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Strong decays and dipion transitions of $\Upsilon(5S)$

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ABSTRACT

Dipion transitions of $\Upsilon(nS)$ with $n=5$, $n'=1,2,3$ are studied using the Field Correlator Method, applied previously to dipion transitions with $n=2,3,4$. The only two parameters of effective Lagrangian were fixed in that earlier study, and total widths $\Gamma_{\pi\pi}(5,n')$ as well as pionless decay widths $\Gamma_{BB}(5S)$, $\Gamma_{BB^*}(5S)$, $\Gamma_{B^*B^*}(5S)$ and $\Gamma_{KK}(5,n')$ were calculated and are in a reasonable agreement with experiment. The experimental $\pi\pi$ spectra for $(5,1)$ and $(5,2)$ transitions are well reproduced taking into account FSI in the $\pi\pi$.

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1. Introduction

In a recent series of papers [1–4] we have studied the (n,n') bottomonium dipion transitions $\Upsilon(n)\to\Upsilon(n')\pi\pi$ and decays $\Upsilon(n)\to B\bar{B}, B\bar{B}\pi$ using effective Lagrangian derived in the framework of the Field Correlator Method (FCM) [5]. This Lagrangian, as was understood in [3], contains two effective masses, playing the role of decay vertices, M_ω for pionless $q\bar{q}$ pair creation, and M_{br} for $q\bar{q}$ accompanied by one or two pions (kaons). It was found that M_ω is responsible for pionless decays of the type $\Upsilon(n)\to BB, BB^*, B^*B^*$, while M_{br} enters into pionic decay transitions $\Upsilon(n)\to BB\pi$. These are the only free parameters of the method. It was shown in [4], that both pionless and dipion transition widths are reasonably well described by the method for $n=4,3,2$ and $n'=1,2,3$ when theoretically sound values $M_\omega\sim\omega\approx 0.58$ GeV (average light quark energy in B) and $M_{br}\sim f_\pi\approx 93$ MeV were used.

The results of [1–3] allowed to describe the $\pi\pi$ spectrum in dipion (n,n') transitions, for $n=2,3$ in [1,2] and $n=2,3,4$ and $n'=1,2$ in [3]. It was stressed in [1–3], that the structure of the (n,n') transition with BB, BB^*, B^*B^* intermediate states contains two types of amplitudes: "a" for consecutive one-pion emission and "b" for zero-pion–two-pion emission, and the Adler Zero Requirement (AZR) establishes connection between "a" and "b". In this way the long-standing problem of the theoretical description of all (n,n') transition spectra, found in experiment [6–8] was approximately resolved. One should stress, however, that all (n,n') dipion transitions in [1–3] with $n\leqslant 4$ refer to the subthreshold case, for $n=4$ the BB threshold is only 20 MeV below the $\Upsilon(4S)$ mass. For $\Upsilon(5S)$ the situation is different: all three channels BB, BB^*, B^*B^* and three others with B_s mesons are open and the corresponding imaginary parts are large due to large accessible energy. The final state $\pi\pi$ interaction is operative for the open channel amplitudes and one should calculate explicitly all terms in the amplitude, while AZR sets limits on the soft part of spectrum.

The decays and transitions of $\Upsilon(5S)$ are a good check of our method, since no new parameters are involved, and the $5S$ realistic wave function was accurately calculated [9]. At the same time the new experimental data on $5S$ decays [10] present several questions for the theory:

(1) The dipion widths $\Gamma_{\pi\pi}(5,1)$, $\Gamma_{\pi\pi}(5,2)$, $\Gamma_{\pi\pi}(5,3)$ are ~ 1000 times larger than the corresponding widths for $\Gamma_{\pi\pi}(nn')$ with $n=2,3,4$.
(2) The hierarchy of the widths $\Gamma_{BB}(5S)<\Gamma_{BB^*}(5S)<\Gamma_{B^*B^*}(5S)$ occurs in experiment with $\Gamma_{\text{tot}}(5S)\sim 0(100\text{ MeV})$.
(3) Dikaon width of $\Upsilon(5S)$ is $\sim 1/10$ of the dipion width.
(4) The dipion spectra in $(5,1)$, $(5,2)$ transitions are not similar to spectra found for $n=2,3,4$, showing a possible role of $\pi\pi$ FSI.

It is a purpose of the present Letter to study the $\Upsilon(5S)$ decays and transitions using the same method as in [1–3] without introducing any new parameters. We shall give quantitative answers to questions (1)–(4), finding a reasonable order of magnitude agreement for all observables, however also a strong sensitivity to the properties of the $5S$ wave function. The Letter is organized as follows. In Section 2 general equations of the method from [1–3] are written for the case of $\Upsilon(5S)$. In Section 3 pionless decay widths are computed and compared to experiment, whereas in Section 4 total dipion and dikaon widths are discussed. The dipion spectra with and without $\pi\pi$ FSI factors are given in Section 5. Main re-

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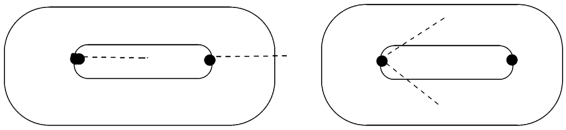

(a) (b)

Fig. 1. (a) Subsequent one-pion emission. (b) Two-pion emission.

