes at N^{2}LO [2],

$$\beta_{M2} = \frac{\alpha_{\mathrm{em}}(\mu^{(1)})^2}{2M_N\gamma^2}\left[1 + \frac{M_N\gamma}{2\pi}\mathcal{A}_{-1}^{(^1S_0)}(B)\right]. \tag{23}$$

However, it is very large numerically, $0.195\ \mathrm{fm}^3$, because of the large isovector magnetic moment. (Again, the $(\mu^{(0)})^2$ dependence in Ref. [2] should be absent.)

For odd-J, one can write down a dispersion relation without subtraction

$$f_J(\omega) = \frac{2}{\pi}\omega\int_0^\infty d\omega' \frac{\mathrm{Im}\, f_J(\omega')}{\omega'^2 - \omega^2}. \tag{24}$$

Again using the optical theorem, one has [6,7]

$$f_J(\omega) = \frac{\omega}{2\pi^2}\int_0^\infty d\omega'\, \omega' \frac{\sigma_J(\omega')}{\omega'^2 - \omega^2}. \tag{25}$$

For $J = 1$, the scattering amplitude has a low-energy expansion

$$f_1 = -\frac{\alpha_{\mathrm{em}}\kappa^2}{4S^2M^2}\omega + 2\gamma\omega^3 + \cdots, \tag{26}$$

where the first term corresponds to the famous low-energy theorem [14] with the anomalous magnetic

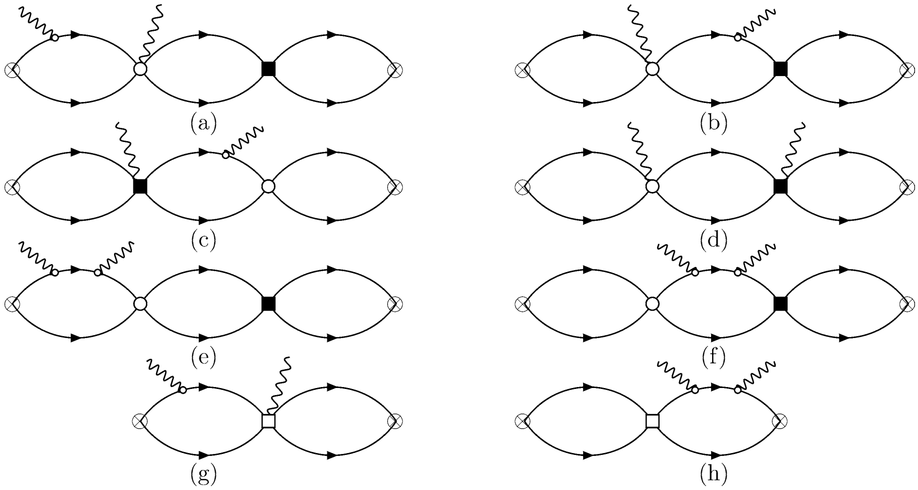

Fig. 2. The next-to-leading order contribution to the electric tensor polarizability of the deuteron. The black box represents the triplet $C_2^{(^3S_1)}$. The small circle denotes the electric current coupling. The large circle represents the S–D mixing interaction $C_0^{(sd)}$. The blank square box represents the NLO S–D mixing vertex: $C_2^{(sd)}$ and $\tilde{C}_2^{(sd)}$. The attachment of the different symbols to a photon comes from the gauge symmetry. The crossing circles represent the deuteron interpolating fields. The photon cross diagrams are not shown.

moment κ defined as $\mu - 2S$ [15], where μ is the magnetic moment in unit of $e\hbar/2Mc$. The next term defines the forward spin-polarizability γ. Substituting the above into Eq. (25), the first term yields the famous Drell–Hearn–Gerasimov (DHG) sum rule, now extended to target of any spin S,

$$\frac{\alpha_{\rm em}\kappa^2}{4S^2M^2} = \frac{1}{2\pi^2}\int_0^\infty d\omega' \, \frac{\sigma_1(\omega')}{\omega'}, \tag{27}$$

where $\sigma_1 = [3/S(S+1)(2S+1)]\sum_{m_s} m_S\sigma_{m_S}$. For a discussion about the DHG sum rule for the deuteron, see Ref. [7].

