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Analytical self-dual solutions in a nonstandard Yang–Mills–Higgs scenario

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ABSTRACT

We have found analytical self-dual solutions within the generalized Yang–Mills–Higgs model introduced in R. Casana et al. (2012) [1]. Such solutions are magnetic monopoles satisfying Bogomol'nyi–Prasad–Sommerfield (BPS) equations and usual finite energy boundary conditions. Moreover, the new solutions are classified in two different types according to their capability of recovering (or not) the usual 't Hooft–Polyakov monopole. Finally, we compare the profiles of the solutions we found with the standard ones, from which we comment about the main features exhibited by the new configurations.

1. Introduction

Configurations supporting a nontrivial topology have been intensively studied in connection with many areas of physics [2]. In particular, in the context of High Energy Physics such configurations are described as the static solutions inherent to some classical field models, which are supposed to be endowed with a symmetry breaking potential for the self-interacting scalar-matter sector. Consequently, these topological solutions usually arise as the result of a symmetry breaking or a phase transition.

The most common topological configuration is the kink [3], which stands for the static solution inherent to a $(1+1)$-dimensional model containing only one self-interacting real Higgs field.

Regarding higher-dimensional models, other examples of topological structures include the vortex [4] and the magnetic monopole [5]. While the vortices are defined in $(1+2)$-dimensional gauge models, such as the Maxwell–Higgs theory, the magnetic monopole solutions appear in a $(1+3)$-dimensional non-Abelian–Higgs gauge scenario. Specifically, the monopoles arise as well-behaved finite energy solutions in a SO(3) Yang–Mills–Higgs model, representing the interaction between a gauge and a real scalar triplets [6]. In a very special situation (i.e., in the absence of the Higgs potential), the monopole solution turns out as a BPS structure [7] supported by a set of first-order differential equations whose analytical solution is the 't Hooft–Polyakov monopole [5].

During the last years, a new class of topological solutions, called topological k-defects, has been intensively investigated in the context of the field theories presenting modified dynamics (k-field theories). The idea of noncanonical dynamics is inspired in string theories where it arises in a natural way. Such models have been used in several distinct physical scenarios, with interesting results involving studies of the accelerated inflationary phase of the universe [8], strong gravitational waves [9], tachyon matter [10], dark matter [11], and others [12]. In this context, some of us have studied the self-dual frameworks engendered by some k-field theories [13]. Such BPS k-configurations, in general, have asymptotic behavior (when $r \to 0$ and for $r \to \infty$) similar as their standard counterparts. However, the generalized dynamics can induce variations in the defect amplitude, in the characteristic length, and in the profile shape. Additional investigations regarding the topological k-structures and their main features can also be found in Ref. [14]. Concerning the searching of BPS solitons in new models, one has also considered generalized theories mimicking the usual defect solutions, in the so-called twinlike models [15], which provide the very same solutions obtained by the usual model taken as the starting point.

In a recent paper [1], some of us have introduced the self-dual framework inherent to a generalized Yang–Mills–Higgs model whose noncanonical self-dual solutions also constitute magnetic monopoles. At this first moment, our attention was focused in attaining numerical solutions. Now, one interesting question naturally arises about the existence of generalized Yang–Mills–Higgs models endowed with analytical BPS monopole solutions. The purpose of the present Letter is to go further into this issue by introducing some effective non-Abelian gauge models whose self-dual equations can be analytically solved. Such models here considered are divided into two different classes, according to their capability of recovering or not the 't Hooft–Polyakov monopole solution.

In order to present our results, this Letter is organized as follows: in the next section, we briefly review the unusual Yang–Mills–Higgs model studied in Ref. [1], including its self-dual

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structure, from which one gets the generalized BPS equations to be investigated. In Section 3, we achieve the main goal of this work by introducing the aforecited non-Abelian models, the corresponding exact self-dual solutions being explicitly presented. Then, we depict the related analytical profiles, from which we verify that the new solutions are well-behaved. Furthermore, we compare these solutions with the usual 't Hooft–Polyakov analytical ones, commenting on the main features of the nonstandard configurations. Finally, in Section 4, we present our ending comments and perspectives regarding future investigations.