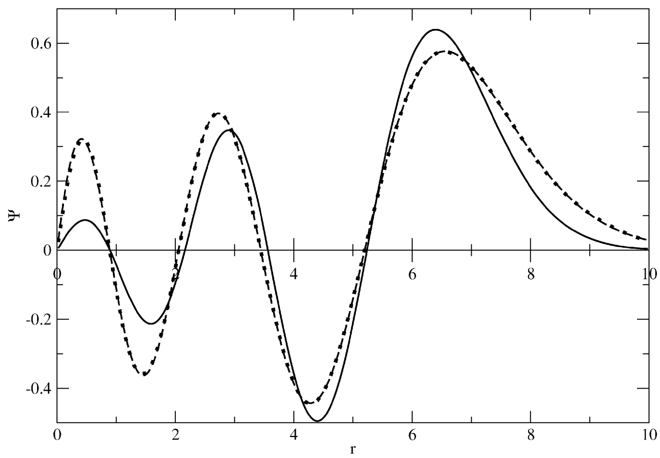

Fig. 2. Realistic w.f. of $\Upsilon(5S)$ (broken line), the series of oscillator functions with $k_{\max}=15$ (dotted line), $k_{\max}=5$ (solid). Note that the dotted curve is almost indistinguishable from the broken one.

sults are discussed in the concluding section together with a short summary and perspective.

2. General formalism for $\Upsilon(5S)$ decays and transitions

The amplitude of the dipion transition (n,m) with pion momenta $\mathbf{k}_1,\mathbf{k}_2$ can be written according to [3] as a sum of two terms, see Fig. 1(a), (b).

$$
\begin{aligned}
&w^{(\pi\pi)}_{nm}(E)\\
&\quad\equiv a-b\\
&\quad=\frac{1}{N_c}\left\{\sum_{n_2n_3}\int\frac{d^3p}{(2\pi)^3}\frac{J^{(1)}_{nn_2n_3}(\mathbf{p},\mathbf{k}_1)J^{*(1)}_{mn_2n_3}(\mathbf{p},\mathbf{k}_2)}{E-E_{n_2n_3}(\mathbf{p})-E_\pi(\mathbf{k}_1)}+(\mathbf{k}_1\leftrightarrow\mathbf{k}_2)\right.\\
&\qquad-\sum_{n_2'n_3'}\int\frac{d^3p}{(2\pi)^3}\frac{J^{(2)}_{nn_2'n_3'}(\mathbf{p},\mathbf{k}_1,\mathbf{k}_2)J^{*(0)}_{mn_2'n_3'}(\mathbf{p})}{E-E_{n_2'n_3'}(\mathbf{p})-E(\mathbf{k}_1,\mathbf{k}_2)}\\
&\qquad\left.-\sum_{n_2''n_3''}\int\frac{d^3p}{(2\pi)^3}\frac{J^{(0)}_{nn_2''n_3''}(\mathbf{p})J^{(2)*}_{mn_2''n_3''}(\mathbf{p},\mathbf{k}_1,\mathbf{k}_2)}{E-E_{n_2''n_3''}(\mathbf{p})}\right\},
\end{aligned}\tag{1}
$$

where $J^{(1)}(\mathbf{p},\mathbf{k})$, $J^{(2)}(\mathbf{p},\mathbf{k}_1,\mathbf{k}_2)$ are the overlap matrix elements between wave functions $\Psi(\mathbf{q})$ of $\Upsilon(5S)$ and $\varphi(\mathbf{q}_1)\varphi(\mathbf{q}_2)$ of $B(B^*)$ mesons.

It is convenient to approximate $\Psi(\mathbf{q}),\varphi(\mathbf{q})$ by a series of oscillator wave functions; indeed in Fig. 2 we show the quality of fitting of $\Psi(r)$ by series of 5 and 15 terms. In this case the dependence on $\mathbf{k}_1,\mathbf{k}_2$ as shown below simplifies. For the pionless overlap matrix element one can write

$$
\begin{aligned}
J^{(0)}_{n,11}(\mathbf{p})&=\int\frac{d^3\mathbf{q}}{(2\pi)^3}\bar y_{123}(p,q)\sum_{k=1}^{N_{\max}}c^{(n)}_k\varphi_k(\beta_1,\mathbf{q}+c\mathbf{p})\varphi_1^2(\beta_2,\mathbf{q})\\
&=\frac{ip_i}{\omega}e^{-\frac{\mathbf{p}^2}{\Delta}}\,{}^{(1)}I_{n,11}(\mathbf{p}).
\end{aligned}\tag{2}
$$

Here $c\approx1$, $c^{(n)}_k$ are χ^2 fitting coefficients and φ_k – oscillator functions for $\Psi(q)$ and φ_1 – for B,B^* mesons, and β_1,β_2 are oscillator parameters for $\Upsilon(5S)$ and B,B^* found from fitting. The factor $\bar y_{123}$ defined in [3] takes into account the Dirac trace structure of the overlap vertex.

