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Transverse radial expansion in nuclear collisions and two particle correlations

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Abstract

At the very first stage of an ultra-relativistic nucleus–nucleus collision new particles are produced in individual nucleon–nucleon collisions. In the transverse plane, all particles from a single NN collision are initially located at the same position. The subsequent thermalization and transverse radial expansion of the system create strong position-momentum correlations and lead to characteristic rapidity, transverse momentum, and azimuthal correlations among the produced particles.

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The physics of the high energy heavy ion collisions attracts strong attention of the physics community as creation of a new type of matter, the quark–gluon plasma, is expected in such collisions. During the last few years of Au + Au collisions at the BNL Relativistic Heavy Ion Collider (RHIC) many new phenomena has been observed, such as strong elliptic flow [1] and suppression of the high transverse momentum two particle back-to-back correlations [2]. These observations strongly indicate that a dense partonic matter has been created in such high energy nuclear collisions. Parton re-interactions lead to pressure build-up and the system undergoes longitudinal and transverse expansion, the latter leading to an increase in the particle final transverse momenta. The thermalization time of the system is estimated to be smaller than 1 fm/c [3]. Here, we mostly discuss the effect of the transverse *radial* expansion, neglecting a possible azimuthal dependence in the transverse expansion velocity field, which is mostly important for the anisotropic flow study. Usually the transverse expansion is studied via detailed analysis of single particle transverse momentum spectra, most often using thermal parameterization suggested in [4]. Transverse expansion also causes the transverse momentum dependence of the HBT radii [5]. In this Letter we note that the transverse radial expansion should also lead to characteristic rapidity, transverse momentum, and azimuthal angle (long range, as opposed to the HBT like scale) two-particle correlations. Those correlations are of a totally new type (compared to what is usually discussed) having the nature of the correlations among particles placed at the same spatial location in the rapidly expanding dense medium.

The origin of these correlations can be described as follows: at the first stage of an AA collision many individual nucleon–nucleon collision happen. The strings stretched and new particles are produced (partons are freed; the exact mechanism of the particle production is not important for the presentation of the main idea of this Letter). Due to a rapid thermalization, the newly produced particles become “frozen in” the medium, preserving all the correlations between them, e.g. the correlations due to the conservation laws of electric and/or baryon charges, strangeness, etc. The relative particle position (in the transverse plane) could be only modified by particle diffusion during the system evolution before the final freeze-out (for a more detailed discussion of the role of the thermalization time and diffusion see later in the Letter). Then the transverse expansion of the system would create strong position-momentum correlations in the transverse plane: in general, farther from the center axis of the system a particle

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is produced initially, on average the larger (transverse, in radial outward direction) push it receives from other particles during the system evolution. It is common [3] that the radial expansion collective velocity is parameterized by monotonically increasing function of the radial distance, though some deviations from that could be possible at very large distances due to low density at the periphery of the system. All particles produced from the same string (from the same NN collision) have initially the *same spatial* position in the transverse plane. Consequently, all particle from the same string gets *on average* the same push and thus become correlated. Correlated pairs of particles, being produced at the same place, for example containing s- and $\bar{s}$-quarks, would be boosted in the same direction and become azimuthally correlated, etc.

Note that in rapidity space the correlations due to described mechanism could extend up to the length of the entire string. For simplicity, in the most of the discussion below we assume the radial expansion of the system to be boost invariant. The 'push' is in the transverse direction, and on average does not affects the longitudinal momentum component.

Based on the above mentioned ideas it appears a simple picture of AA collisions, which could be considered in some sense as a next order approximation to a simple superposition of independent NN collisions often use as a base line in correlation studies of AA collisions. In this picture, the particles produced in each independent collision are boosted in the radial direction depending to the location of the collision in the transverse plane and the transverse expansion velocity profile. It corresponds to non-zero space–momentum correlations in the transverse plane, $\langle x_i, \Delta p_{t,i}\rangle \neq 0$ (see [6] as an example of the role of space–momentum correlations in the development of directed flow). As discussed below this picture leads to many distinctive phenomena, most of which can be studied by means of two- (and many-)particle correlations.