2. The theoretical model

We begin by reviewing the nonstandard Yang–Mills–Higgs model introduced in Ref. [1], whose dimensionless Lagrangian density is

$$\mathcal{L} = -\frac{g(\phi^a\phi^a)}{4}F^b_{\mu\nu}F^{\mu\nu,b} + \frac{f(\phi^a\phi^a)}{2}D_\mu\phi^b D^\mu\phi^b. \tag{1}$$

Here, $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc}A^b_\mu A^c_\nu$ is the Yang–Mills field strength tensor, $D_\mu\phi^a = \partial_\mu\phi^a + e\epsilon^{abc}A^b_\mu\phi^c$ stands for the non-Abelian covariant derivative, and ϵ^{abc} is the antisymmetric Levi-Civita symbol (with $\epsilon^{123}=1$). Moreover, $g(\phi^a\phi^a)$ and $f(\phi^a\phi^a)$ are positive arbitrary functions which change the dynamics of the non-Abelian fields in an exotic way.

In this Letter, we focus our attention on the spherically symmetric configurations arising from Lagrangian (1). In this sense, we look for static solutions described by the standard *Ansatz*

$$\phi^a = x^a\frac{H(r)}{r} \quad \text{and} \quad A^a_0 = 0, \tag{2}$$

$$A^a_i = \epsilon_{iak}x_k\frac{W(r)-1}{er^2}, \tag{3}$$

where $r^2 = x^a x^a$. The functions $H(r)$ and $W(r)$ are supposed to behave according the finite energy boundary conditions

$$H(0) = 0 \quad \text{and} \quad W(0) = 1, \tag{4}$$

$$H(\infty) = \mp 1 \quad \text{and} \quad W(\infty) = 0, \tag{5}$$

which also guarantee the breaking of the SO(3) symmetry inherent to the Lagrangian (1).

In general, given the arbitrariness of the generalizing functions $f(\phi^a\phi^a)$ and $g(\phi^a\phi^a)$, the corresponding Euler–Lagrange equations for $H(r)$ and $W(r)$ can be extremely hard to solve, even in the presence of suitable boundary conditions. Notwithstanding, spite of the complicated scenario, the system also admits finite energy BPS structures, i.e., legitimate field configurations obtained as the solutions of a set of first-order differential equations. In this sense, whereas the standard approach [7] states that the BPS equations arise by requiring the minimization of the energy of the overall model, one has to consider the spherically symmetric expression for the energy density inherent to the non-Abelian Lagrangian (1):

$$\varepsilon = \frac{g}{e^2r^2}\left(\left(\frac{dW}{dr}\right)^2 + \frac{(1-W^2)^2}{2r^2}\right) + f\left(\frac{1}{2}\left(\frac{dH}{dr}\right)^2 + \left(\frac{HW}{r}\right)^2\right). \tag{6}$$

Here, it is worthwhile to point out that, as already verified in Ref. [1], the self-duality of the resulting model only holds when one imposes the following constraint:

$$g = \frac{1}{f}. \tag{7}$$

In more details, taking Eq. (7) into account, Eq. (6) can be written in the form

$$\varepsilon = \frac{f}{2}\left(\frac{dH}{dr} \pm \frac{1-W^2}{er^2 f}\right)^2 + \frac{1}{e^2r^2 f}\left(\frac{dW}{dr} \mp efHW\right)^2 \mp \frac{1}{er^2}\frac{d}{dr}\left(H\left(1-W^2\right)\right). \tag{8}$$

The minimization procedure then leads to the first-order equations

$$\frac{dH}{dr} = \mp\frac{1-W^2}{er^2 f}, \tag{9}$$

$$\frac{dW}{dr} = \pm efHW, \tag{10}$$

which are the BPS equations of the model. Therefore, the energy density of the BPS states is

$$\varepsilon_{bps} = \mp\frac{1}{er^2}\frac{d}{dr}\left(H\left(1-W^2\right)\right), \tag{11}$$

while the total energy is reduced to

$$E_{bps} = 4\pi\int r^2\varepsilon_{bps}\,dr = \frac{4\pi}{e}, \tag{12}$$

whenever (4) and (5) are satisfied.