In a similar way one can define $J^{(1)}_n$, $J^{(2)}_n$ for one- and two-pion emission integrals ($\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2$)

$$
J^{(1)}_{n,11}(\mathbf{p},\mathbf{k})=e^{-\frac{\mathbf{p}^2}{\Delta}-\frac{\mathbf{K}^2}{4\beta_2^2}}I_{n,11}(\mathbf{p})\bar y^{(\pi)}_{123},\tag{3}
$$

$$
J^{(2)}_{n,11}(\mathbf{p},\mathbf{k}_1,\mathbf{k}_2)=e^{-\frac{\mathbf{p}^2}{\Delta}-\frac{\mathbf{K}^2}{4\beta_2^2}}\,{}^{(1)}I_{n,11}(\mathbf{p})\bar y^{(\pi\pi)}_{123}p_i.\tag{4}
$$

Here $\bar y^{(\pi)}_{123}$, $\bar y^{(\pi\pi)}_{123}$ are defined by the Dirac traces of the amplitudes and are given in [3]. As a result, the total amplitude is written as

$$
\begin{aligned}
\mathcal{M}&=\exp\left(-\frac{\mathbf{k}_1^2+\mathbf{k}_2^2}{4\beta_2^2}\right)\left(\frac{M_{br}}{f_\pi}\right)^2\mathcal{M}_1\\
&\quad-\exp\left(-\frac{\mathbf{K}^2}{4\beta_2^2}\right)\frac{M_{br}M_\omega}{f_\pi^2}\mathcal{M}_2.
\end{aligned}\tag{5}
$$

Here $\mathcal{M}_1\sim a,\mathcal{M}_2\sim b$, explicit expressions for $\mathcal{M}_1$, $\mathcal{M}_2$ in terms of the integrals of overlap matrix elements $J^{(1)}$, $J^{(2)}$, $J^{(0)}$, as in (1), are given in [3], and here we only quote results of numerical computations of $\mathcal{M}_1,\mathcal{M}_2$ for $(5,1)$, $(5,2)$ and $(5,3)$ transitions. As will be seen, both $\mathcal{M}_1$ and $\mathcal{M}_2$ do not depend strongly on $\cos\theta$ and x, so that the main dependence of $\mathcal{M}(x,\cos\theta)$ on arguments comes from two exponential factors in (5) (some exclusion is imaginary part of $\mathcal{M}_1$, which is peaked near $|\cos\theta|=1$).

The differential probability of dipion transition is given by

$$
\frac{dw_{\pi\pi}(n,n')}{dq\,d\cos\theta}=C_0\mu^2\sqrt{x(1-x)}|\mathcal{M}|^2,\tag{6}
$$

where we introduced variables $q\equiv M_{\pi\pi}$, $q^2=(\omega_\pi(k_1)+\omega_\pi(k_2))^2-(\mathbf{k}_1+\mathbf{k}_2)^2$, $x=\frac{q^2-4m_\pi^2}{\mu^2}$, $\mu^2\equiv(\Delta E)^2-4m_\pi^2$; and numerical factor $C_0=\frac{1}{32\pi^3N_c^2}=1.12\times10^{-4}$. Here $\Delta E\equiv M(\Upsilon(nS))-M(\Upsilon(n'S))$; explicit values of μ and ΔE for $(5,n')$ transitions are the following (in GeV); $\Delta E(5,1)=1.4$; $\mu(5,1)=1.37$; $\Delta E(5,2)=0.837$, $\mu(5,2)=0.788$; $\Delta E(5,3)=0.505$, $\mu(5,3)=0.418$. Finally the total dipion width is given by

$$
\Gamma_{\pi\pi}(n,n')=C_0\mu^3\int_0^1dx\sqrt{\frac{x(1-x)}{x+\frac{4m_\pi^2}{\mu^2}}}\int_{-1}^{+1}\left|\mathcal{M}(x,\cos\theta)\right|^2\frac{d\cos\theta}{2}.\tag{7}
$$

3. The B-meson decays of $\Upsilon(5S)$

In this section we study the pionless decays of $\Upsilon(5S)$, namely into $B\bar B$, $B\bar B^*+\text{c.c.}$, $B^*\bar B^*$, $B_s\bar B_s$, $B_s\bar B_s^*+\text{c.c.}$, $B_s^*\bar B_s^*$ to which we ascribe numbers $k=1,2,\dots,6$. The corresponding formula for the width was derived in [3], namely

$$
\Gamma\big(\Upsilon(nS)\to(B\bar B)^k\big)=\left(\frac{M_\omega}{2\omega}\right)^2\frac{p_k^3M_k}{6\pi N_c}(Z_k)^2\big|J^{BB}_n(p_k)\big|^2.\tag{8}
$$

