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Quantum scale invariance, cosmological constant and hierarchy problem

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ABSTRACT

We construct a class of theories which are scale-invariant on quantum level in all orders of perturbation theory. In a subclass of these models scale invariance is spontaneously broken, leading to the existence of a massless dilaton. The applications of these results to the problem of stability of the electroweak scale against quantum corrections, to the cosmological constant problem and to dark energy are discussed.

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1. Introduction

If in any theory all dimensionfull parameters (generically denoted by M), including masses of elementary particles, Newton's gravitational constant, Λ_{QCD} and alike are rescaled by the same amount $M \rightarrow M\sigma$, this cannot be measured by any observation. Indeed, this change, supplemented by a dilatation of space–time coordinates $x^\mu \rightarrow \sigma x^\mu$ and an appropriate redefinition of the fields does not change the complete quantum effective action of the theory. However, the symmetry transformations in quantum field theory only act on fields and not on parameters of the Lagrangian. The realization of scale invariance happens to be a non-trivial problem. A classical field theory which does not contain any dimensionfull parameters is invariant under the substitution

$$\Phi(x) \rightarrow \sigma^n \Phi(\sigma x), \quad (1)$$

where n is the canonical mass dimension of the field Φ . This dilatational symmetry turns out to be anomalous on quantum level for all realistic renormalizable quantum field theories (for a review see [1]). The divergence of the dilatation current J_μ is non-zero

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and is proportional to the β -functions of the couplings. For example, in pure gluodynamics, scale-invariant on the classical level, one has

$$\partial_\mu J^\mu \propto \beta(g) G_{\alpha\beta}^a G^{\alpha\beta a}, \quad (2)$$

where $G_{\alpha\beta}^a$ is the non-Abelian gauge field strength.

At the same time, it is very tempting to have a theory which is scale-invariant (SI) on the quantum level, as this would solve a number of puzzles in high energy physics. Most notably, these problems include two tremendous fine-tunings, facing the Standard Model (SM). The first one is related to the stability of the Higgs mass against radiative corrections and the second one to the cosmological constant problem. If the full quantum theory, including gravity, is indeed scale-invariant, and SI is broken spontaneously, the Higgs mass is protected from radiative corrections by an exact dilatational symmetry.

Moreover, as we have shown in [2], the classical theory with SI broken spontaneously and given by the action (we omit from the Lagrangian of [2] all degrees of freedom which are irrelevant for the present discussion and keep only the gravity part, the Higgs field h and the dilaton χ):

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_G + \mathcal{L}, \quad (3)$$

where

$$\mathcal{L}_G = -(\xi_\chi \chi^2 + \xi_h h^2) \frac{R}{2}, \quad (4)$$

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu \chi)^2 + (\partial_\mu h)^2] - \lambda(h^2 - \zeta^2 \chi^2)^2, \quad (5)$$

not only has zero cosmological constant but also gives a source for dynamical dark energy, provided that gravity is unimodular, i.e. the determinant of the metric is fixed to be -1 . (Here R is the scalar curvature and ξ_χ , ξ_h , λ and ζ are dimensionless coupling constants.) In this theory all mass parameters (on the tree level) come from one and the same source – the vacuum expectation value of the dilaton field $\langle \chi \rangle = \chi_0$, which is exactly massless. In addition, the primordial inflation is a natural consequence of (3), with a Higgs field playing the role of the inflaton [3].

It looks like all these findings are ruined by quantum corrections. The aim of this Letter is to show that this is not the case. We will construct a class of effective field theories, which obey the following properties:

- (i) Scale invariance is preserved on quantum level in all orders of perturbation theory.
- (ii) Scale invariance is broken spontaneously, leading to a massless dilaton.
- (iii) The effective running of coupling constants is automatically reproduced at low energies.

In other words, the benefits of classical SI theories (no corrections to the Higgs mass, zero cosmological constant, presence of dark energy and primordial inflation) can all be present on the quantum level. At the same time, the standard results of quantum field theory, such as the running of coupling constants, remain in place. Whether the theories we construct are renormalizable¹ and unitary is not known to us (though we will formulate some conjectures on this point). However, the renormalizability is not essential for the validity of the results.

