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Existence and uniqueness of invariant measures for a class of transition semigroups on Hilbert spaces ☆

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ABSTRACT

We prove a smoothing property and the irreducibility of transition semigroups corresponding to a class of semilinear stochastic equations on a separable Hilbert space H. Existence and uniqueness of invariant measures are discussed as well.

1. Introduction and preliminaries

Consider the semilinear stochastic differential equation

$$\begin{cases} dX(t) = \big(AX(t) + (-A)^{\frac{1}{2}}F\big(X(t)\big)\big)\,dt + dW_t, & t \geqslant 0, \\ X(0) = x \end{cases} \tag{1}$$

in a separable Hilbert space H. Here A is a self adjoint operator with negative type ω and compact resolvent A^{-1} on H, F is a nonlinear function. The process $(W_t)_{t\geqslant 0}$ is a standard cylindrical Wiener process on H defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geqslant 0}, \mathbb{P})$. Under appropriate assumptions the solution to (1) is given by the formula

$$X(t) = e^{tA}x + \int_0^t e^{(t-s)A}(-A)^{\frac{1}{2}}F\big(X(s)\big)\,ds + \int_0^t e^{(t-s)A}\,dW_s, \quad t \geqslant 0.$$

For $\varphi \in B_b(H)$ (space of all bounded measurable functions on H), we define the transition semigroup $(P_t)_{t\geqslant 0}$ by

$$P_t\varphi(x) = \mathbb{E}\big(\varphi\big(X(t,x)\big)\big), \quad x \in H,\ t \geqslant 0.$$

We are concerned with regularity properties of the function $P_t\varphi$. One of our main aims is to show that, under appropriate assumptions the function $P_t\varphi$ is globally Lipschitz on H which means that the semigroup $(P_t)_{t\geqslant 0}$ is strong Feller. In the framework of infinite dimensional stochastic equations, the strong Feller property of transition semigroups related to stochastic evolution equations have been addressed by many authors. They consider mostly equations of type (1) without the term $(-A)^{\frac{1}{2}}$ in front of the nonlinear drift F and they discuss equations of the form

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$$\begin{cases} dX(t) = \big(AX(t) + G\big(X(t)\big)\big)\,dt + dW_t, & t \geqslant 0, \\ X(0) = x. \end{cases} \tag{2}$$

If G is bounded on H, it has been proved in [2,4] that $(P_t)_{t\geqslant 0}$ is strong Feller by solving a mild form of the Kolmogorov equation corresponding to (2). In [11] the results of [4] are extended to cover in particular, the case of a nonlinear local Lipschitz function G and later in [9] this result was extended also for general drift G with weak regularity properties. In [12] by generalizing an earlier formula, due to Elworthy [7] to the infinite dimensional setting, the strong Feller property of $(P_t)_{t\geqslant 0}$ was proved for equations with multiplicative noise and global Lipschitz nonlinear drifts. Recently in [10], a similar equation to (1) was treated on space of 2π-periodic square integrable real functions. The following equation was considered

$$\begin{cases} dX(t) = \big(\big(D_\xi^2 - \mathrm{I_d}\big)X(t) + D_\xi F\big(X(t)\big)\big)\,dt + dW_t, & t \geqslant 0, \\ X(0) = x, \end{cases} \tag{3}$$

where F is a C^1-function on $L^2[0,2\pi]$ with bounded derivative. It has been proved that the transition semigroup associated to (3) is strong Feller and irreducible and there exists a unique invariant measure for (3). In the present paper we generalize the result in [10] in a more abstract setting, by supposing the nonlinear function F to be only global Lipschitz we prove the strong Feller property of $(P_t)_{t\geqslant 0}$ in Theorem 2.3. Our method is based on an approximation argument as in [12]. Using similar techniques as in [10] we prove the irreducibility of the semigroup $(P_t)_{t\geqslant 0}$. Moreover, if we suppose the nonlinear function F to be dissipative, we can prove the existence and uniqueness of an invariant measure μ of (1). We shall remark that in our case only F need to be dissipative not $(-A)^{\frac{1}{2}}F$ for the existence of μ. In the rest of this introductory section let us fix some notations and our main assumptions. For $\gamma \in [0,1]$ let

$$V_\gamma := \big(D\big((-A)^\gamma\big), \langle\cdot,\cdot\rangle_\gamma\big), \quad \text{where } \langle x, y\rangle_\gamma = \big\langle(-A)^\gamma x, (-A)^\gamma y\big\rangle \text{ for } x, y \in V_\gamma.$$

Note that, since A has a compact resolvent, the imbedding $V_\gamma \hookrightarrow H$ is compact. In the following $\|\cdot\|_{\mathrm{HS}}$ denotes the Hilbert–Schmidt operator norm on the space H. We shall formulate our assumptions:

(H_0) A is selfadjoint and $\|e^{tA}\| \leqslant e^{-\omega t}$ for certain $\omega > 0$.
(H_1) There exist $\alpha \in]0, \frac{1}{2}[$ such that for all $t > 0$

$$\int_0^t s^{-2\alpha}\big\|e^{sA}\big\|_{\mathrm{HS}}^2\,ds < \infty.$$

(H_2) F maps H into $D((-A)^{\frac{1}{2}})$ and

$$\big|F(x) - F(y)\big| \leqslant L|x-y|, \quad x, y \in H.$$

Definition 1.1. A mild solution of Eq. (1) is an $\mathcal{F}_t$-adapted process which satisfies the following integral equation

$$X(t) = e^{tA}x + \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A} F\big(X(s)\big)\,ds + \int_0^t e^{(t-s)A}\,dW_s, \quad t \geqslant 0.$$

For $T > 0$ and $\frac{1}{\alpha} > p > 2$ we denote by $\mathcal{H}_{p,T}$ the Banach space of all adapted processes in $L^p(\Omega, C([0,T],H)) \cap L^\infty([0,T],H)$ endowed with the norm

$$\|Y\|_{p,T}^p = \mathbb{E}\sup_{t\in[0,T]}\big(\big|Y(t)\big|^p\big).$$

Theorem 1.2. *Under hypotheses* (H_1) *and* (H_2), *for any $x \in H$, Eq.* (1) *has a unique mild solution $X(\cdot,x) \in \mathcal{H}_{p,T}$.*