M_k is twice the reduced mass in channel k. The corresponding coefficients Z_k account for spin and isospin multiplicities and (cf. similar coefficients in [11]) are as follows:

$$
Z_1^2=2Z_4^2=1,\qquad Z_2^2=2Z_5^2=4,\qquad Z_3^2=2Z_6^2=7.\tag{9}
$$

Here $J^{BB}_n(p_k)$ are overlap matrix elements

$$
\frac{p_i}{\omega}J^{BB}_n(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}(q_i-\bar c p_i)\Psi^*_n(\mathbf{p}+\mathbf{q})\varphi_B^2(\mathbf{q}),\tag{10}
$$

Table 1
The values of two-body decay widths Γ_k calculated with realistic $5S$ wave function.

k	1, $B\bar{B}$	2, $B\bar{B}^*$	3, $B^*\bar{B}^*$	4, $B_s\bar{B}_s$	5, $B_s\bar{B}_s^*$	6, $B_s^*\bar{B}_s^*$
p_k, GeV	1.26	1.16	1.05	0.835	0.683	0.482
M_k, GeV	5.28	5.30	5.32	5.37	5.39	5.41
Z_k	1	4	7	1/2	4/2	7/2
$\Gamma_k/(\frac{M_\omega}{2\omega})^2$ MeV	11	57	65	0.08	10	18

where $\bar{c} = \frac{\omega}{2(\omega+\Omega)}$, and ω, Ω are average energies of light and heavy quarks in B meson, computed in [12], $\omega \approx 0.587$ GeV, $\Omega = 4.827$ GeV, $\omega_s = 0.639$ GeV, $\Omega_s = 4.83$ GeV, see Table 4 in [1].

Expanding Ψ_n, φ_B in series of oscillator functions as in [3], one obtains the form $J_n^{BB}(\mathbf{p}) = e^{-\frac{\mathbf{p}^2}{\Delta}}\,{}^{(1)}I_{n11}(\mathbf{p})$, where ${}^{(1)}I_{n,11}(\mathbf{p})$ is a polynomial in p^2, $\Delta = 2\beta_1^2 + \beta_2^2$ and β_1, β_2 are oscillator parameters for $\Upsilon(nS)$ and B meson respectively, found from the χ^2 fitting procedure to the realistic wave function calculated in [9], and for $1S$, $2S$, $5S$ states and B meson one finds respectively $\beta_1(1S) = 1.08$ GeV, $\beta_1(2S) = 0.81$ GeV, $\beta_1(5S) = 0.75$ GeV, $\beta_2 = 0.48$ GeV.

Denoting $\Gamma_k \equiv \Gamma_{th}(\Upsilon(5S) \to channel(k))$, one has

$$\left(\frac{2\omega}{M_\omega}\right)^2 \Gamma_k = 0.0177 p_k^3 M_k Z_k^2 \left|J_5(p_k)\right|^2, \tag{11}$$

where $J_5(p) = {}^{(1)}I_{5,11}(p)e^{-\frac{p^2}{\Delta}}$, and ${}^{(1)}I_{5,11}$ is given in Eq. (2). Below in Table 1 the computed values of Γ_k for $k = 1,\dots,6$ and with $k_{\max} = 5$, i.e. five oscillator terms approximating wave function of $\Upsilon(5S)$ are given. Computing ${}^{(1)}I_{5,11}(p)$ for different number of oscillator terms $k_{\max}$, one can see, that values of $I_{5,11}(t)$, $t = \frac{p^2}{\beta_0^2}$, $\beta_0 \approx 0.886$ GeV, in the interval $0.2 \leqslant t \leqslant 2$ are sensitive to $k_{\max}$ and vary around the value $|I_{5,11}| \approx 1$ GeV$^{3/2}$. We choose this value to estimate the variation of Γ_k and find that for the dominant channel 3 the width changes by 6%, while Γ_4 can change by a factor of 10.

We now can compare our predicted theoretical values for Γ_k with experimental data from [13]. First of all the total width of $\Upsilon(5S)$ is known with 10% accuracy, $\Gamma_{\text{tot}}^{\text{exp}} = 110 \pm 13$ MeV [13], and some relations were established [13]

$$\frac{\Gamma_1^{\text{exp}}}{\Gamma_2^{\text{exp}}} < 0.92; \qquad \frac{\Gamma_1^{\text{exp}}}{\Gamma_3^{\text{exp}}} < 0.3; \qquad \frac{\Gamma_2^{\text{exp}}}{\Gamma_3^{\text{exp}}} = 0.324. \tag{12}$$

For channels with B_s, B_s^* one has [13]

$$\frac{\Gamma_4^{\text{exp}} + \Gamma_5^{\text{exp}} + \Gamma_6^{\text{exp}}}{\Gamma_{\text{tot}}} = 0.16 \pm 0.02 \pm 0.058 \tag{13}$$

and also

$$\frac{\Gamma_4^{\text{exp}}}{\Gamma_6^{\text{exp}}} < 0.16; \qquad \frac{\Gamma_5^{\text{exp}}}{\Gamma_6^{\text{exp}}} < 0.16. \tag{14}$$

Calculating Γ_{tot} from Table 1, one has $\Gamma_{\text{tot}} \simeq (\frac{M_\omega}{2\omega})^2 160$ MeV and choosing $(\frac{M_\omega}{2\omega})^2 = 0.6$ one can approximately reproduce the decay $\Upsilon(4S) \to B\bar{B}$ $\Gamma_{\text{tot}} \simeq 26$ MeV *vs* $\Gamma_{\text{exp}} = 20.5 \pm 2.5$ MeV (see [3]), while for $\Gamma_{\text{tot}}(5S)$ one has $\Gamma_{\text{tot}} = 113$ MeV, which is not far from the experimental value $\Gamma_{\text{tot}}^{\text{exp}} = (110 \pm 13)$ MeV. However for more accurate calculation of Γ_k one needs better knowledge of the wave function.