The Letter is organized as follows. In Section 2 we explain our main idea with the use of a simple model of two scalar fields. In Section 3 we describe its generalization to an arbitrary case. In Section 4 we discuss the inclusion of gravity and present our conclusions in Section 5.

2. Scalar field example

We will explain our idea using the example of a simple system containing two scalar fields and described in classical theory by the Lagrangian (5) without gravity. The construction is essentially perturbative and based on the dimensional regularization of 't Hooft and Veltman [4] (for a discussion of the hierarchy problem within this scheme see, e.g. [5]).

At the classical level the theory (5) is scale-invariant. In fact, the requirement of dilatational symmetry does not forbid the presence of an additional term $\beta \chi^4$ in (5). If $\beta < 0$, the theory does not have a stable ground state, for $\beta > 0$ the ground state is unique and corresponds to $h = \chi = 0$. At the classical level one would conclude that the theory contains two scalar massless excitations for the ground state respecting scale-invariance. For the case $\beta = 0$ the potential contains two flat directions $h = \pm \zeta \chi$ and the vacuum is degenerate. If $\chi = \chi_0 \neq 0$, the dilatational invariance is spontaneously broken. Then the theory contains a massive Higgs boson, $m_H^2(\chi_0) = 2\lambda\zeta^2(1 + \zeta^2)\chi_0^2$ and a massless dilaton. So, the only choice for β , interesting for phenomenology, is $\beta = 0$, otherwise the vacuum does not exist or the theory does not contain any massive particles². In what follows we will also

assume that $\zeta \ll 1$, which is true for phenomenological applications: $\chi_0 \sim M_P = 2.44 \times 10^{18}$ GeV is related to the Planck scale, and $h_0 = \zeta \chi_0 \sim M_W \sim 100$ GeV to the electroweak scale. However, the smallness of ζ is not essential for the theoretical construction.

It is well known what happens in this theory if the *standard* renormalization procedure is applied. In d -dimensional space–time (we use the convention $d = 4 - 2\epsilon$) the mass dimension of the scalar fields is $1 - \epsilon$, and that of the coupling constant λ is 2ϵ . Introducing a (finite) dimensionless coupling λ_R , one can write

$$\lambda = \mu^{2\epsilon} \left[\lambda_R + \sum_{n=1}^{\infty} \frac{a_n}{\epsilon^n} \right], \quad (6)$$

where μ is a dimensionfull parameter and the Laurent series in ϵ corresponds to counter-terms. The parameters a_n are to be fixed by the requirement that renormalized Green's functions are finite in every order of perturbation theory. Similar replacements are to be done with other parameters of the theory, and the factors Z_χ , Z_h , related to the renormalization of fields must be introduced (they do not appear at one-loop level in our scalar theory). Then, in the $\overline{\text{MS}}$ subtraction scheme, the one-loop effective potential along the flat direction has the form

$$V_1(\chi) = \frac{m_H^4(\chi)}{64\pi^2} \left[\log \frac{m_H^2(\chi)}{\mu^2} - \frac{3}{2} \right], \quad (7)$$

spoiling its degeneracy, and leading thus to explicit breaking of the dilatational symmetry. The vacuum expectation value of the field χ can be fixed by renormalization conditions [6]. The dilaton acquires a nonzero mass. It is the mismatch in mass dimensions of bare (λ) and renormalized couplings (λ_R) which leads to the dilatational anomaly and thus to explicit breaking of scale invariance (see [7] for a recent discussion).

Let us now use another prescription, which we will call the “SI prescription”.³ Replace $\mu^{2\epsilon}$ in (6) and in all other similar relations by (different, in general) combinations of fields χ and h , which have the correct mass dimension:

$$\mu^{2\epsilon} \rightarrow \chi^{\frac{2\epsilon}{1-\epsilon}} F_\epsilon(x), \quad (8)$$

where $x = h/\chi$ and $F_\epsilon(x)$ is a function depending on the parameter ϵ with the property $F_0(x) = 1$. In principle, one can use different functions $F_\epsilon(x)$ for the various couplings. The resulting field theory, by construction, is scale-invariant for any number of space–time dimensions d . This means, that if for instance the $\overline{\text{MS}}$ subtraction scheme is used for calculations, the renormalized theory is also scale-invariant in any order of perturbation theory.