Proof. We define the mapping $\mathcal{K}$ on $\mathcal{H}_{p,T}$ by

$$\mathcal{K}(t,X) = e^{tA}x + \int_0^t e^{(t-s)A}(-A)^{\frac{1}{2}} F\big(X(s)\big)\,ds + \int_0^t e^{(t-s)A}\,dW_s, \quad t \in [0,T].$$

First we remark that for $X \in \mathcal{H}_{p,T}$ we have $\mathcal{K}(\cdot,X) \in \mathcal{H}_{p,T}$. Indeed,

$$\big|\mathcal{K}(t,X)\big|^p \leqslant 3^{p-1}\left(\big\|e^{tA}\big\|^p|x|^p + \left(\int_0^t \big|e^{(t-s)A}(-A)^{\frac{1}{2}}F\big(X(s)\big)\big|\,ds\right)^p + \left|\int_0^t e^{(t-s)A}\,dW_s\right|^p\right).$$

Hypothesis (H_1) implies that the stochastic convolution $W_A(t) := \int_0^t e^{(t-s)A}\,dW_s$ is well defined in H and by [4, Proposition 7.9] there exists a constant $c_p(T) > 0$ such that

$$\mathbb{E}\biggl(\sup_{t\in[0,T]}\biggl(\int_0^t e^{(t-s)A}\,dW_s\biggr)^p\biggr) \leqslant c_p(T)\biggl|\int_0^T s^{-2\alpha}\,\|e^{sA}\|_{\mathrm{HS}}^2\,ds\biggr|^{\frac{p}{2}} < \infty.$$

Since F is global Lipschitz we have

$$\biggl(\int_0^t \bigl|e^{(t-s)A}(-A)^{\frac{1}{2}}F\bigl(X(s)\bigr)\bigr|\,ds\biggr)^p \leqslant c^p_{\frac{1}{2}}\biggl(\int_0^t (t-s)^{-\frac{p}{2(p-1)}}\,ds\biggr)^{(p-1)}\cdot\int_0^t \bigl\|F\bigl(X(s)\bigr)\bigr\|^p$$
$$\leqslant \tilde{c}_p(T)\cdot\Bigl(1+\sup_{0\leqslant t\leqslant T}\bigl|X(t)\bigr|^p\Bigr),$$

where we put $\tilde{c}_p(T) := 2^p c^p_{\frac{1}{2}} c^p (\frac{2(p-1)}{(p-2)})^{p-1} T^{\frac{p}{2}}$ and we used

$$\bigl\|(-A)^{\frac{1}{2}}e^{tA}\bigr\| \leqslant c_{\frac{1}{2}}t^{-\frac{1}{2}} \quad\text{and}\quad \bigl|F(x)\bigr| \leqslant c\cdot\bigl(1+|x|\bigr),\quad x\in H.$$

Hence

$$\mathbb{E}\biggl(\sup_{t\in[0,T]}\biggl(\int_0^t \bigl|e^{(t-s)A}(-A)^{\frac{1}{2}}F\bigl(X(s)\bigr)\bigr|\,ds\biggr)^p\biggr) \leqslant \tilde{c}_p(T)\cdot\Bigl(1+\mathbb{E}\Bigl(\sup_{0\leqslant t\leqslant T}\bigl|X(t)\bigr|^p\Bigr)\Bigr).$$

Therefore, we have for some constants c_1, c_2, c_3,

$$\mathbb{E}\Bigl(\sup_{t\in[0,T]}\bigl|\mathcal{K}(t,X)\bigr|^p\Bigr) \leqslant c_1 + c_2|x|^p + c_3\mathbb{E}\Bigl(\sup_{t\in[0,T]}\bigl|X(t)\bigr|^p\Bigr).$$

Thus $\mathcal{K}(\cdot,X) \in \mathcal{H}_{p,T}$.

In the same way, we obtain for $X_1, X_2 \in \mathcal{H}_{p,T}$

$$\mathbb{E}\Bigl(\sup_{t\in[0,T]}\bigl|\mathcal{K}(t,X_1)-\mathcal{K}(t,X_2)\bigr|^p\Bigr) \leqslant c_3\mathbb{E}\Bigl(\sup_{t\in[0,T]}\bigl|X_1(t)-X_2(t)\bigr|^p\Bigr).$$

We remark that if T is small enough, then $c_3 < 1$ and consequently, by the Banach fixed point theorem, Eq. (1) has a unique solution in $\mathcal{H}_{p,T}$. The case of general $T > 0$ can be treated by considering the equation in intervals $[0,\widetilde{T}]$, $[\widetilde{T},2\widetilde{T}]$, … for small $\widetilde{T}$. □

2. Strong Feller property

In this section we discuss the strong Feller property of the semigroup $(P_t)_{t\geqslant 0}$. We start with the case when F is regular and assume that $F \in C_b^2(H,H)$. In the following we prove that the mild solution $X(t,x)$ is differentiable with respect to x and for any $h \in H$ it holds

$$DX(t,x)\cdot h = \eta^h(t,x),$$

where $\eta^h(t,x)$ is the mild solution of the equation

$$\begin{cases} \dfrac{d}{dt}\eta^h(t,x) = A\eta^h(t,x) + (-A)^{\frac{1}{2}}DF\bigl(X(t,x)\bigr)\cdot\eta^h(t,x),\\ \eta^h(0,x) = h \in H. \end{cases} \tag{4}$$

This means that $\eta^h(t,x)$ is the solution of the integral equation

$$\eta^h(t,x) = e^{tA}h + \int_0^t (-A)^{\frac{1}{2}}e^{(t-s)A}DF\bigl(X(s)\bigr)\cdot\eta^h(s,x)\,ds,\quad t\geqslant 0. \tag{5}$$