Two particle transverse momentum correlations. The single particle spectra are affected by radial flow in such a way that the effective slope as well as the mean transverse momentum are mostly sensitive to the *average* expansion velocity squared (the validity of this approximation is discussed in more detail below) and to much lesser extend to the actual velocity profile (dependence of the expansion velocity on the radial distance from the center axis of the system). The two-particle transverse momentum correlations [7], $\langle p_{t,1}p_{t,2}\rangle - \langle p_{t,1}\rangle\langle p_{t,2}\rangle \equiv \langle \delta p_{t,1}\delta p_{t,2}\rangle$, would be sensitive mostly to the *variance* in collective transverse expansion velocity, and thus are more sensitive to the actual velocity profile. Simple estimates show that the corresponding contribution could be comparable in magnitude to the primordial correlations existed in NN collisions. For a rough estimate one can use a relation

$$\langle p_t\rangle_{AA} \approx \langle p_t\rangle_{NN} + \alpha\langle v^2\rangle, \tag{1}$$

where the coefficient α is of the order of typical particle mass, and v being the expansion velocity. Then the correlation between transverse momenta of two particles would be

$$\langle \delta p_{t,1}\delta p_{t,2}\rangle_{AA} \approx D_{N_{\rm coll}}\left(\langle \delta p_{t,1}\delta p_{t,2}\rangle_{NN} + \alpha^2\sigma^2_{v^2}\right). \tag{2}$$

The factor D,

$$D_{N_{\rm coll}} = \frac{\langle n(n-1)\rangle_{NN}}{(N_{\rm coll}-1)\langle n\rangle^2_{NN} + \langle n(n-1)\rangle_{NN}}, \tag{3}$$

takes into account the dilution of the correlations due to a mixture of particles from $N_{\rm coll}$ uncorrelated NN collisions, and that in an individual NN collision the mean number of particle pairs, $\langle n(n-1)\rangle_{NN}$, on average is larger than the mean multiplicity squared, $\langle n\rangle^2_{NN}$ (see also [8,9]). For a linear velocity profile, $\sigma^2_{v^2} = \langle v^2\rangle^2/3$. Taking into account that the increase in the average transverse momentum due to the radial expansion is of the order of 20–30%, one concludes that the two terms in Eq. (2) could be of the same order of magnitude: an increase in the relative correlations, $\alpha^2\sigma^2_{v^2}/\langle p_t\rangle^2$, could be of the order of 1–3%, similar to the value of $\langle \delta p_{t,1}\delta p_{t,2}\rangle_{NN}/\langle p_t\rangle^2 \approx (0.12)^2$, measured at ISR [10]. Thus, the correlations due to transverse expansion could be the major part in the observed centrality dependence of the mean p_t fluctuations/correlations observed at the SPS and RHIC (for recent results, see [11]).

More accurate estimates can be obtained employing a thermal model presented in [4]. In this approach particles are produced by freeze-out of the thermalized matter at temperature T, approximated by a boosted Boltzmann distribution. Assuming boost-invariant longitudinal expansion and freeze-out at constant proper time, one finds, up to irrelevant constants, for single particle spectra:

$$\frac{d^2N}{dp_t^2\,d\phi} \sim \int_0^R r\,dr \int_0^{2\pi} d\phi_b\, m_t K_1(\beta_t) e^{\alpha_t \cos(\phi_b-\phi)}, \tag{4}$$

where p_t is the transverse flow rapidity, ϕ_b is the boost direction, $\alpha_t = (p_t/T)\sinh(p_t)$, and $\beta_t = (m_t/T)\cosh(\rho_t)$. It also assumes a uniform matter density within a cylinder, $r < R$, and a power law transverse rapidity flow profile $\rho_t = \rho_{t,\max}(r/R)^n$.

In such a model, the mean transverse momentum is given by expression:

$$\langle p_t\rangle = \frac{\int d\mathbf{p}_t \int d\rho_t\, d\phi_b\, \rho_t^{2/n-1} p_t m_t K_1(\beta_t) e^{\alpha_t\cos(\phi_b-\phi)}}{\int d\mathbf{p}_t \int d\rho_t\, d\phi_b\, \rho_t^{2/n-1} m_t K_1(\beta_t) e^{\alpha_t\cos(\phi_b-\phi)}}. \tag{5}$$

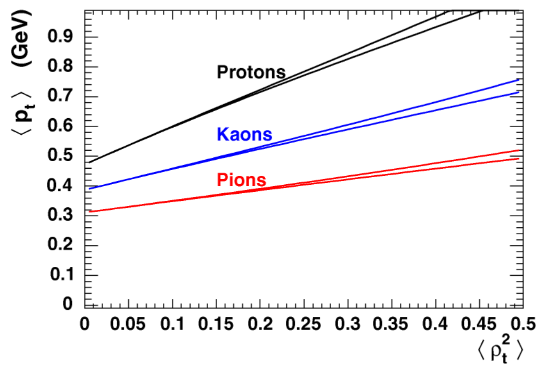

Fig. 1. (Color online.) Mean transverse momentum in the blast wave calculations. $T = 110$ MeV.