The requirement of scale invariance itself does not fix the details of the prescription. However, the form of the couplings of the scalar fields χ and h to gravity as in Eq. (4) indicates that the combination

$$\xi_\chi \chi^2 + \xi_h h^2 \equiv \omega^2 \quad (9)$$

plays a special role, being the effective Planck constant. Therefore, we arrive to a simple “GR–SI prescription”, in which

$$\mu^{2\epsilon} \rightarrow [\omega^2]^{\frac{\epsilon}{1-\epsilon}}, \quad (10)$$

corresponding to the choice of the function $F_\epsilon(x) = (\xi_\chi + \xi_h x^2)^{\frac{\epsilon}{1-\epsilon}}$. We will apply the GR–SI prescription to the one-loop analysis of our scalar theory below. In Appendix A we will consider a modified variant of the procedure.

The SI construction is entirely perturbative and can in fact be used *only* if SI is spontaneously broken. In other words, in order to

¹ The precise sense of this word in the present context will be specified later.

² More discussion of $\beta > 0$ case will be given at the end of this section.

³ A similar procedure was suggested in [8] in connection with the conformal anomaly. We thank Thomas Hertog who pointed out this reference to us after our work has been submitted to hep-th.

use the GR–SI prescription the ground state has to be $(h_0, \chi_0) \neq (0, 0)$, because otherwise it is impossible to perform an expansion of (10). Indeed, consider the exact effective potential $V_{\text{eff}}(h, \chi)$ of our theory, constructed using the prescription (8) or (10) in the limit $\epsilon \rightarrow 0$. Because of exact SI, it can be written as

$$V_{\text{eff}}(h, \chi) = \chi^4 V_\chi(x) = h^4 V_h(x). \quad (11)$$

For the ground state to exist, we must have $V_\chi(x) \geq 0$ (or, what is the same, $V_h(x) \geq 0$) for all x . For the minimum of $V_{\text{eff}}(h, \chi)$ to lie in the region where $\chi \neq 0$ (or $h \neq 0$), we must have $V_\chi(x_0) = 0$ ($V_h(x_0) = 0$), where x_0 is a solution of $V'_\chi(x_0) = 0$ ($V'_h(x_0) = 0$) and prime denotes the derivative with respect to x . If these conditions are satisfied, the theory has an infinite set of ground states corresponding to the spontaneous breakdown of dilatational invariance. The dilaton is massless in all orders of perturbation theory. In this case one can develop the perturbation theory around the vacuum state corresponding to $\chi_0 \neq 0$, $h_0 = x_0 \chi_0$ with arbitrary χ_0 (or $h_0 \neq 0$, $\chi_0 = h/x_0$ with arbitrary h_0).

To summarize: the use of prescriptions (8) or (10) supplemented by the requirement $V_{\chi,h}(x_0) = 0$ leads to a new class of theories exhibiting spontaneously broken scale invariance, which is exact on quantum level. These theories can be called renormalizable if the introduction of a finite number of counter-terms is sufficient to remove all divergences and guarantee the existence of a flat direction in the potential. The check whether this is indeed the case goes beyond the scope of the present Letter. In principle, we cannot exclude the possibility that, in order to remove all divergences, a new type of counter-terms containing non-polynomial interactions (such as h^6/χ^2) is required. But, even if this is the case, scale invariance is maintained in all orders of perturbation theory and can be spontaneously broken. Another potential issue is unitarity. We do not know whether higher derivative terms in the effective action, dangerous from this point of view, would require the introduction of corresponding counter-terms. However, the functional arbitrariness in the choice of $F_\epsilon(x)$ for potential and kinetic terms may give enough freedom to remove the unwanted contributions.

The theories we construct are quite different from ordinary renormalizable theories. Their physics is determined not only by the values of “classical” coupling constants (λ and ζ in our case), but also by “hidden” parameters contained in the functions $F_\epsilon(x)$. Still, as we will see shortly, for the SI–GR prescription, in the limit $\zeta \ll 1$ and for small energies $E \ll \chi_0$, only “classical” parameters matter. Moreover, they automatically acquire the necessary renormalization group running.