In Ref. (1), for a specific choice of f, the BPS equations (9) and (10) were numerically solved fulfilling the finite energy boundary conditions (4) and (5). The attained profiles describe BPS magnetic monopole solutions with total energy given by Eq. (12) within the nonstandard Yang–Mills–Higgs scenario (1). In the next section, we will deal with the attainment of analytical solution for such a generalized model.

3. Analytical solutions

In this section, we accomplish the main goal of this work by introducing some effective models for which the BPS equations (9) and (10) can be solved analytically, providing well-behaved solutions endowed with finite energy. Furthermore, we depict the corresponding profiles choosing $e=1$ and considering only the lower signs in Eqs. (5), (9), (10) and (11). We also determine the profiles for the BPS energy density (11) and for $r^2\varepsilon_{bps}$ (the integrand of Eq. (12)). Then, by comparing the new solutions and standard (analytical) one, we comment on the main features of the generalized monopoles here presented.

Firstly, it is important to note that the usual Yang–Mills–Higgs scenario is easily recovered by setting $f=1$, for which the BPS equations generate the well-known 't Hooft–Polyakov analytical solution (already written in accordance with our conventions):

$$H_{tHP}(r) = \frac{1}{\tanh r} - \frac{1}{r}, \tag{13}$$

$$W_{tHP}(r) = \frac{r}{\sinh r}. \tag{14}$$

In the sequel, we present the profiles of the new solutions and we make a comparison between them and the 't Hooft–Polyakov monopole solution, commenting about the main features and differences among them (see Figs. 1, 2, 3 and 4 below). The non-canonical models to be examined in this Letter are divided into two different classes. The first class is related to those models recovering the usual 't Hooft–Polyakov result (given an appropriated limit), while the second one includes the models which do not.

All these solutions fulfill the finite energy boundary conditions, as expected.

Here, in order to introduce our results, we first point out that the BPS equations (9) and (10) can be combined into a single equation, i.e.,

$$\frac{dW}{dr}\frac{dH}{dr} = \frac{(W^2-1)HW}{r^2} \tag{15}$$

which relates the solution for $H(r)$ to that for $W(r)$. In this sense, for a given $H(r)$, Eq. (15) can be integrated to give the corresponding solution for $W(r)$, and vice versa. Here, it is worthwhile to note that such strategy can be used even to describe nonphysical scenarios, i.e., those for which $H(r)$ and/or $W(r)$ dot not behave as (4) and (5).

In this work, as we are interested in the physical solutions only, we adopt the following prescription: firstly, we choose an analytical solution for $H(r)$ satisfying the boundary conditions (4) and (5). Then, we calculate the corresponding solution for $W(r)$ by integrating Eq. (15) explicitly (as the reader can verify, the solutions we have found this way automatically obey (4) and (5)). A posteriori, we use such expressions to attain the one for $f(r)$ via the BPS equations (9) and (10). Moreover, we also depict the corresponding exact profiles for the BPS energy density Eq. (11) and for $r^2\varepsilon_{bps}$. Here, it is important to say that all the solutions we have obtained for $f(r)$ and ε_{bps} are positive, as desired; see Eq. (11). In addition, all the noncanonical scenarios we have discovered exhibit the very same total energy, i.e., $E_{bps} = 4\pi$; see Eq. (12).

At the first moment, the question about the generalization of the usual 't Hooft–Polyakov solutions (13) and (14) arises in a rather natural way. Indeed, we have verified that such generalization is possible, the resulting model belonging to the first class. In this sense, taking

$$H(r) = \frac{1}{\tanh r} - \frac{1}{r}, \tag{16}$$

Eq. (15) can be integrated to attain

$$W(r) = \frac{\sqrt{1-C_1}\,r}{\sqrt{\sinh^2 r - C_1 r^2}}, \tag{17}$$

where C_1 stands for a real constant such that $C_1 < 1$ (note that $C_1 = 0$ leads us back to the standard theory). In addition, using (16) and (17), Eqs. (9) and (10) can be solved for $f(r)$, the result being

$$f(r) = \frac{\sinh^2 r}{\sinh^2 r - C_1 r^2}. \tag{18}$$

Here, despite the noncanonical form of (18), the solution for $H(r)$, given in Eq. (16), is the same one of the usual scenario (see Eq. (13)). On the other hand, the solution for $W(r)$ exhibits a generalized structure, which reduces to Eq. (14) when $C_1 = 0$.