Theorem 2.1. *The mild solution $X(t,x)$ of Eq. (1) is differentiable with respect to x $\mathbb{P}$-a.s., and for any $h\in H$, we have*

$$DX(t,x)\cdot h = \eta^h(t,x),\quad \mathbb{P}\text{-a.s.} \tag{6}$$

and

$$\bigl|\eta^h(t,x)\bigr| \leqslant e^{t\frac{L^2}{4}}|h|,\quad t\geqslant 0. \tag{7}$$

Proof. Similarly as in the proof of Theorem 1.2, we can see that Eq. (4) has a unique mild solution $\eta^h(t,x)$ in $\mathcal{H}_{p,T}$. Let us prove (7). For $\lambda \in \rho(A)$, we set $R(\lambda, A) := \lambda(\lambda - A)^{-1}$ and consider the approximation sequence $\eta^h_\lambda(t,x) := \lambda R(\lambda, A)\eta^h(t,x)$, $\lambda \in \rho(A)$. By multiplying both sides of (4) by $\lambda R(\lambda, A)$, we obtain

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\left|\eta^h_\lambda(t,x)\right|^2 &= \left\langle A\eta^h_\lambda(t,x), \eta^h_\lambda(t,x)\right\rangle + \left\langle(-A)^{\frac{1}{2}} DF\big(X(t,x)\big)\eta^h(t,x), (-A)^{\frac{1}{2}}\eta^h_\lambda(t,x)\right\rangle \\ &= \left\langle A\eta^h_\lambda(t,x), \eta^h_\lambda(t,x)\right\rangle + \frac{\|DF\|^2_\infty}{4}\left|\eta^h(t,x)\right|^2 + \left|(-A)^{\frac{1}{2}}\eta^h_\lambda(t,x)\right|^2 \\ &= \frac{\|DF\|^2_\infty}{4}\left|\eta^h(t,x)\right|^2.\end{aligned}$$

Therefore,

$$\frac{1}{2}\frac{d}{dt}\left|\eta^h_\lambda(t,x)\right|^2 \leqslant \frac{1}{2}\left|\lambda R(\lambda, A)h\right|^2 + \frac{\|DF\|^2_\infty}{4}\int_0^t \left|\eta^h(t,x)\right|^2 ds.$$

Letting $\lambda \to +\infty$ we get

$$\frac{1}{2}\left|\eta^h(t,x)\right|^2 \leqslant \frac{1}{2}|h|^2 + \frac{\|DF\|^2_\infty}{4}\int_0^t \left|\eta^h(t,x)\right|^2 ds.$$

Then (7) follows now by Gronwall's lemma.

We now prove that $\eta^h(t,x)$ fulfills (6). The argument we follow here is similar to the proof of Theorem 5.4.1 in [5]. Fix $T > 0$, $x, h \in H$ such that $|h| \leqslant 1$. Setting

$$\Delta_h(t,x) = X(t, x+h) - X(t,x) - \eta^h(t,x),$$

we have

$$\begin{aligned}\Delta_h(t,x) = &\int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A}\big(\widehat{F}\big(X(s,x+h)\big) - \widehat{F}\big(X(s,x)\big)\big)\,ds \\ &- \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A} D\widehat{F}\big(X(s,x)\big)\cdot \eta^h(s,x)\,ds.\end{aligned}$$

Consequently,

$$\begin{aligned}\Delta_h(t,x) &= \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A}\int_0^1 D\widehat{F}\big(\rho(\xi,s)\big)\,d\xi\cdot\big(X(s,x+h) - X(s,x)\big)\,ds \\ &\quad - \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A} D\widehat{F}\big(X(s,x)\big)\cdot\eta^h(s,x)\,ds \\ &= \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A}\int_0^1 D\widehat{F}\big(\rho(\xi,s)\big)\,d\xi\cdot\Delta_h(s,x)\,ds \\ &\quad + \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A}\int_0^1 \big(D\widehat{F}\big(\rho(\xi,s)\big) - D\widehat{F}\big(X(s,x)\big)\big)\,d\xi\cdot\eta^h(s,x)\,ds,\end{aligned}$$

where $\rho(\xi,s) = \xi X(s,x+h) + (1-\xi)X(s,x)$. Since $\widehat{F} \in C^1_b(H,H)$ and $X(t,x)$ is continuous with respect to x uniformly in $[0,T]$, we have

$$\left|D\widehat{F}\big(\rho(\xi,s)\big) - D\widehat{F}\big(X(s,x)\big)\right| \leqslant \delta_T(h),$$

for some function $\delta_T \to 0$ as $h \to 0$.

Hence using $\|(-A)^{\frac{1}{2}}e^{tA}\| \leqslant c_{\frac{1}{2}}t^{-\frac{1}{2}}$ we deduce

$$\left|\int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A} \int_0^1 \left(D\widehat{F}\big(\rho(\xi,s)\big) - D\widehat{F}\big(X(s,x)\big)\right) d\xi \cdot \eta^h(s,x)\, ds\right|$$
$$\leqslant c_{\frac{1}{2}} \int_0^t (t-s)^{-\frac{1}{2}}\, ds\, \delta_T(h) \left|\eta^h(s,x)\right| \leqslant 2c_{\frac{1}{2}} \sqrt{T} \delta_T(h) \sup_{t\in[0,T]} e^{t\frac{L^2}{4}} |h|.$$

It follows that

$$\left|\Delta_h(t,x)\right| \leqslant \|D\widehat{F}\|_\infty \int_0^t (t-s)^{-\frac{1}{2}} \left|\Delta_h(s,x)\right| ds + 2c_{\frac{1}{2}} \sqrt{T} \delta_T(h) e^{T\frac{L^2}{4}} |h|$$
$$\leqslant L \int_0^t (t-s)^{-\frac{1}{2}} \left|\Delta_h(s,x)\right| ds + 2c_{\frac{1}{2}} \sqrt{T} \delta_T(h) e^{T\frac{L^2}{4}} |h|.$$