To this end, we carry out a one-loop analysis of the theory (5) with the GR–SI prescription. We write the d -dimensional generalization of the classical potential as⁴

$$U = \frac{\lambda_R}{4} [\omega^2]^{\frac{\epsilon}{1-\epsilon}} [h^2 - \zeta_R^2 \chi^2]^2, \quad (12)$$

and introduce the counter-terms

$$U_{cc} = [\omega^2]^{\frac{\epsilon}{1-\epsilon}} \left[Ah^2 \chi^2 \left(\frac{1}{\epsilon} + a \right) + B \chi^4 \left(\frac{1}{\epsilon} + b \right) + C h^4 \left(\frac{1}{\epsilon} + c \right) \right], \quad (13)$$

where $\frac{1}{\epsilon} = \frac{1}{d-4} - \gamma + \log(4\pi)$, γ is the Euler constant and a, b, c, A, B , and C are arbitrary for the moment. We do not introduce any modification of the kinetic terms since no wave function renormalization is expected at the one loop level.

⁴ If we define the parameters $\alpha \equiv \sqrt{\lambda}$ and $\beta \equiv \sqrt{\lambda} \zeta^2$, the classical potential takes the form $U = \frac{1}{4} (\alpha h^2 - \beta \chi^2)^2$. In this notation the GR–SI prescription corresponds to the substitutions $\alpha \rightarrow [\omega^2]^{\frac{\epsilon}{2(1-\epsilon)}} \alpha_R$ and $\beta \rightarrow [\omega^2]^{\frac{\epsilon}{2(1-\epsilon)}} \beta_R$.

It is straightforward to find the one-loop effective potential for this theory. The counter-terms removing the divergences coincide with those of the standard prescription and are given by:

$$\begin{aligned} A &\rightarrow -\lambda_R^2 \zeta_R^2 \frac{3\zeta_R^4 - 4\zeta_R^2 + 3}{32\pi^2}, \\ B &\rightarrow \lambda_R^2 \zeta_R^4 \frac{9\zeta_R^4 + 1}{64\pi^2}, \quad C \rightarrow \lambda_R^2 \frac{\zeta_R^4 + 9}{64\pi^2}. \end{aligned} \quad (14)$$

The potential itself has a generic form $U_1 = \chi^4 W_1(x)$ and is given by a rather lengthy expression (we do not present it here, since it is not very illuminating), which also depends on the “hidden” parameters. For a generic choice of a, b , and c the classical flat direction $x_0 = \zeta_R$ is lifted by quantum effects. However, the requirement $W_1(\zeta_R) = W'_1(\zeta_R) = 0$ allows to fix two of these parameters in a way such that the one-loop potential has exactly the same flat direction. For $\zeta_R \ll 1$ this requirement leads to⁵

$$\begin{aligned} b &= 3a + 2 \log \left(\frac{2\lambda_R \zeta_R^2}{\xi_\chi} \right) + \mathcal{O}(\zeta_R^2), \\ c &= \frac{1}{3} \left[a + 2 - 2 \log \left(\frac{2\lambda_R \zeta_R^2}{\xi_\chi} \right) \right] + \mathcal{O}(\zeta_R^2). \end{aligned} \quad (15)$$

The function $W_1(x)$ is positive near the flat direction, provided $a + 2 + 2 \log(2\lambda_R \zeta_R^2 / \xi_\chi) > 0$.

It is interesting to look at the one-loop effective potential as a function of h for $\chi = \chi_0$, $h \sim \zeta_R \chi_0 \equiv v$ and $\zeta \ll 1$, i.e. $h_0 \ll \chi_0$. One finds

$$\begin{aligned} U_1 &= \frac{m^4(h)}{64\pi^2} \left[\log \frac{m^2(h)}{v^2} + \mathcal{O}(\zeta_R^2) \right] \\ &\quad + \frac{\lambda_R^2}{64\pi^2} [C_0 v^4 + C_2 v^2 h^2 + C_4 h^4] + \mathcal{O}\left(\frac{h^6}{\chi^2}\right), \end{aligned} \quad (16)$$