Now, we use our prescription to introduce two examples of effective models belonging to the second class, i.e., standing for new families of analytical monopole solutions. Here, in order to define the first family, we choose the analytical solution for $H(r)$ as

$$H(r) = \frac{r}{r+1}, \tag{19}$$

which indeed obeys the boundary conditions (4) and (5). In the sequel, by solving Eq. (15), one gets that the corresponding nonusual profile for $W(r)$ is

$$W(r) = \frac{1}{\sqrt{1 + C_2 r^2 e^{2r}}}, \tag{20}$$

which also behaves according (4) and (5), C_2 being a positive real constant. Furthermore, taking (19) and (20) into account, the self-dual equations (9) and (10) give

$$f(r) = \frac{C_2(r+1)^2 e^{2r}}{1 + C_2 r^2 e^{2r}}. \tag{21}$$

In this case, we note that one has $f \neq 1$ for any value of C_2. This explains why the solutions (19) and (20) are always different from the usual ones, (13) and (14), respectively.

The last model to be studied is a little bit more sophisticated than the previous ones. Even in this case, one still gets well-behaved solutions which support the model itself. In this sense, the second family of models, which do not recover the usual 't Hooft–Polyakov solution, is defined by

$$H(r) = e^{h(r)}. \tag{22}$$

Here, $h(r)$ is given by

$$h(r) \equiv \frac{e^{-2r}(r^2+4r+1) - 2r - 1 - 2r^2\,\mathrm{Ei}(1,2r)}{2r^2}, \tag{23}$$

with the function $\mathrm{Ei}(1,r)$ being the exponential integral

$$\mathrm{Ei}(1,r) \equiv \int_1^\infty \frac{e^{-rx}}{x}\,dx. \tag{24}$$

Also in this case, and despite the highly nonlinear structure of $H(r)$, Eq. (15) still can be integrated exactly, the result being a relatively simple analytical expression for $W(r)$, i.e.,

$$W(r) = \frac{\sqrt{C_3+1}\,(r+1)}{\sqrt{C_3(r+1)^2 + e^{2r}}}, \tag{25}$$

where C_3 is a real constant such that $C_3 > -1$. In addition, Eqs. (9) and (10) give

$$f(r) = \frac{re^{2r-h(r)}}{(r+1)(C_3(r+1)^2 + e^{2r})}. \tag{26}$$

Moreover, one clearly see that both (22) and (25) behave according (4) and (5), as expected.

Now, once we have introduced the noncanonical solutions for $H(r)$ and $W(r)$, we compare their profiles by plotting them in Figs. 1 and 2. Also, in Figs. 3 and 4, we show the corresponding solutions for ε_{bps} and $r^2\varepsilon_{bps}$, respectively. The standard results are also shown, for comparison. In what follows, we choose $C_1 = C_2 = C_3 = 0.5$.

In Fig. 1, we depict the analytical solutions for $H(r)$. The solution (19) is shown as a dotted red line, the dashed blue line standing for solution (22). The usual profile, that of Eq. (13), is also shown (solid black line). It describes the solution inherent to the choice (18), given by Eq. (16). One also notes how close is the solution (22) to the standard profile. The overall conclusion is that the solutions behave in the same general way: starting from zero at the origin, they monotonically reach the asymptotic condition in the limit $r \to \infty$.

In Fig. 2, we show the solutions for $W(r)$, with Eq. (17) being represented by the dash-dotted green line. Also in this case, the solutions have the same general profile, being lumps centered at the origin, monotonically decreasing, and vanishing in the asymptotic limit. A comparison between the new solutions with the standard one reveals that the first ones present smaller characteristic lengths, the solution coming from the choice (21) being the one with the smallest range.