Comparing partial widths from the Table 1 with experimental limits (12)–(14), one can see, that all inequalities except the last right ones in (12) and (14) are satisfied by our theoretical values, however more work on theoretical side (explicit form of $5S$ wave function) and in experiment is needed.

Table 2
The total dipion and dikaon widths for the models 1 and 2 (from top to bottom) in comparison with experimental widths from [10].

Transition (n, n')	51	52	53	51, KK
$\Gamma_{\pi\pi}^{\text{AZI}}/(\frac{M_{br}}{f_\pi})^4$, MeV	1.4	0.67	0.032	0.12
$\Gamma_{\pi\pi}^{\text{FSI}}/(\frac{M_{br}}{f_\pi})^4$, MeV	2.0	1.67	0.23	0.18
$\Gamma_{\pi\pi}^{\text{exp}}(n, n')$, MeV	0.59 ± 0.04 ± 0.09	0.85 ± 0.07 ± 0.16	$0.52^{+0.20}_{-0.17}$ ± 0.10	$0.067^{+0.017}_{-0.015}$ ± 0.013

4. Dipion and dikaon transitions of $\Upsilon(5S)$

In this section we discuss dipion spectra and angular distributions for the transitions $(5,1)$, $(5,2)$ and $(5,3)$, as well as total dipion and dikaon widths, given by Eq. (11). The differential probability $\frac{dw_{\pi\pi}}{dq\, d\cos\theta}$ is given in (6), and integrating over dx or over $d\cos\theta$ we obtain one-dimensional spectrum

$$\frac{dw}{dq} = C_0 \mu^2 \sqrt{x(1-x)} \int_{-1}^{+1} |\mathcal{M}|^2\, d\cos\theta \tag{15}$$

and angular distribution

$$\frac{dw}{d\cos\theta} = \frac{1}{2} C_0 \mu^3 \int_0^1 dx \sqrt{\frac{x(1-x)}{x + \frac{4m_\pi^2}{\mu^2}}} \left|\mathcal{M}(x, \cos\theta)\right|^2. \tag{16}$$

The values of $\mathcal{M}$, Eq. (5), were calculated using $\frac{M_\omega}{M_{br}} = 6$ and for $\mathcal{M}_1$, $\mathcal{M}_2$ the same equations (23)–(25) from [3] were used as for $\Upsilon(nS)$ transitions with $n \leqslant 4$.

At this point we impose on the amplitude $\mathcal{M}$ the soft pion property, and use the AZR to rewrite Eq. (5) in the form

$$\mathcal{M} = \bar{\mathcal{M}}\big(\exp_1 - \exp_2 f(q)\big), \tag{17}$$

where $\exp_1$ and $\exp_2$ refer to the exponential factors in (5) and the factor $f(q)$, later used for the FSI effects, obeys the condition $f(q^2 = m_\pi^2) = 1$. Normalizing $\bar{\mathcal{M}}$ to $\mathcal{M}_2$, so that $\bar{\mathcal{M}} = \frac{M_{br}M_\omega}{f_\pi^2} M_2$, one can insert (17) in (15) to obtain $\Gamma_{\pi\pi}$. The corresponding values without FSI, i.e. for $f(q) \equiv 1$ are given in Table 2, upper line, and called the model 1.

For the dikaon $(5,1)$ transition one can in first approximation neglect the change of m_π to m_K in matrix element (5), and take it into account in phase space, also remembering that $\mathcal{M}$ is $O(\frac{1}{f_\pi^2})$, which should be replaced by $O(\frac{1}{f_K^2})$. In the total width Γ_{KK} $(5,1)$ one can write similarly to (7)

$$\Gamma_{KK}(5,1) = C_0 \mu_K^3 \int_0^1 dx \sqrt{\frac{x(1-x)}{x + \frac{4m_K^2}{\mu_K^2}}} \int_{-1}^{+1} \frac{d\cos\theta}{2} |\mathcal{M}_k|^2. \tag{18}$$

Here $\mu_K^2 = (\Delta E)^2 - 4m_K^2 = 0.985$ GeV^2, $\mu_K = 0.992$ GeV.