where $m^2(h) = \lambda_R(3h^2 - v^2)$ and

$$\begin{aligned} C_0 &= \frac{3}{2} \left[2a - 1 + 2 \log \left(\frac{\zeta_R^2}{\xi_\chi} \right) + \frac{4}{3} \log 2\lambda_R + \mathcal{O}(\zeta_R^2) \right], \\ C_2 &= -3 \left[2a - 3 + 2 \log \left(\frac{\zeta_R^2}{\xi_\chi} \right) + \mathcal{O}(\zeta_R^2) \right], \\ C_4 &= \frac{3}{2} \left[2a - 5 + 2 \log \left(\frac{\zeta_R^2}{\xi_\chi} \right) - 4 \log 2\lambda_R + \mathcal{O}(\zeta_R^2) \right]. \end{aligned} \quad (17)$$

The first term in (16) is exactly the standard effective potential for the theory (5) with the dynamical field χ replaced by a constant χ_0 , while the rest is a quartic polynomial of h and comes from our GR–SI prescription, leading to redefinition of coupling constants, masses, and the vacuum energy.

One can see from (16) that the quantum corrections to the Higgs mass are proportional to $v^2 \propto \zeta_R^2 \chi^2$. This means that they are small compared to the classical value. Moreover, the potentially dangerous corrections of the type $\lambda^n \chi_0^2$ to the Higgs mass cannot appear in higher orders of perturbation theory. Indeed, for $\zeta = 0$ the Higgs field decouples from the dilaton at the classical level and the dilaton field is described by a free theory. Therefore, if $\zeta = 0$ the value of the (large) field χ can appear only through log’s in the effective potential, coming from the expansion of $[\omega^2]^{\epsilon/1-\epsilon}$ in Eq. (12), or at most as $\zeta_R^2 \chi^2$ if $\zeta \neq 0$. Hence, in this theory there is no problem of instability of the Higgs mass against quantum corrections, appearing in the Standard Model.

⁵ The truncation only serves to shorten the expressions. There is no difficulty in finding the exact relations.

Consider now the high energy ($\sqrt{s} \gg v$ but $\sqrt{s} \ll \chi_0$) behavior of scattering amplitudes with the example of Higgs–Higgs scattering (assuming, as usual, that $\zeta_R \ll 1$). It is easy to see that in one-loop approximation one gets for the 4-point function

$$\Gamma_4 = \lambda_R + \frac{9\lambda_R^2}{64\pi^2} \left[\log\left(\frac{s}{\xi_\chi \chi_0^2}\right) + \text{const} \right] + \mathcal{O}(\zeta_R^2). \quad (18)$$

This implies that at $v \ll \sqrt{s} \ll \chi_0$ the effective Higgs self-coupling runs in a way prescribed by the ordinary renormalization group. Not only the tree Higgs mass is determined by the vev of the dilaton, but also all Λ_{QCD} -like parameters. We expect that these results remain valid in higher orders of perturbation theory.

Let us comment now on the case when the flat direction does not exist at the quantum level (classically this corresponds to $\beta > 0$). Then the ground state of the theory is scale-invariant. Theories of this type do not in general contain asymptotic particle states (for a review see, e.g. [10]). If they do (this would correspond to anomalous dimensions for the fields equal to zero), the propagators will coincide with the free ones, leading to a theory with a trivial S-matrix [11,12]. In other words, the requirement that the scale-invariant quantum field theory can be used for the description of interacting particles, existing as asymptotic states, singles out the class of theories with spontaneous breaking of scale-invariance.

3. Scale-invariant quantum field theory: General formulation

It is straightforward to generalize the construction presented above to the case of theories containing fermions and gauge fields, such as the Standard Model. The mass dimension of a fermionic field is $\frac{3}{2} - \epsilon$, leading to the dimension of bare Yukawa couplings F_B equal to ϵ . The mass dimension of the gauge field can be fixed to 1 for any number of space–time dimensions d , leading to the dimensionality of the bare gauge coupling g_B equal to ϵ . So, in the standard procedure one chooses $F_B \propto \mu^\epsilon F_R$, $g_B \propto \mu^\epsilon g_R$, where the index R refers to renormalized couplings. For the SI or GR–SI prescription one replaces μ^ϵ by a combination of scalar fields of appropriate dimension, as in (8) or in (10). For the perturbation theory to make sense, one has to choose counter-terms in such a way that the full effective potential has a flat direction corresponding to spontaneously broken dilatational invariance.