Using a singular Gronwall inequality (see Amann [1, Section II.3.3]) we have

$$\left|\Delta_h(t,x)\right| \leqslant C_T \cdot \delta_T(h)|h| \quad \text{for some } C_T > 0.$$

Hence

$$\frac{|\Delta_h(t,x)|}{|h|} \leqslant C_T \cdot \delta_T(h)$$

which implies (6). □

Consider now the approximation problem

$$\begin{cases} dX(t) = \big(AX(t) + A_n F\big(X(t)\big)\big)\, dt + dW_t, & t \geqslant 0, \\ X(0) = x, \end{cases} \tag{8}$$

where $A_n := (-A)^{\frac{1}{2}} nR(n,A) = -n(-A)^{-\frac{1}{2}} AR(n,A)$. So A_n are bounded operators converging pointwise to $(-A)^{\frac{1}{2}}$ (see [8, Section 4.10]) and commuting with A. Notice that $A_n \circ F : H \to H$ is a nonlinear Lipschitz continuous function, hence the corresponding problem (8) has a unique mild solution $X_n(t,x)$ in $\mathcal{H}_{p,T}$ which is the fixed point of the following mapping

$$\mathcal{K}_n(Y)(t) := e^{tA}x + \int_0^t e^{(t-s)A}\, dW(s) + \int_0^t e^{(t-s)A} A_n F\big(Y(s)\big)\, ds,$$

on the space $\mathcal{H}_{p,T}$. Moreover

$$\lim_{n\to\infty} X_n(\cdot,x) = X(\cdot,x) \quad \text{in } \mathcal{H}_{p,T}. \tag{9}$$

Indeed, if we write

$$\mathcal{K}(X)(t) := e^{tA}x + \int_0^t e^{(t-s)A}\, dW(s) + \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A} F\big(X(s)\big)\, ds,$$

then is straightforward that $\mathcal{K}_n \to \mathcal{K}$ strongly in $\mathcal{H}_{p,T}$. Similar computation as in the proof of Theorem 1.2 shows that the Lipschitz constants of $\mathcal{K}$ and $\mathcal{K}_n$ can be chose identic and equal to some $\alpha \in (0,1)$ uniformly with respect to $n \in \mathbb{N}$, if $T > 0$ is small enough. Indeed, one has only to notice that $\|A_n e^{(t-s)A}\| = \|(-A)^{\frac{1}{2}} nR(n,A) e^{(t-s)A}\| \leqslant \|(-A)^{\frac{1}{2}} e^{(t-s)A}\|$ for $s \in [0,t)$, and the repeat the arguments as in the proof of Theorem 1.2. By Theorem 7.1.1 in [6] we obtain that $\mathcal{K}$ and $\mathcal{K}_n$ have unique fixed points $X(t,x)$ and $X_n(t,x)$, $n \geqslant 1$ respectively. Further, Theorem 7.1.5 in [6] shows that $X_n \to X$ in $\mathcal{H}_{p,T}$.

We denote by $\eta_n^h(t,x)$ the mild solution of problem

$$\begin{cases} \dfrac{d}{dt}\eta_n^h(t,x) = A\eta_n^h(t,x) + A_n DF\big(X(t,x)\big) \cdot \eta_n^h(t,x), \\ \eta_n^h(0,x) = h \in H. \end{cases} \tag{10}$$

It is well known that the solution $X_n(t,x)$ of problem (8) is differentiable with respect to x $\mathbb{P}$-a.s. (see [4, Section 9.1.1]), and that

$$DX_n(t,x) \cdot h = \eta_n^h(t,x), \quad h \in H,\ t > 0.$$

Moreover using the contractivity of the semigroup $(e^{tA})_{t\geqslant 0}$, it is straightforward that estimate (7) still holds for $\eta_n^h(t,x)$ (cf., Theorem 2.1). Furthermore we have

$$\lim_{n\to\infty} \eta_n^h(\cdot,x) = \eta^h(\cdot,x) \quad \text{in } \mathcal{H}_{p,T}. \tag{11}$$

We now consider the approximating semigroup

$$P_t^n\varphi(x) = \mathbb{E}\big(\varphi\big(X_n(t,x)\big)\big), \quad \varphi \in B_b(H),\ t \geqslant 0,\ x \in H,$$

for $n \in N$, where $X_n(t,x)$ is the solution of (8). By Lebesgue's theorem we have

$$\lim_{n\to\infty} P_t^n\varphi(x) = P_t\varphi(x), \quad \varphi \in C_b(H),\ x \in H.$$

Hence by Theorem 2.1, we have that for all $\varphi \in C_b^1(H)$, $P_t^n\varphi$ and $P_t\varphi$ are differentiable with respect to x and it holds

$$\big\langle DP_t\varphi(x), h\big\rangle = \mathbb{E}\big\langle D\varphi\big(X(t,x)\big), \eta^h(t,x)\big\rangle, \quad h \in H,$$
$$\big\langle DP_t^n\varphi(x), h\big\rangle = \mathbb{E}\big\langle D\varphi\big(X_n(t,x)\big), \eta_n^h(t,x)\big\rangle, \quad h \in H.$$

Moreover, by Eqs. (9) and (11), it follows that for all $\varphi \in C_b^1(H)$, $h \in H$,

$$\lim_{n\to\infty}\big\langle DP_t^n\varphi(x), h\big\rangle = \big\langle DP_t\varphi(x), h\big\rangle \quad \text{in } C\big([0,T],\mathbb{R}\big).$$

We now are in the position to prove the following lemma.