As a result, approximating the ratio of integrals over dx as $1/2$, one obtains

$$\frac{\Gamma_{KK}(5,1)}{\Gamma_{\pi\pi}(5,1)} = \frac{1}{2}\left(\frac{\mu_K}{\mu}\right)^3 \left(\frac{f_\pi}{f_k}\right)^4 = 0.194 \left(\frac{f_\pi}{f_K}\right)^4 = 0.092 \approx 1/10, \tag{19}$$

where we have used $f_\pi = 93$ MeV, $f_K = 112$ MeV [13].

Correspondingly one obtains the last column in Table 2 from the second one, using (19).

5. Final state interaction in $(5, n')$ transitions

One of the important new features of $(5S)$ (and higher states like $(6S)$) transitions is that a large phase space is available where both σ and f_0 resonances can be seen. In $(4, 1)$ transitions f_0 is at the edge of phase space while σ in most transitions lies near the region $x = \eta$, where amplitudes vanish and therefore no strong FSI effects are visible in (n, n') for $n \leqslant 4$.

In $(5, 1)$, $(5, 2)$ transitions the situation is different and e.g. in the $(5, 1)$ transition the f_0 resonance is well inside the available q region.

At this point it is necessary to stress that the FSI acts differently on one-pion ("a" or $\mathcal{M}_1$) amplitude and two-pion ("b" or $\mathcal{M}_2$) amplitude. Namely, for the case of $\mathcal{M}_1$, where two pions are emitted from two points separated by distance $L \sim 1/\Gamma$, $\Gamma \lesssim 0(10$ MeV), the $\pi\pi$ interaction of range $r_0 \lesssim 0.6$–0.8 fm is damped by a factor of the order of $r_0/L \sim O(1/10)$. E.g. in the FSI description in [14–16], the relative weight of $\pi\pi$ amplitudes with and without FSI was estimated as $\sim (1/7)$.

Completely different situation occurs in $b, (\mathcal{M}_2)$, where a pair of s-wave pions with $I = 0$ is emitted from a point (or, rather, a region of the order of $\lambda \sim 0.1$ fm, λ-gluonic correlation length of QCD vacuum). Here FSI is obligatory and is given by the Omnès–Muskhelishvili solution $f(q) = \frac{P(q^2)}{D(q^2)}$; with $P(q^2)$ – a polynomial normalizing $f(q^2)$ at some point: we shall use normalization $f(q^2 = (2m_\pi)^2) = 1$; a very close result is obtained for the Adler zero normalization $f(q = m_\pi) = 1$. Hence one can write $f(q^2)$ as follows (cf. the corresponding factors in [14,15]).

$$f(q) = \alpha f_\sigma(q) + \beta f_{f_0}(q), \tag{20}$$

$$f_i\big(q^2\big) = \frac{D_i(q^2 = 4m_\pi^2)}{D_i(q^2)},$$

$$D_i\big(q^2\big) = \exp\left(-\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dq'^2\,\delta_i(q'^2)}{q'^2(q'^2 - q^2)}\right), \quad i = \sigma, f_0 \tag{21}$$

and $\delta_i(q^2)$ is the $\pi\pi$ phase due to the ith resonance.

In the simplest approximation one can write

$$f_\sigma(q) = \left[\frac{(m_\sigma^2 - m_\pi^2)^2 + \gamma_\sigma^2}{(m_\sigma^2 - q^2)^2 + \gamma_\sigma^2}\right]^{1/2},$$

$$f_{f_0}(q) = \left[\frac{(m_{f_0}^2 - m_\pi^2)^2 + \gamma_{f_0}^2}{(m_{f_0}^2 - q^2)^2 + \gamma_{f_0}^2}\right]^{1/2} \operatorname{sign}(m_{f_0} - q). \tag{22}$$

The factors, corresponding to the resonances yield peaks, in (22) the σ peak is a wide structure, while f_0 produces a sharp peak near 1 GeV. Another feature of $f_{f_0}(q)$, Eq. (22), is that it changes sign just above position of f_0 due to the jump of $\delta(q^2)$ nearly equal to π, near $q = 1$ GeV, [14,15].

We have fitted the experimental $(5, 1)$ and $(5, 2)$ $\pi\pi$ spectra using the form (17) with $f(q)$ given in (20) and obtain the following values of parameters: $m_\sigma = 0.5$ GeV, $m_{f_0} = 1.15$ GeV, $\gamma_\sigma = 0.35$ GeV, $\gamma_{f_0} = 0.1$ GeV; $\alpha = 1$, $\beta = 0.01$. We call this fit the model 2.

The resulting curves (solid lines) are given in Figs. 3 and 4 for $(5, 1)$ and in Figs. 5 and 6 for the $(5, 2)$ cases, together with the curves for the model 1 ($f \equiv 1$, no FSI), shown by broken lines. Note, that in Figs. 3–6 theoretical curves were fitted to the experimental width $\Gamma_{\pi\pi}^{\exp}$, which means that M_{br}/f_π were varied in the interval 1–0.75.