The BPS energy density ε_{bps} of the analytical solutions are plotted in Fig. 3. The profiles related to the choices (18) and (26)

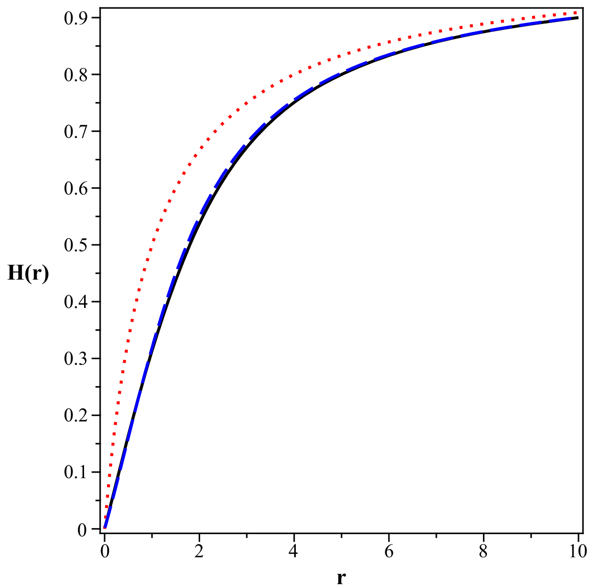

Fig. 1. Solutions to $H(r)$ given by (13) (usual case, solid black line), (19) (dotted red line), and (22) (dashed blue line). Here, (16) mimics the standard result. (For interpretation of the references to color in this figure, the reader is referred to the web version of this Letter.)

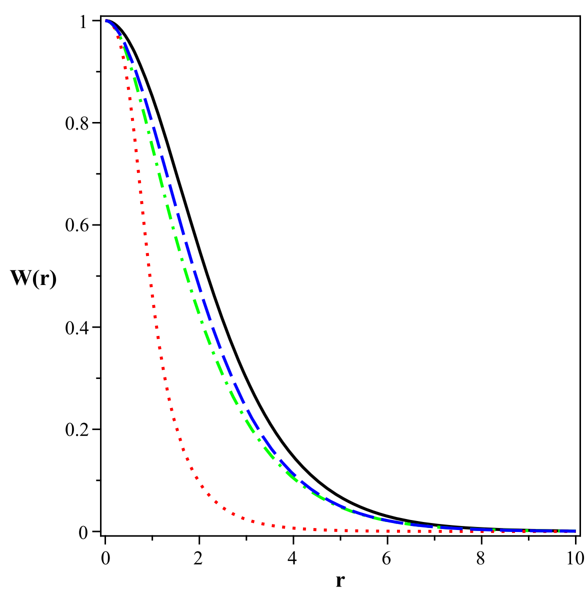

Fig. 2. Solutions to $W(r)$. Conventions as in Fig. 1. Here, (17) is represented by the dash-dotted green line. (For interpretation of the references to color in this figure, the reader is referred to the web version of this Letter.)

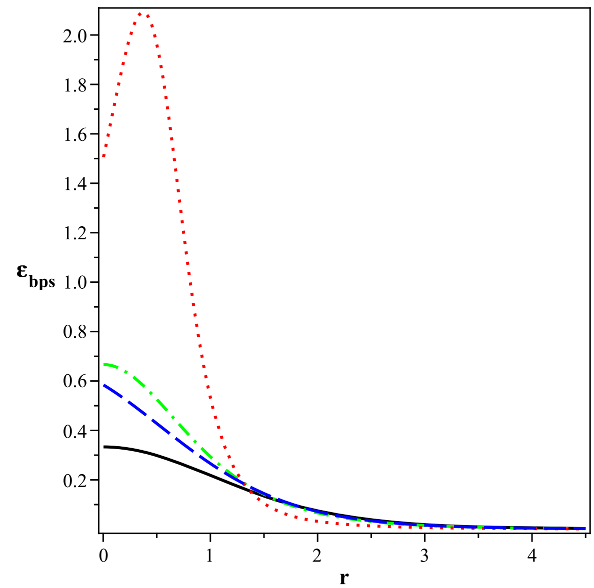

Fig. 3. Solutions to ε_{bps}. Conventions as in the previous figures.