The sum rule for the forward spin polarizability reads

$$2\gamma = \frac{1}{2\pi^2}\int_0^\infty d\omega' \, \frac{\sigma_1(\omega')}{\omega'^3}. \tag{28}$$

Specializing to the deuteron, we have

$$\gamma = -\frac{1}{8\pi^2}\int_0^\infty d\omega' \, \frac{\sigma^{(1)} - \sigma^{(-1)}}{\omega'^3}. \tag{29}$$

This is the sum rule potentially useful for extracting the forward spin polarizability from the polarized photo-production cross section.

The relevant Feynman diagrams for the forward Compton scattering are shown in Fig. 3. Taking into account the crossing symmetry, the result for the leading-order spin polarizability is

$$\gamma^{\rm LO} = \frac{\alpha_{\rm em}(\mu^{(1)})^2}{16\gamma^4}\left[1 + \frac{M_N\gamma}{2\pi}\mathcal{A}_{-1}^{(^1S_0)}(-B) \times \left(1 + \frac{M_N\gamma}{4\pi}\mathcal{A}_{-1}^{(^1S_0)}(-B)\right)\right] + \frac{\alpha_{\rm em}(4\mu^{(1)} - 1)}{128\gamma^4}. \tag{30}$$

The numerical value of γ at this order is 3.762 fm^4. The effect of diagram 3(a) on the deuteron hyperfine splitting has been studied in Ref. [16], and our result indicates a much bigger effect from diagram 3(c).

At the next-to-leading order, there are contributions from the C_2 coupling in the singlet and triplet channels

$$\begin{aligned}\mathcal{L}' = {} & -C_2^{(^1S_0)}(\mu)\frac{1}{8}\Big[\big(N^T\bar{P}_iN\big)^\dagger \\ & \times \big(N^T\big[\bar{P}_i\overrightarrow{D}^2 + \overleftarrow{D}^2\bar{P}_i - 2\overleftarrow{D}\bar{P}_i\overrightarrow{D}\big]N\big) + \text{h.c.}\Big] \\ & - C_2^{(^3S_1)}(\mu)\frac{1}{8}\Big[\big(N^TP_iN\big)^\dagger \\ & \times \big(N^T\big[P_i\overrightarrow{D}^2 + \overleftarrow{D}^2P_i - 2\overleftarrow{D}P_i\overrightarrow{D}\big]N\big) + \text{h.c.}\Big],\end{aligned} \tag{31}$$

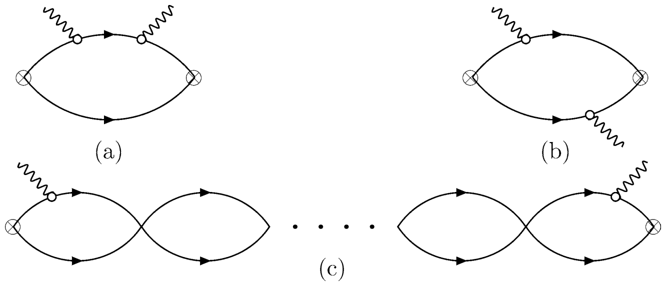

Fig. 3. Leading order contribution to the forward vector polarizability of deuteron. The gray circle denotes the magnetic moment coupling, or spin–orbit interaction, or electric coupling in Eq. (12). The chain bubble in (b) includes insertions of both triplet and singlet types of C_0: $C_0^{(^3S_1)}$ and $C_0^{(^1S_0)}$. The crossing circles represent the deuteron interpolating fields. The photon cross diagrams are not shown.