Lemma 2.2. *If $F \in C_b^2(H,H)$, the transition semigroup $(P_t)_{t\geqslant 0}$ is strong Feller.*

Proof. It is well known from [3] (see also [12]) that the semigroup $(P_t^n)_{t\geqslant 0}$ satisfies the following Bismut–Elworthy formula

$$\big\langle DP_t^n\varphi(x), h\big\rangle = \frac{1}{t}\mathbb{E}\left(\varphi\big(X_n(t,x)\big)\int_0^t \big\langle \eta_n^h(t,x), dW_s\big\rangle\right) \quad \text{for all } \varphi \in C_b^2(H). \tag{12}$$

Hence if $\varphi \in C_b^2(H)$, $t > 0$, we can use (12) and Hölder inequality and recall (7) to obtain

$$\begin{aligned}
\big|DP_t^n\varphi(x), h\big|^2 &\leqslant t^{-2}\|\varphi\|_\infty^2 \int_0^t \big|\eta_n^h(s,x)\big|^2\,ds \\
&\leqslant t^{-2}\|\varphi\|_\infty^2 \int_0^t e^{\frac{L^2}{2}s}|h|^2\,ds \\
&\leqslant t^{-2}\|\varphi\|_\infty^2 \frac{2}{L^2}\big(e^{\frac{L^2}{2}t}-1\big)|h|^2 \quad \text{for all } n \in \mathbb{N}.
\end{aligned}$$

Now, letting $n \to \infty$ and from the arbitrariness of h we get

$$\big|DP_t\varphi(x)\big| \leqslant t^{-1}\frac{\sqrt{2}}{L}\big(e^{\frac{L^2}{2}t}-1\big)^{\frac{1}{2}}\|\varphi\|_\infty. \tag{13}$$

We now claim that for any $\varphi \in B_b(H)$, $t > 0$, and $x, y \in H$, it holds

$$\big|P_t\varphi(x) - P_t\varphi(y)\big| \leqslant t^{-1}\frac{\sqrt{2}}{L}\big(e^{\frac{L^2}{2}t}-1\big)^{\frac{1}{2}}\|\varphi\|_\infty|x-y|. \tag{14}$$

To see this, we define $\mathcal{M}_1 := \{\varphi \in C_b(H)\colon \|\varphi\|_\infty \leqslant 1\}$ and $\mathcal{M}_2 := \{\varphi \in C_b^2(H)\colon \|\varphi\|_\infty \leqslant 1\}$. Since each function in $C_b(H)$ can be approximated pointwise by a sequence of functions in $C_b^2(H)$, we have

$$\sup_{\varphi\in\mathcal{M}_1}\big|P_t\varphi(x) - P_t\varphi(y)\big| = \sup_{\varphi\in\mathcal{M}_2}\big|P_t\varphi(x) - P_t\varphi(y)\big| \quad \text{for all } x, y \in H.$$

As a consequence of the Hahn decomposition theorem we have

$$\sup_{\varphi\in\mathcal{M}_1}\big|P_t\varphi(x) - P_t\varphi(y)\big| = \mathrm{Var}\big(P_t(x,\cdot) - P_t(y,\cdot)\big),$$

where $P_t(x,U) = P_t\mathbf{1}_U(x)$ for $U \in \mathcal{B}(H)$. Therefore by (13) we have for all $x, y \in H$

$$\operatorname{Var}\big(P_t(x,\cdot)-P_t(y,\cdot)\big)\leqslant t^{-1}\frac{\sqrt{2}}{L}\big(e^{\frac{L^2}{2}t}-1\big)^{\frac{1}{2}}|x-y|$$

and consequently for all $\varphi\in B_b(H)$

$$\begin{aligned}\big|P_t\varphi(x)-P_t\varphi(y)\big| &= \left|\int_H \varphi(u)\big(P_t(x,du)-P_t(y,du)\big)\right| \\ &\leqslant \|\varphi\|_\infty \operatorname{Var}\big(P_t(x,\cdot)-P_t(y,\cdot)\big) \\ &\leqslant t^{-1}\frac{\sqrt{2}}{L}\big(e^{\frac{L^2}{2}t}-1\big)^{\frac{1}{2}}\|\varphi\|_\infty|x-y|.\end{aligned}$$

Hence (14) is proved. □

Our main theorem in this section is the following.

Theorem 2.3. *The transition semigroup $(P_t)_{t\geqslant 0}$ corresponding to (1) is strong Feller.*

Proof. We are going to approximate F by a sequence of $C_b^1(H)$ function. For this purpose we consider a sequence of nonnegative twice differentiable functions $(\rho_n)_{n\in\mathbb{N}}$ such that

$$\operatorname{supp}(\rho_n)\subset\big\{\xi\in\mathbb{R}^n\colon\ |\xi|_{\mathbb{R}^n}\leqslant 1/n\big\}\quad\text{and}\quad\int_{\mathbb{R}^n}\rho_n(\xi)\,d\xi=1.$$

Let $(e_n)_{n\in\mathbb{N}}$ be an orthonormal basis in H, and for $n\in\mathbb{N}$ denote by P_n the orthogonal projection from H onto $\operatorname{span}\{e_1,\dots,e_n\}$ which we identify with $\mathbb{R}^n$, hence $P_n: H\to\mathbb{R}^n$ and the Euclidean inner product on $\mathbb{R}^n$ is just $\langle\cdot,\cdot\rangle$ (i.e. the one induced by $(H,\langle\cdot,\cdot\rangle)$ on $\operatorname{span}\{e_1,\dots,e_n\}$). Similar to [12] we define $F_n: H\to H$ by

$$F_n(x)=\int_{\mathbb{R}^n}\rho_n(\xi-P_nx)F\left(\sum_{i=1}^n\xi_ie_i\right)d\xi.$$

The sequence $(F_n)_{n\in\mathbb{N}}$ converges pointwise to F. Moreover for every $n\in\mathbb{N}$, F_n is a twice Fréchet differentiable function with bounded and continuous derivatives. Furthermore, for all x, y and $n\in\mathbb{N}$,

$$\begin{aligned}\big|F_n(x)-F_n(y)\big| &= \left|\int_{\mathbb{R}^n}\rho_n(\xi)\left(F\left(\sum_{i=1}^n\xi_ie_i+P_nx\right)-F\left(\sum_{i=1}^n\xi_ie_i+P_ny\right)\right)d\xi\right| \\ &\leqslant L\big|P_n(x-y)\big|\int_{\mathbb{R}^n}\rho_n(\xi)\,d\xi\leqslant L|x-y|.\end{aligned}$$