6. Results and discussion

We start with the BB widths of $\Upsilon(5S)$ given in Table 1. It is clear that the values Γ_k give only a rough estimate and ac-

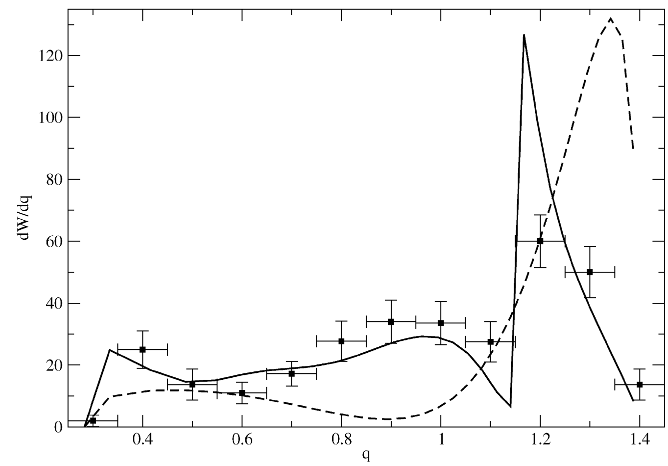

Fig. 3. Comparison of theoretical predictions, Eqs. (17), (20) with experiment [10] for the dipion spectrum, $\frac{dw}{dq}$, in the $\Upsilon(5, 1)\pi\pi$ transition. Theory: Eq. (17) with $f \equiv 1$ – broken curve, Eq. (17) with f as in Eq. (20) (parameters given in the text) – solid line. Theoretical curve is normalized to the total experimental width $\Gamma_{\pi\pi}^{\exp} = \frac{dw}{dq}dq$.

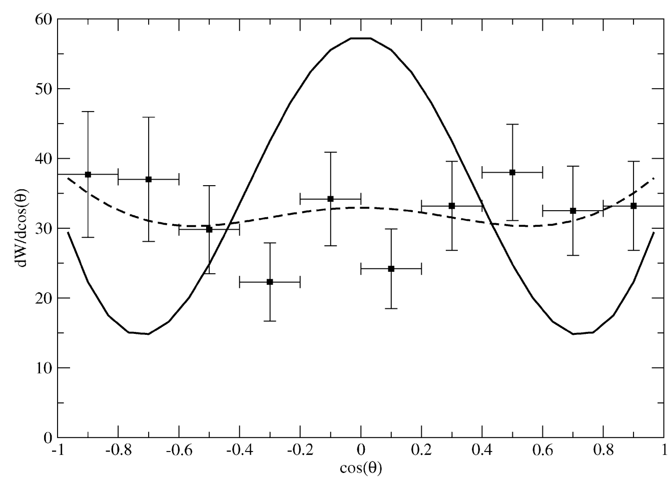

Fig. 4. The same as in Fig. 3, for the angular distribution $\frac{dw}{d\cos\theta}$ in the $\Upsilon(5, 1)\pi\pi$ transition.

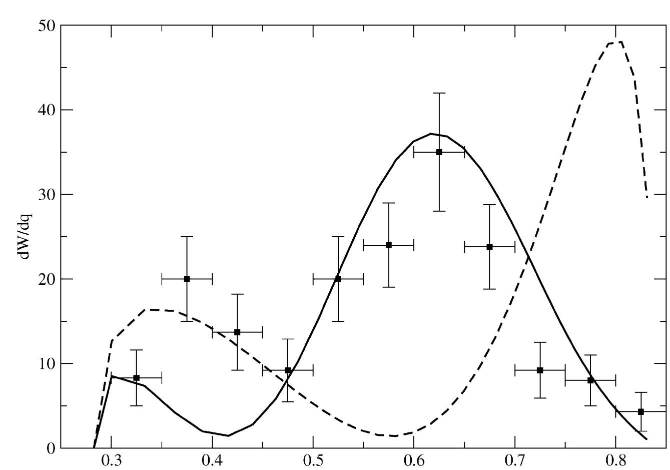

Fig. 5. The same as in Fig. 3, for the dipion spectrum $\frac{dw}{dq}$ in the $\Upsilon(5, 2)\pi\pi$ transition.

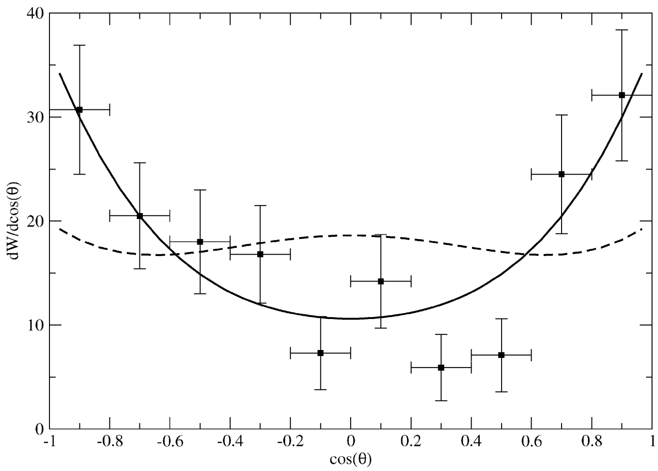

Fig. 6. The same as in Fig. 3, for the angular distribution $\frac{dw}{d\cos\theta}$ in the $\Upsilon(5,2)\pi\pi$ transition.