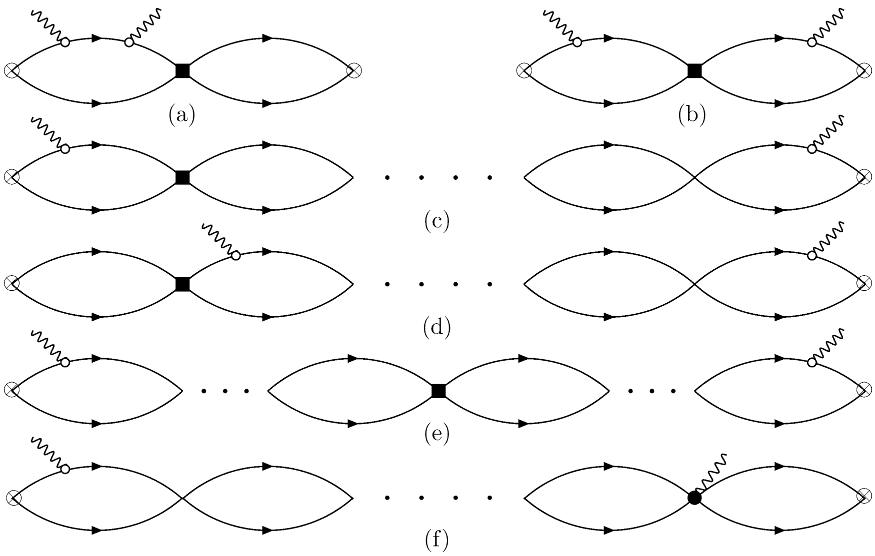

Fig. 4. The next-to-leading order contribution to the forward vector polarizability of deuteron. The square black box represent both triplet and singlet types of C_2: $C_2^{(^3S_1)}$ and $C_2^{(^1S_0)}$. The large black dot denotes the L_1 coupling. The gray circle denotes the magnetic moment coupling. The chain bubble in (c)–(e) includes insertions of both triplet and singlet types of C_0: $C_0^{(^3S_1)}$ and $C_0^{(^1S_0)}$, and that in (f) singlet only. The crossing circles represent the deuteron interpolating fields. The photon cross diagrams are not shown.

where

$$C_2^{(^1S_0)}(\mu) = \frac{4\pi}{M_N}\frac{r_0}{2}\frac{1}{(\mu - 1/a^{(^1S_0)})^2},$$
$$C_2^{(^3S_1)}(\mu) = \frac{2\pi}{M_N}\frac{\rho_d}{(\mu-\gamma)^2}, \qquad (32)$$

with $r_0 = 2.73$ fm. There are also contributions from the following electromagnetic counter term [17]

$$\mathcal{L}'' = eL_1\big(N^T P_i N\big)^\dagger \big(N^T \bar{P}_3 N\big) B_i. \qquad (33)$$

The relevant next-to-leading order Feynman diagrams for the Compton amplitude are shown in Fig. 4. The calculated vector polarizability is

$$\gamma^{\rm NLO} = \frac{\alpha_{\rm em}(\mu^{(1)})^2 M_N^2 C_2^{(^1S_0)}(\mu)}{2(8\pi\gamma)^2}\mathcal{A}_{-1}^{(^1S_0)}(-B)$$
$$\times \big(\mu - 1/a^{(^1S_0)}\big)^2$$
$$\times \bigg[\frac{3\gamma-\mu}{\mu - 1/a^{(^1S_0)}} - \frac{3M_N\gamma}{4\pi}\mathcal{A}_{-1}^{(^1S_0)}(-B)$$

$$
\begin{aligned}
&\quad + \frac{(M_N\gamma)^2}{8\pi^2}\mathcal{A}_{-1}^{(^1S_0)}(-B)^2\Bigg] \\
&+ \frac{\alpha_{\rm em}(\mu^{(1)})^2 M_N C_2^{(^3S_1)}(\mu)}{32\pi\gamma^3}(\mu-\gamma) \\
&\times\left[\mu - 2\gamma + (\mu-3\gamma)\frac{M_N\gamma}{4\pi}\mathcal{A}_{-1}^{(^1S_0)}(-B)\right] \\
&+ \frac{\alpha_{\rm em}(\mu^{(1)})L_1(\mu)M_N^2}{(8\pi)^2\gamma^2}(\mu-\gamma) \\
&\times\left(\mu - 1/a^{(^1S_0)}\right) \\
&\times\mathcal{A}_{-1}^{(^1S_0)}(-B)\left[1+\frac{M_N\gamma}{2\pi}\mathcal{A}_{-1}^{(^1S_0)}(-B)\right] \\
&+ \frac{\alpha_{\rm em}(4\mu^{(1)}-1)\rho_d}{128\gamma^3}. \qquad (34)
\end{aligned}
$$

The last term in the first square bracket can be added to the leading-order result if one replaces $\mathcal{A}_{-1}^{(^1S_0)}(-B)$ with $\mathcal{A}_{-1}^{(^1S_0)}(-B) + \mathcal{A}_{0}^{(^1S_0)}(-B)$. It is easy to verify that if the following equation holds,

$$
\mu\frac{d}{d\mu}\left[\frac{L_1(\mu) - \frac{1}{2}\mu^{(1)}(C_2^{(^1S_0)}(\mu) + C_2^{(^3S_1)}(\mu))}{C_0^{(^1S_0)}(\mu)C_0^{(^3S_1)}(\mu)}\right] = 0, \qquad (35)
$$

the above result is independent of the renormalization scale μ. This is consistent with the *np* capture calculation [3,17].