We now consider the solution $X_n(t,x)$ of the equation

$$\begin{cases} dX_n(t)=\big(AX_n(t)+(-A)^{\frac{1}{2}}F_n\big(X_n(t)\big)\big)\,dt+dW_t, & t\geqslant 0,\\ X_n(0)=x.\end{cases}\tag{15}$$

Clearly,

$$\lim_{n\to\infty}X_n(\cdot,x)=X(\cdot,x)\quad\text{in }\mathcal{H}_{p,T}.$$

Let $(P_t^n)_{t\geqslant 0}$ be the corresponding transition semigroup and take $\varphi\in C_b^2(H)$. By (14) we have for the semigroup $(P_t^n)_{t\geqslant 0}$

$$\big|P_t^n\varphi(x)-P_t^n\varphi(y)\big|\leqslant t^{-1}\frac{\sqrt{2}}{L}\big(e^{\frac{L^2}{2}t}-1\big)^{\frac{1}{2}}\|\varphi\|_\infty|x-y|.$$

By letting $n\to\infty$ and applying Lebesgue's theorem we get

$$\big|P_t\varphi(x)-P_t\varphi(y)\big|\leqslant t^{-1}\frac{\sqrt{2}}{L}\big(e^{\frac{L^2}{2}t}-1\big)^{\frac{1}{2}}\|\varphi\|_\infty|x-y|,$$

which proves the theorem. □

3. Irreducibility

In this section we discuss the irreducibility of the semigroup $(P_t)_{t\geqslant 0}$. To this end, we need to check first that the deterministic control problem corresponding to (1) is approximatively controllable. We shall prove that given $x_1, x_2 \in H$, $T > 0$ and $\varepsilon > 0$ then there exists a control $u(s) \in L^2([0,T], H)$ such that the solution of

$$Y(t) = e^{tA}x_1 + \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A} F\big(Y(s)\big)\,ds + \int_0^t e^{(t-s)A} u(s)\,ds \tag{16}$$

comes within ε of x_2 at time T. Let us define the following operator

$$J : L^2\big([0,T]; H\big) \to C_0\big([0,T], H\big), \qquad Ju = \int_0^t e^{(t-s)A} u(s)\,ds,$$

where

$$C_0\big([0,T]; H\big) := \big\{ f \in C\big([0,T], H\big):\ f(0) = 0 \big\}.$$

The operator J has a dense range. Indeed, take

$$\varphi \in C_0\big([0,T], D(A)\big) := \big\{ f \in C\big([0,T], D(A)\big):\ f(0) = 0 \big\}$$

and set

$$u(t) = \varphi'(t) - A\varphi(t).$$

We can see that $Ju = \varphi$, hence by the denseness of $C_0([0,T], D(A))$ we conclude the denseness of $\operatorname{Im} J$. Let us now consider the path $\gamma(t)$ joining x_1 and x_2 defined by

$$\gamma(t) = \frac{T-t}{T} x_1 + \frac{t}{T} x_2.$$

Set

$$f(t) = \gamma(t) - e^{tA}x_1 - \int_0^t (-A)^{\frac{1}{2}} e^{(t-s)A} F\big(\gamma(s)\big)\,ds, \quad t \in [0,T].$$

We have $f \in C_0([0,T]; H)$, hence for any $\delta > 0$ there exists $u \in L^2([0,T], H)$ such that

$$|Ju - f| \leqslant \delta \quad \text{for some } \delta > 0.$$

Now, given $\varepsilon > 0$, if $Y(\cdot, x_1, u)$ is the solution of (16), we have

$$\begin{aligned}\big|Y(t) - \gamma(t)\big| &\leqslant \int_0^t \big|(-A)^{\frac{1}{2}} e^{(t-s)A}\big(F\big(Y(s)\big) - F\big(\gamma(s)\big)\big)\big|\,ds + \big|Ju(t) - f(t)\big| \\ &\leqslant c_{1/2} L \int_0^t \frac{1}{\sqrt{t-s}} \big|Y(s) - \gamma(s)\big|\,ds + \big|Ju(t) - f(t)\big|.\end{aligned}$$

Hence by singular Gronwall inequality (see Amann [1, Section II.3.3]) we obtain

$$\big|Y(t) - \gamma(t)\big| \leqslant \delta \cdot C_T \quad \text{for some constant } C_T > 0.$$

It follows that

$$\big|Y(T) - x_2\big| \leqslant \delta \cdot C_T.$$

It is now enough to choose $\delta < \frac{\varepsilon}{C_T}$. We have thus proven the following.

Lemma 3.1. *Given $x_1, x_2 \in H$, $T > 0$ and $\varepsilon > 0$ then there exists a control $u \in L^2([0,T], H)$ such that the solution of*

$$Y(t)=e^{tA}x_1+\int_0^t(-A)^{\frac{1}{2}}e^{(t-s)A}F\big(Y(s)\big)\,ds+\int_0^t e^{(t-s)A}u(s)\,ds \tag{17}$$

comes within ε of x_2 at time T.

After this preparation we are now able the prove the following theorem.