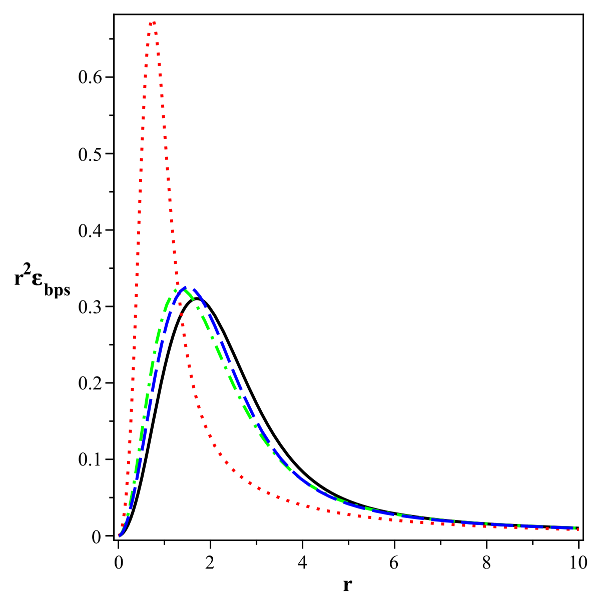

Fig. 4. Solutions to $r^2\varepsilon_{bps}$. Conventions as in the previous figures.

behave as the standard one, that is, as a lump centered at $r=0$. On the other hand, the solution inherent to (21) reaches its maximum value at some finite distance R from the origin, implying a ringlike energy distribution in the plane. In addition, the solutions vanish asymptotically, since the condition $\varepsilon_{bps}(r\to\infty)\to 0$ arises in a rather natural way from the boundary conditions (5).

Finally, in Fig. 4 we present the profiles for $r^2\varepsilon_{bps}$. It clearly depicts a compensatory effect related to the profiles already discussed in Ref. [1]: different solutions enclose the same area (equal to the unity, according our conventions). As a consequence, the resulting configurations have the very same total energy, given by $E_{bps}=4\pi$.

4. Ending comments

In this Letter, we have extended a previous work [1] by introducing non-Abelian effective models for which the resulting BPS equations can be solved analytically. The starting point of such investigation was the first-order formalism developed within a nonstandard Yang–Mills–Higgs theory [1], whose dynamic is controlled by two positive generalizing functions, $g(\phi^a\phi^a)$ and $f(\phi^a\phi^a)$. The non-Abelian fields were supposed to be described by the standard spherically symmetric *Ansatz* (2) and (3), where the functions $H(r)$ and $W(r)$ must behave according the finite energy boundary conditions, (4) and (5). Our goal was to introduce effective Yang–Mills–Higgs models whose corresponding BPS equations yield analytical solutions. Here, the nonstandard models were divided into two different classes: the ones which do recover the usual 't Hooft–Polyakov result (given the appropriate limit), and the ones which do not; the last ones standing for new families of analytical monopole solutions.

The profiles of the new solutions were depicted in Figs. 1, 2, 3 and 4. The overall conclusion is that the effective models provide consistent and well-behaved self-dual solutions which strongly support the models themselves. Moreover, we have identified a particular family of nonstandard models for which the BPS energy density exhibits a different profile (see Fig. 4), with a ring-like energy distribution centered at $r \neq 0$. Thus, we have shown that starting from a generalized Yang–Mills–Higgs framework we can attain analytical self-dual solutions for non-Abelian magnetic monopoles.

Regarding future investigations, an interesting issue is the search for an analytical description for the non-charged BPS vortices arising in the generalized Maxwell–Higgs model proposed in Ref. [16]. Moreover, taking as the starting point the solutions we have presented in this work, we intend to generate new self-dual profiles based on an appropriate deformation prescription [17]. These issues are now under consideration, with expected interesting results for a future contribution.

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