Using the counter term determined from the neutron capture, $L_1(m_\pi) = 7.24\ \text{fm}^4$ [3], we find $\gamma^{\rm NLO} = 0.50\ \text{fm}^4$ which is about 10% of the leading-order result. Therefore the effective field theory expansion seems to converge well.

In summary, we have presented sum rules for the vector and tensor polarizabilities of the deuteron. We have evaluated both quantities in the pionless effective field theory to the next-to-leading order. The rate of convergence in the Q-expansion is fast for the vector polarizability, and is similar between the scalar and tensor ones.

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References

[1] J.L. Friar, G.L. Payne, Phys. Rev. C 55 (1997) 2764;
M.I. Levchuk, A.I. Lvov, Few-Body Systems Suppl. 9 (1995) 439;
P.T. Wilbois, H. Arenhovel, Few-Body Systems Suppl. 9 (1995) 263;
A. Karchenko, Ukr. J. Phys. 42 (1997) 18.
[2] J.W. Chen, H.W. Grießhammer, M.J. Savage, R.P. Springer, Nucl. Phys. A 644 (1998) 221.
[3] J.W. Chen, G. Rupak, M.J. Savage, Nucl. Phys. A 653 (1999) 386.
[4] N.L. Rodning, L.D. Knutson, W.G. Lynch, M.B. Tsang, Phys. Rev. Lett. 49 (1982) 909.
[5] J.L. Friar, et al., Phys. Rev. C 27 (1983) 1364.
[6] M. Weyrauch, H. Arenhovel, Nucl. Phys. A 408 (1983) 425;
M. Weyrauch, Phys. Rev. C 38 (1988) 611;
M. Weyrauch, Phys. Rev. C 41 (1990) 880.
[7] H. Arenhövel, nucl-th/0006083.
[8] J.W. Chen, H.W. Grießhammer, M.J. Savage, R.P. Springer, Nucl. Phys. A 644 (1998) 245;
J.W. Chen, Nucl. Phys. A 653 (1999) 375.
[9] H.R. Weller, in: Proc. for International Symposium on Electromagnetic Interactions in Nuclear and Hadron Physics (EMI 2001), Osaka, Ibaraki, Japan, 4–7 December, 2001, p. 227;
H.R. Weller, in: Proc. for the 2nd International Symposium on the Gerasimov–Drell–Hearn Sum Rule and the Spin Structure of the Nucleon (GDH 2002), Genoa, Italy, 3–6 July, 2002, p. 173.
[10] D.B. Kaplan, M.J. Savage, M.B. Wise, Phys. Lett. B 424 (1998) 390;
D.B. Kaplan, M.J. Savage, M.B. Wise, Nucl. Phys. B 534 (1998) 329.
[11] S.R. Beane, P.F. Bedaque, W.C. Haxton, D.R. Phillips, M.J. Savage, nucl-th/0008064.
[12] D.R. Phillips, G. Rupak, M.J. Savage, Phys. Lett. B 473 (2000) 209.
[13] J.W. Chen, G. Rupak, M.J. Savage, Phys. Lett. B 464 (1999) 1.
[14] F.E. Low, Phys. Rev. 96 (1954) 1428;
M. Gell-Mann, M.L. Goldberger, Phys. Rev. 96 (1954) 1433.
[15] A. Pais, Phys. Rev. Lett. 19 (1967) 544;
S. Weinberg, in: S. Deser, M. Grisaru, H. Pendleton (Eds.), Lectures on Elementary Particles and Quantum Field Theory, in: Brandeis Lecture in Quantum Field Theory, vol. 1, MIT Press, 1970.
[16] A.I. Milshtein, I.B. Khriplovich, S.S. Petrosyan, JETP 82 (1996) 616.
[17] M.J. Savage, K.A. Scaldeferri, M.B. Wise, Nucl. Phys. A 652 (1999) 273.