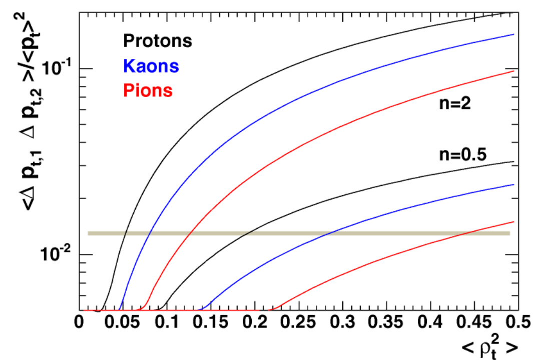

Fig. 2. (Color online.) Two particle transverse momentum correlations in the blast wave calculations. $T = 110$ MeV. The gray line indicate the level of primordial correlation in NN collision [10].

The contribution of the transverse expansion to the two particle mean transverse momentum correlations (both particles are from the same NN collision, the dilution factor, Eq. (3), to be applied afterward) can be written as:

$$\langle \delta p_{t,1} \delta p_{t,2} \rangle = \frac{\int d\rho_t \, d\phi_b \, \rho_t^{2/n-1} \int d\mathbf{p}_{t,1} \int d\mathbf{p}_{t,2} \, (\delta p_{t,1} \delta p_{t,2}) m_{t,i} K_1(\beta_{t,1}) e^{\alpha_{t,1}\cos(\phi_b-\phi_1)} m_{t,2} K_1(\beta_{t,2}) e^{\alpha_{t,2}\cos(\phi_b-\phi_2)}}{\int d\rho_t \, d\phi_b \, \rho_t^{2/n-1} \int d\mathbf{p}_{t,1} \int d\mathbf{p}_{t,2} \, m_{t,1} K_1(\beta_{t,1}) e^{\alpha_{t,1}\cos(\phi_b-\phi_1)} m_{t,2} K_1(\beta_{t,2}) e^{\alpha_{t,2}\cos(\phi_b-\phi_2)}}. \tag{6}$$

This equation additionally assumes that during the expansion time (before the freeze-out) the particles originated from the same NN collision do not diffuse far one from another compared to the system size. We discuss this assumption in more detail later in this Letter.

The results of the numerical calculations based on the above equations are presented in Figs. 1 and 2 as function of $\langle p_t^2 \rangle = \langle p_t \rangle^2 (4n+4)/(2+n)^2$. The results are shown for two different velocity (transverse rapidity) profiles, $n = 2$, and $n = 0.5$. One observes that indeed for all the particle types presented, $\langle p_t \rangle$ depends very weakly on the actual profile. On opposite, the correlations are drastically different for two cases presented.

Rapidity correlations. Charge balance functions. The transverse expansion 'push' consists of many individual collisions. It leads not only to the increase of the transverse momentum but also to the particle diffusion in the rapidity space. We do not consider the effect of such diffusion in this discussion concentrating only on the effect of the transverse radial push. We define the balance function as

$$B_{ab}(x_b; x_a) = \rho_{2,ab}(x_a, x_b)/\rho_{1,a}(x_a) - \rho_{1,b}(x_b). \tag{7}$$

Very roughly, $B_{ab}(x_b; x_a)$ has a meaning of a distribution of the 'associated' particles b under condition of a 'trigger' particle a to be found at the location x_a. (The *charge balance function* introduced in [12] can be written as $B(b|a) = (1/2)(B_{+-} + B_{-+} - B_{++} - B_{--})$.) Note that due to the 'normalization' to the number of 'trigger' particles, the balance function is the same for any superposition of independent NN collisions. The transverse expansion leads to the narrowing of the charge balance function [13]. In our simple picture the width of the balance function would be roughly inversely proportional to the transverse mass as follows from the relation

$$\Delta p_z = m_t \sinh(\Delta y) \approx m_t \Delta y. \tag{8}$$

The width decreases as the mean transverse mass increases due to transverse expansion. This effect is consistent with the experimentally observed narrowing of the balance function with centrality [14]. Note that because the charge balance function is normalized to unity, the narrowing of the correlations means an increase in the magnitude of the two particle correlation function. In its turn it means an enhancement of the net charge multiplicity fluctuations if measured in a rapidity region compared or smaller than the correlation length (1–2 units of rapidity). This observation might be an explanation for the centrality dependence of the net charge fluctuations measured at RHIC [15].