4. Inclusion of gravity

The inclusion of scale-invariant gravity is carried out precisely along the same lines. The metric tensor $g_{\mu\nu}$ is dimensionless for any number of space–time dimensions and R always has mass dimension 2. Therefore, the non-minimal couplings ξ_χ , ξ_h (see Eq. (4)) are dimensionless and thus can only be multiplied by functions $F_\epsilon(x)$ of the type defined in (8). In addition to (4), the gravitational action may contain the operators R^2 , $R_{\mu\nu}R^{\mu\nu}$, $\square R$ and $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, multiplied by $\chi^{\frac{-2\epsilon}{1-\epsilon}} F_\epsilon(x)$ (here $R_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}$ are the Ricci and Riemann curvature tensors). These operators are actually needed for renormalization of field theory in curved space–time (for a review see [9]).

The presence of gravity is crucial for phenomenological applications. Since Newton’s constant is dynamically generated, the dilaton decouples from the particles of the Standard Model [2,13–16], and thus satisfies all laboratory and astrophysical constraints. As we found in [2], if gravity is unimodular, the absence of a cosmological constant and the existence of dynamical dark energy are automatic consequences of the theory. It is interesting to note that the action of unimodular gravity is polynomial with respect to the

metric tensor. This leads us to the conjecture that the SI unimodular gravity with matter fields may happen to be a renormalizable theory in the sense described in Section 2.

5. Conclusions

In this Letter we constructed a class of theories, which are scale-invariant on the quantum level. If dilatational symmetry is spontaneously broken, all mass scales in these models are generated simultaneously and originate from one and the same source. In these theories the effective cutoff scale depends on the background dilaton field, as was already proposed in [16], which is essential for inflation [3] and dark energy [2]. The cosmological constant is absent and the mass of the Higgs boson is protected from large radiative corrections by the dilatational symmetry. Dynamical dark energy is a remnant of initial conditions in unimodular gravity.

There are still many questions to be understood. Here is a partial list of them. Our construction is essentially perturbative. How to make it work non-perturbatively?⁶ Though the stability of the electroweak scale against quantum corrections is achieved, it is absolutely unclear why the electroweak scale is so much smaller than the Planck scale (or why $\zeta \ll 1$). It remains to be seen if this new class of theories is renormalizable and unitary (note, though, that renormalizability is not essential for the construction). At large momentum transfer $p \gtrsim M_P$ the perturbation theory diverges and thus is not applicable. What is the high energy limit of these theories?

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Appendix A

For the GR–SI prescription considered in the Letter physics well below the Planck scale associated with the dilaton vev χ_0 was the same as for the ordinary renormalizable scalar theory containing the Higgs field h only. This is not necessarily the case if the SI prescription given by Eq. (8) is used. Indeed, consider now a distinct way of continuing the scalar potential to d -dimensional space–time:⁷

$$U = \frac{\lambda_R}{4} \left[h^{\frac{2-\epsilon}{1-\epsilon}} x^{a_1\epsilon} - \zeta_R^2 \chi^{\frac{2-\epsilon}{1-\epsilon}} x^{b_1\epsilon} \right]^2, \quad (19)$$

and introduce counter-terms for all terms appearing in the potential:

$$U_{cc} = \left[A \left(\frac{1}{\epsilon} + a \right) h^{\frac{2-\epsilon}{1-\epsilon}} \chi^{\frac{2-\epsilon}{1-\epsilon}} x^{(a_1+b_1)\epsilon} + B \left(\frac{1}{\epsilon} + b \right) \chi^{\frac{4-2\epsilon}{1-\epsilon}} x^{2b_1\epsilon} + C \left(\frac{1}{\epsilon} + c \right) h^{\frac{4-2\epsilon}{1-\epsilon}} x^{2a_1\epsilon} \right]. \quad (20)$$

As before, we do not introduce any modification of the kinetic terms. Now we have more freedom in comparison with the GR–SI prescription due to existence of new arbitrary parameters a_1 and b_1 .