tual values Γ_k depends strongly on the behaviour of the $\Upsilon(5S)$ wave function. This is certainly true for the Eq. (8), derived for the wave function in the one-channel approximation. In the next orders, given by the equation

$$\det\left(\left(E - E_n^{(0)}\right)\delta_{nm} - w_{nm}(E)\right) = 0,$$

this sensitivity should be weaker, since the wave function becomes complex and does not have zeros. Hence one might hope that the values Γ_k yield the correct order of magnitude for all channels $k = 1, \dots, 6$, with the value $(\frac{M_\omega}{2\omega})^2 \approx 1/2$ as deduced from $\Gamma_{\text{tot}}(\Upsilon(4S))$. Comparing $\bar{\Gamma}_k$ with the widths Γ_k obtained for the $5S$ wave function approximated by 5 oscillator functions, one finds a reasonable agreement in magnitude, except for Γ_4 which is small due to nearby zero of $J_5(p)$.

Coming now to the total dipion widths in Table 2, one can notice, that our general expression (5), without FSI, yields reasonable order of magnitude for $\Gamma_{\pi\pi}$ and Γ_{KK} if $(\frac{M_{br}}{f_\pi}) \approx 1$. Here again strong dependence on the $\Upsilon(5S)$ wave function persists and results for $k_{\max} = 5$ and $k_{\max} = 15$ differ several times. In view of this it is not surprising that in Table 2 theoretical widths for $(5,1)$ and $(5,2)$ dipion transitions have a hierarchy different from that of experimental widths; however the smallness of $\Gamma_{th}(5,3)$ is well explained by a small phase space factor $\mu^3 : \mu^3(5,3)/\mu^3(5,1) \approx 2.8 \times 10^{-2}$ and it is not clear, why $\Gamma_{\exp}(5,3) \approx \Gamma_{\exp}(5,1)$.

Similar results for $\Gamma_{\pi\pi}, \Gamma_{KK}$ are obtained when both FSI and AZR are taken into account.

Turning to the $\pi\pi$ spectra, one observes that the spectra without FSI (model 1) in Figs. 3, 5 have less structure in contrast to the experimental data [10], where peaks in spectra at $q = 0.6$ GeV for $(5,2)$ and at $q \cong 1.2$ GeV for $(5,1)$ are clearly seen and strong $\cos\theta$ dependence is observed for the $(5,2)$ transition.

The situation is much better for the FSI − AZR approximation (model 2) in Figs. 3, 5 where the σ and f_0 peaks are seen in $(5,2)$ and $(5,1)$ cases, and also the experimental U-form of the $\cos\theta$ distribution is produced in the $(5,2)$ transition. However the much weaker experimental $\cos\theta$ dependence, Fig. 4 for the $(5,1)$ case is better reproduced in the model 1.

As a whole, it seems, that the spectrum, especially its lower enhancement at $q \approx 0.4$ GeV in both $(5,1)$ and $(5,2)$ transitions, can be well described by the AZR + FSI form, where the lower peak at $q \approx 0.4$ GeV is due to cancellation of two terms in (17), i.e. mainly due to AZR.

Summarizing, we have used the theory developed in previous papers [1–3] and applied in [3] to the subthreshold transitions (n, n'), $n \leqslant 4$. This theory does not contain free parameters, the only ones M_ω and M_{br} are defined previously in [3].

Exploiting this theory, we have calculated six BB-type widths of $\Upsilon(5S)$, Γ_k, $k = 1, \dots, 6$ total dipion widths of $(5, n')$, $n' = 1, 2, 3$ transition, and dipion spectra and $\cos\theta$ distributions of $(5, n')$ transitions. We have succeeded in explaining approximately all 4 points, mentioned in introduction:

1. Total widths $\Gamma_{\pi\pi}(5, n')$ are $O(1\text{ MeV})$.
2. The sequence of inequalities between Γ_{BB}, Γ_{BB^*}, $\Gamma_{B^*B^*}$ and corresponding widths for $B_s B_s$, occur naturally.
3. Dikaon width of $(5,1)$ is $\approx 1/10$ of the corresponding dipion width.
4. Dipion spectra of $(5,1)$, $(5,2)$ transitions require inclusion of FSI with σ and f_0 peaks and the appearance of the peak at $M_{\pi\pi} \approx 0.4$ GeV is possible due to a nearby zero of amplitude. We stress, that our method allows to reproduce the sophisticated $(5,1)$ spectrum in Fig. 3 with good accuracy, using the same FSI parameters as for the $(5,2)$ spectrum in Fig. 5.
5. In addition the unusual (U-type) $\cos\theta$ dependence is quantitatively explained for the $(5,2)$ transition as consequence of FSI.

We have observed strong dependence of all results on the properties of the $\Upsilon(5S)$ wave function, in particular on the position of its zeros, which in turn may serve to derive it from the total set of experimental data.

As a whole, our method allows to understand the basic features of all $\Upsilon(nS)$ transitions and decays, however more work is needed to explain all data in detail.

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