Theorem 3.2. *The transition semigroup $(P_t)_{t\geqslant 0}$ corresponding to (1) is irreducible.*

Proof. Let $B(x_0,r)\subseteq H$ be an open ball. We show $P_t\mathbf{1}_{B(x_0,r)}=\mathbb{P}(\{|X(t,x)-x_0|<r\})>0$ for all $t>0$, where $X(t,x)$ is the solution of (1). We choose a control $u\in L^2([0,T],H)$ such that $|Y(T,x,u)-x_0|\leqslant\frac{r}{2}$, where $Y(T,x,u)$ is the solution of (17). Then we have

$$\mathbb{P}\big(\big\{\big|X(T,x)-x_0\big|<r\big\}\big)\geqslant\mathbb{P}\big(\big\{\big|X(T,x)-Y(T,x)\big|<r/2\big\}\big). \tag{18}$$

On the other hand,

$$\begin{aligned}\big|Y(t,x,u)-X(t,x)\big|&\leqslant\int_0^t\big|(-A)^{\frac{1}{2}}e^{(t-s)A}\big(F\big(Y(s)\big)-F\big(X(s)\big)\big)\big|\,ds+\big|W_A(t)-Ju(t)\big|\\&\leqslant c_{1/2}L\int_0^t\frac{1}{\sqrt{t-s}}\big|Y(s)-X(s)\big|\,ds+\sup_{t\in[0,T]}\big|W_A(t)-Ju(t)\big|.\end{aligned}$$

Hence by singular Gronwall inequality we obtain

$$\big|X(t,x)-Y(t,x,u)\big|\leqslant C_T\cdot\sup_{t\in[0,T]}\big|W_A(t)-Ju(t)\big|.$$

Moreover, since $W_A(\cdot)$ is a nondegenerate continuous Gaussian random variable, we have that

$$\mathbb{P}\left\{\sup_{t\in[0,T]}\big|W_A(t)-Ju(t)\big|<\frac{r}{2C_T}\right\}>0.$$

This implies that

$$\mathbb{P}\big(\big\{\big|X(T,x)-Y(T,x)\big|<r/2\big\}\big)>0.$$

Therefore estimate (18) implies the irreducibility of the semigroup $(P_t)_{t\geqslant 0}$. □

4. Invariant measure

In this section we discuss the existence and uniqueness of the invariant measure μ of the semigroup $(P_t)_{t\geqslant 0}$. For this purpose we will use Krylov–Bogoliubov's theorem. Since $(P_t)_{t\geqslant 0}$ is strong Feller, in order to obtain the existence of an invariant measure it is sufficient to check tightness of the set of probability measures $\{\mu_T:=\frac{1}{T}\int_0^T\mu_{X(t,x)}\,dt,\ T\geqslant 1\}$. Here $\mu_{X(t,x)}$ denotes the distribution of $X(t,x)$, $t\geqslant 0$. Indeed, using [5, Theorem 3.1.1] any limit point μ of some weakly convergent subsequence of $(\mu_T)_{T\geqslant 1}$ will be an invariant measure for (1).

We now set

$$Y(t):=X(t)-W_A(t).$$

We shall assume further assumption

$$(\mathrm{H}_3)\qquad \big\langle F(x)-F(y),x-y\big\rangle\leqslant 0,\quad x,y\in H.$$

Since the semigroup generated by A is analytic, $(Y(t))_{t\geqslant 0}$ is differentiable on $V_{\frac{1}{4}}$ for $t>0$ with derivative

$$Y'(t)=AY(t)+(-A)^{\frac{1}{2}}F\big(Y(t)+W_A(t)\big),\quad t>0.$$

Using the dissipativity of F in (H_3) we have the following estimate for the process $(Y(t))_{t\geqslant 0}$.

Lemma 4.1. *Assume that assumptions* (H_0), (H_1) *and* (H_3) *hold. Then there exists* $C > 0$ *and* $\alpha > 0$ *such that*

$$\mathbb{E}\left(\frac{1}{2}\left\|(-A)^{-\frac{1}{4}}\big(Y(t)\big)\right\|^2 + \alpha\int_0^t \left\|Y(s)\right\|_{\frac{1}{4}}^2\,ds\right) \leqslant C(t+1) \quad \text{for } t \geqslant 0.$$

Proof. Let $t > 0$. We have

$$\begin{aligned}
\frac{1}{2}\frac{d}{dt}\left\|(-A)^{-\frac{1}{4}}\big(Y(t)\big)\right\|^2 &= \left\langle AY(t) + (-A)^{\frac{1}{2}}F\big(Y(t)+W_A(t)\big), (-A)^{-\frac{1}{2}}\big(Y(t)\big)\right\rangle \\
&= \left\langle -(-A)^{\frac{1}{2}}Y(t) + F\big(Y(t)+W_A(t)\big), Y(t)\right\rangle \\
&= -\left\|Y(t)\right\|_{\frac{1}{4}}^2 + \left\langle F\big(Y(t)+W_A(t)\big) - F\big(W_A(t)\big), Y(t)\right\rangle + \left\langle F\big(W_A(t)\big), Y(t)\right\rangle \\
&\leqslant -\left\|Y(t)\right\|_{\frac{1}{4}}^2 + \left\langle F\big(W_A(t)\big), Y(t)\right\rangle \\
&\leqslant -\left\|Y(t)\right\|_{\frac{1}{4}}^2 + \sigma\left\|Y(t)\right\|^2 + \frac{1}{4\sigma}c\big(1+\left\|W_A(t)\right\|^2\big).
\end{aligned}$$

Since $\|y\|_{\frac{1}{4}}^2 \geqslant \sqrt{\omega}\|y\|^2$, we obtain

$$\frac{1}{2}\frac{d}{dt}\left\|(-A)^{-\frac{1}{4}}\big(Y(t)\big)\right\|^2 \leqslant -\left(1-\frac{\sigma}{\sqrt{\omega}}\right)\left\|Y(t)\right\|_{\frac{1}{4}}^2 + \frac{1}{4\sigma}c\big(1+\left\|W_A(t)\right\|^2\big).$$