The coefficients A , B , and C are fixed as in Eq. (14). The parameters a_1 and b_1 can be chosen in such a way that the one-loop effective potential does not contain terms χ^6/h^2 and h^6/χ^2 , which are singular at $(0, 0)$. These conditions lead to $a_1 = 0$, $b_1 = 0$. Then

⁶ A proposal based on lattice regularization has been recently discussed in [17].

⁷ In the notation with $\alpha \equiv \sqrt{\lambda}$ and $\beta \equiv \sqrt{\lambda}\zeta^2$, the prescription used here corresponds to the substitutions $\alpha \rightarrow h^{\frac{2-\epsilon}{1-\epsilon}} x^{a_1\epsilon} \alpha_R$ and $\beta \rightarrow \chi^{\frac{2-\epsilon}{1-\epsilon}} x^{b_1\epsilon} \beta_R$.

the requirement that the classical flat direction $x_0 = \zeta$ is not lifted by quantum effects gives (for $\zeta \lll 1$):

$$\begin{aligned} b &= 3a - 7 + 2 \log(2\lambda_R) + \mathcal{O}(\zeta_R^2), \\ c &= \frac{1}{3}[a + 7 - 2 \log(2\lambda_R)] + \mathcal{O}(\zeta_R^2). \end{aligned} \quad (21)$$

With all these conditions satisfied the one-loop effective potential as a function of h for $\chi = \chi_0$ fixed, $h \sim \zeta \chi_0 = v$ and $\zeta \lll 1$ is different from that in Eq. (16):

$$U_1 = \frac{m^4(h)}{64\pi^2} \left[\log \frac{m^2(h)}{v^2} + \mathcal{O}(\zeta_R^2) \right] + P_1 \log \frac{h^2}{v^2} + P_2, \quad (22)$$

where P_1, P_2 are quadratic polynomials of h^2 and v^2 . Though the first term is exactly the standard effective potential for the theory (5) with the dynamical field χ replaced by a constant χ_0 , the rest is not simply a redefinition of the coupling constants of the theory due to the presence of $\log \frac{h^2}{v^2}$. In other words, even the low energy physics is modified in comparison with ordinary renormalizable theories.

References

- [1] S. Coleman, Dilatations, in: Aspects of Symmetry, Cambridge Univ. Press, 1985, p. 67.
- [2] M. Shaposhnikov, D. Zenhäusern, arXiv: 0809.3395 [hep-th].
- [3] F.L. Bezrukov, M. Shaposhnikov, Phys. Lett. B 659 (2008) 703.
- [4] G. 't Hooft, M.J.G. Veltman, Nucl. Phys. B 44 (1972) 189.
- [5] M. Shaposhnikov, arXiv: 0708.3550 [hep-th].
- [6] S.R. Coleman, E. Weinberg, Phys. Rev. D 7 (1973) 1888.
- [7] K.A. Meissner, H. Nicolai, Phys. Lett. B 648 (2007) 312; K.A. Meissner, H. Nicolai, Phys. Lett. B 660 (2008) 260.
- [8] F. Englert, C. Truffin, R. Gastmans, Nucl. Phys. B 117 (1976) 407.
- [9] I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, Effective Action in Quantum Gravity, IOP, Bristol, 1992, 413 p.
- [10] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri, Y. Oz, Phys. Rep. 323 (2000) 183.
- [11] O.W. Greenberg, Ann. Phys. 16 (1961) 158.
- [12] N.N. Bogolubov, A.A. Logunov, I.T. Todorov, Introduction to Axiomatic Quantum Field Theory, W.A. Benjamin, New York, 1975.
- [13] J.J. van der Bij, Acta Phys. Pol. B 25 (1994) 827.
- [14] J.L. Cervantes-Cota, H. Dehnen, Nucl. Phys. B 442 (1995) 391.
- [15] C. Wetterich, Nucl. Phys. B 302 (1988) 645.
- [16] C. Wetterich, Nucl. Phys. B 302 (1988) 668.
- [17] M.E. Shaposhnikov, I.I. Tkachev, arXiv: 0811.1967 [hep-th].