Azimuthal correlations. As all particles from the same NN collisions are pushed in the same direction (radially in the transverse plane) they become correlated in azimuthal space. The correlations can become really strong for large transverse flow as shown in Fig. 2 (again, for particles originated from the *same* spatial position in the transverse plane). Fig. 3 shows the strength of the azimuthal correlation in terms of the first two harmonics. $\langle \cos(\Delta\phi) \rangle$ reaches quite large values (even if one takes into account a dilution factor, $D \sim 1/150$ for central Au + Au collisions). If the average would be taken over all particles including the particles from other NN collisions, the momentum conservation contribution [16] cannot be neglected; it can greatly reduce the effect. Note, however, that the momentum conservation effects are expected to be small for the (azimuthal) charge balance function. In this case

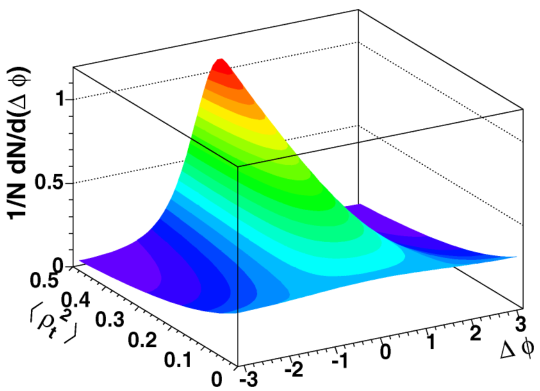

Fig. 3. (Color online.) Two pion $\Delta\phi$ distribution as function of $\langle \rho_t^2 \rangle$ in the blast wave model. Linear velocity profile and $T = 110$ MeV have been assumed.

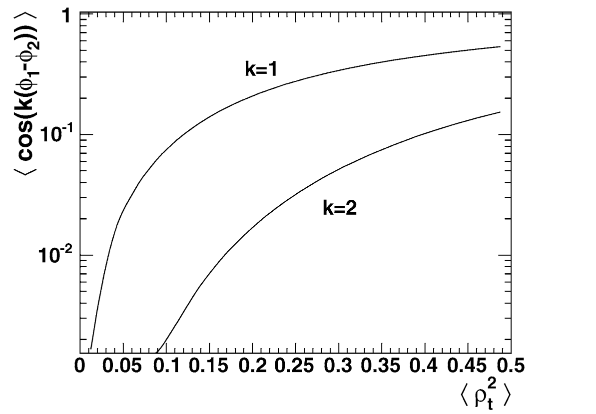

Fig. 4. The average values of $\cos(\Delta\phi)$ and $\cos(2\Delta\phi)$ for the distribution shown in Fig. 3.

both correlated particles originate from the same NN collision and the results presented in Fig. 3 are valid. In our model $\cos(\Delta\phi)$ should strongly depend on the particle mass and its transverse momentum. Identified particle correlation study would be of great interest in this respect.

The second harmonic in the azimuthal correlations generated by radial expansion is of a particular interest as it would contribute to the measurements of elliptic flow. The numbers from Fig. 3 corresponding to $\langle \rho_t^2 \rangle \sim 0.3$ are comparable with the estimates of the strength of non-flow type azimuthal correlation estimates made in [17]. Thus the azimuthal correlations generated by transverse expansion could be a major contributor to the non-flow azimuthal correlations. More importantly, this contribution would depend on centrality (following the development of radial flow), unlike many other non-flow effects. Note, however, that the typical elliptic flow centrality dependence (rise and fall) is different from that of the correlations due to transverse radial expansion (continuous increase of $N_{\text{coll}} \cdot \langle \cos(2\Delta\phi) \rangle$).