Therefore

$$\frac{1}{2}\left\|(-A)^{-\frac{1}{4}}\big(Y(t)\big)\right\|^2 + \left(1-\frac{\sigma}{\sqrt{\omega}}\right)\int_0^t \left\|Y(s)\right\|_{\frac{1}{4}}^2\,ds \leqslant \frac{1}{2}\left\|(-A)^{-\frac{1}{4}}x\right\| + \delta\int_0^t \left\|W_A(s)\right\|^2\,ds + \delta t, \tag{19}$$

where $\delta := \frac{1}{4\sigma}c$. Now hypotheses (H_0) and (H_1) imply that

$$M := \sup_{t\geqslant 0}\mathbb{E}\big(\left\|W_A(t)\right\|_{\gamma_0}^2\big) = \int_0^\infty \left\|(-A)^{\gamma_0}e^{tA}\right\|_{\mathrm{HS}}^2\,dt < \infty, \quad \text{for any } \gamma_0 \leqslant \alpha. \tag{20}$$

Indeed,

$$\begin{aligned}
\int_0^\infty \left\|(-A)^{\gamma_0}e^{tA}\right\|_{\mathrm{HS}}^2\,dt &= \sum_{k=0}^\infty \int_k^{k+1} \left\|(-A)^{\gamma_0}e^{tA}\right\|_{\mathrm{HS}}^2\,dt \\
&= \sum_{k=0}^\infty \int_0^1 \left\|(-A)^{\gamma_0}e^{tA}e^{kA}\right\|_{\mathrm{HS}}^2\,dt \leqslant \sum_{k=0}^\infty \left\|e^{kA}\right\|^2 \int_0^1 \left\|(-A)^{\gamma_0}e^{tA}\right\|_{\mathrm{HS}}^2\,dt \\
&\leqslant \sum_{k=0}^\infty e^{-2\omega k}\int_0^1 \left\|(-A)^{\gamma_0}e^{tA/2}\right\|^2\left\|e^{tA/2}\right\|_{\mathrm{HS}}^2\,dt \\
&\leqslant \sum_{k=0}^\infty e^{-2\omega k}\int_0^1 \frac{c^2}{t^{2\gamma_0}}\left\|e^{tA/2}\right\|_{\mathrm{HS}}^2\,dt < \infty.
\end{aligned}$$

Hence

$$\sup_{t\geqslant 0}\mathbb{E}\big(\left\|W_A(t)\right\|^2\big) \leqslant \frac{1}{\omega^{2\gamma_0}}\sup_{t\geqslant 0}\mathbb{E}\big(\left\|W_A(t)\right\|_{\gamma_0}^2\big) = \frac{M}{\omega^{2\gamma_0}}$$

so that

$$\mathbb{E}\left(\int_0^t \left\|W_A(s)\right\|^2\,ds\right) = \int_0^t \mathbb{E}\big(\left\|W_A(s)\right\|^2\big)\,ds \leqslant \frac{M}{\omega^{2\gamma_0}}\cdot t.$$

Choosing $\sigma > 0$ such that $\alpha := 1 - \frac{\sigma}{\sqrt{\omega}} > 0$ and taking expectation in (19), we obtain that

$$\mathbb{E}\left(\left\|(-A)^{-\frac{1}{4}}\big(Y(t)\big)\right\|^2+\alpha\int_0^t\left\|Y(s)\right\|_{\frac{1}{4}}^2\,ds\right)\leqslant C(t+1)\quad\text{for } t\geqslant 0 \text{ and some constant } C>0. \qquad \square$$

Now we are in the position to state our main theorem in this section.

Theorem 4.2. *Under hypotheses* (H_0), (H_1) *and* (H_3), *there exists a unique invariant measure for the transition semigroup* $(P_t)_{t\geqslant 0}$.

Proof. As we mentioned at the beginning of this section we need to prove tightness of the set of probability measures $\{\mu_T := \frac{1}{T}\int_0^T \mu_{X(t,x)}\,dt,\ T\geqslant 1\}$. Where $\mu_{X(t,x)}$ denotes the distribution of $X(t,x)$, $t\geqslant 0$. Take $\varepsilon>0$ and $\gamma_0<\inf(\alpha,\frac{1}{4})$, since the map $z\mapsto \alpha\|z\|_{\gamma_0}^2$ is coercive on V_{γ_0} (i.e., $\lim_{\|z\|_{\gamma_0}\to\infty}\alpha\|z\|_{\gamma_0}^2=\infty$), there exists $R_\varepsilon>0$ such that

$$\varepsilon\big(\alpha\|y\|_{\gamma_0}^2+\|w\|_{\gamma_0}^2\big)\geqslant 1$$

for $w, y\in V_{\gamma_0}$ with $\|w+y\|_{V_{\gamma_0}}\geqslant R_\varepsilon$. Consequently if we denote by $\overline{B}(0,R_\varepsilon)$ the closed ball of radius R_ε in V_{γ_0}, we have by using Lemma 4.1 and (20)

$$\begin{aligned}\mu_T\big(H\setminus\overline{B}(0,R_\varepsilon)\big)&=\mathbb{E}\left(\frac{1}{T}\int_0^T \mathbf{1}_{\{\|X(s)\|_{V_{\gamma_0}}\geqslant R_\varepsilon\}}\,ds\right)\leqslant\varepsilon\mathbb{E}\left(\frac{1}{T}\int_0^T\alpha\left\|Y(s)\right\|_{\gamma_0}^2+\left\|W_A(s)\right\|_{V_{\gamma_0}}^2ds\right)\\&\leqslant\varepsilon\mathbb{E}\left(\frac{1}{T}\int_0^T\alpha\left\|Y(s)\right\|_{\frac{1}{4}}^2+\left\|W_A(s)\right\|_{V_{\gamma_0}}^2ds\right)\\&\leqslant\varepsilon\left(C\left(1+\frac{1}{T}\right)+M\right)\leqslant\varepsilon(2C+M)\end{aligned}$$

uniformly in $T\geqslant 1$. Since the embedding $V_{\gamma_0}\hookrightarrow H$ is compact, the family of probability measures $\{\mu_T\}_{T\geqslant 1}$ is tight on H. Now, by the Krylov–Bogoliubov theorem, there exists an invariant measure μ for the semigroup $(P_t)_{t\geqslant 0}$. The uniqueness of μ follows from the strong Feller property and the irreducibility of $(P_t)_{t\geqslant 0}$. $\square$

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