Thermalization time. Diffusion. The magnitude of the correlations due to transverse expansion should be sensitive to the system thermalization time and the particle diffusion in the thermalized matter during the expansion; the azimuthal correlations would be the most interesting/useful for such a study. The dependence on the thermalization time comes from the following. Somewhat exaggerating, one can imagine that just after the collision, newly produced particles ('freed' partons) experience free streaming for some period of time before the thermalization happens. After that moment the particles become frozen into hydrodynamic type cell and an expansion starts with all the consequences discussed in the first part of this Letter. The effect of the free streaming phase would be a diffusion in the transverse plane of the particles created initially at the same position (of the typical hadronic size). Such a diffusion would lead to a broadening of the azimuthal correlations. Many current estimates give the thermalization time of the order of or smaller than 1 fm/c [3]. Such a short thermalization time would be difficult to observe, but in any case the corresponding measurements would be of great interest giving an independent limit to this important parameter.

Single jet tomography? We could go even further with our speculations. Jet tomography of nuclear collisions is a popular subject based on the jet quenching phenomena. For such study it would be very useful to find an observable that is correlated to the space point where the hard collision occurred. It seems that the correlations due to transverse expansion provide such a possibility. One has to correlate the jet (high p_t hadron) yield with mean transverse momentum of particles taken at different rapidity (but better at similar azimuthal angle). In the same NN collision where the high p_t particle is emitted, the soft particles are produced as well. Those soft particles experience the transverse 'push' corresponding to the spatial position in the transverse plane where the original NN collision happens to be. Then the mean transverse momentum of the associated particles would provide the information on how close to the center of the system the collision occurred.

Summary. Expansion of the dense system created at the initial stage of a high energy nucleus–nucleus collision leads to strong position-momentum correlations. As all particles from the same NN collision are produced at the same position in the transverse plane, they get a similar radial push during the expansion stage. It creates rapidity, azimuthal angle, and transverse momentum correlations. The correlations extend over wide rapidity range. The picture is not boost-invariant, as the initial geometry of the source and particle densities change with rapidity leading to a rapidity dependent radial expansion.

The above described picture of AA collisions has many interesting observable effects, only a few mentioned in this Letter. The picture become even richer if one looks at the identified particle correlations. Many questions require a detailed model study, but the approach opens a potentially very interesting possibility to address the initial conditions and the subsequent evolution of the system created in an AA collision.

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References

[1] STAR Collaboration, K.H. Ackermann, et al., Phys. Rev. Lett. 86 (2001) 402.
[2] STAR Collaboration, C. Adler, et al., Phys. Rev. Lett. 90 (2003) 082302.
[3] P.F. Kolb, U. Heinz, in: R.C. Hwa, X.N. Wang (Eds.), Quark Gluon Plasma 3, World Scientific, Singapore, 2004;
P.F. Kolb, U. Heinz, nucl-th/0305084.
[4] E. Schnedermann, J. Sollfrank, U. Heinz, Phys. Rev. C 48 (1993) 2462.
[5] U.A. Wiedemann, P. Scotto, U. Heinz, Phys. Rev. C 53 (1996) 918.
[6] R.J.M. Snellings, H. Sorge, S.A. Voloshin, F.Q. Wang, N. Xu, Phys. Rev. Lett. 84 (2000) 2803.
[7] S.A. Voloshin, V. Koch, H.G. Ritter, Phys. Rev. C 60 (1999) 024901.
[8] C. Pruneau, S. Gavin, S. Voloshin, Phys. Rev. C 66 (2002) 044904.
[9] S. Gavin, Phys. Rev. Lett. 92 (2004) 162301.
[10] K. Braune, et al., Phys. Lett. B 123 (1983) 467.
[11] J. Mitchell, J. Phys. G 30 (2004) S819;
H. Appelshäuser, J. Phys. G 30 (2004) S935.
[12] S.A. Bass, P. Danielewicz, S. Pratt, Phys. Rev. Lett. 85 (2000) 2689.
[13] S. Pratt, S. Cheng, Phys. Rev. C 68 (2003) 014907.
[14] STAR Collaboration, J. Adams, et al., Phys. Rev. Lett. 90 (2003) 172301.
[15] STAR Collaboration, J. Adams, et al., Phys. Rev. C 68 (2003) 044905.
[16] N. Borghini, P.M. Dinh, J.-Y. Ollitrault, A.M. Poskanzer, S.A. Voloshin, Phys. Rev. C 66 (2002) 014901.
[17] STAR Collaboration, C. Adler, et al., Phys. Rev. C 66 (2002) 034904.