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The QCD equation of state near T_c within a quasi-particle model

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We dedicate this work to the memory of the late Prof. Dr. Gerhard Soff

Abstract

We present a description of the equation of state of strongly interacting matter within a quasi-particle model. The model is adjusted to lattice QCD data near the deconfinement temperature T_c. We compare in detail the excess pressure at non-vanishing chemical potential and its Taylor expansion coefficients with two-flavor lattice QCD calculations and outline prospects of the extrapolation to large baryon density.

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Due to the recent progress of first principle lattice QCD calculations, the equation of state (EoS) of strongly interacting matter is now at our disposal in some region of temperature T and chemical potential μ. Either the overlap improving multi-parameter reweighting technique [1] or the Taylor expansion or hybrids of them [2,3] deliver the pressure, entropy density, quark density, susceptibilities, etc. The knowledge of these quantities is of primary importance for a hydrodynamical description of relativistic heavy-ion collisions, the confinement transition in the early universe and possible quark cores in compact neutron stars. Knowing the phase boundary [2] and the end point of the first-order deconfinement transition [4] in the region of non-vanishing chemical potential is particularly interesting for the envisaged CBM project at the future accelerator facility FAIR at Darmstadt [5]. In the planned experiments a systematic investigation of phenomena of maximum baryon density reachable in heavy-ion collisions will be attempted.

Apart from lattice QCD calculations as purely numerical technique to obtain the EoS, also analytical approaches have been invented to understand the basic features. We mention dimensional reduction, resummed HTL scheme, Φ functional approach,

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Polyakov loop model, etc. (cf. [6] for a recent survey). A controlled chain of approximations from full QCD to analytical expressions without adjustable parameters describing the lattice data would be of desire. Success has been achieved [7] for $T > 2T_c$. In contrast, the range $T \geqslant T_c$, in particular close to T_c, is covered by phenomenological models [8,9] with parameters adjusted to lattice QCD data at $\mu = 0$. It is the aim of the present Letter to compare in detail the quasi-particle model [8] with the recent lattice QCD data [3] in the region around T_c with the focus on finite baryon density. We present a quasi-particle description of the Taylor expansion coefficients of the excess pressure for the strongly coupled quark–gluon fluid. Only in such a way an adequate and direct comparison with the lattice QCD results [3,10] is possible.

One way to decompose the EoS is writing for the pressure [2,3]

$$p(T,\mu) = p(T,\mu=0) + \Delta p(T,\mu),$$

$$\frac{\Delta p(T,\mu)}{T^4} = \sum_{i=2}^{\infty} c_i \left(\frac{\mu}{T}\right)^i. \tag{1}$$

$p(T,\mu=0)$ was subject of previous lattice QCD calculations (cf. [11] for the two-flavor case), while $\Delta p(T,\mu)$ became accessible only recently [3,4]. $\Delta p(T,\mu)$ is easier calculable, therefore, lattice QCD calculations focus on this quantity, instead of focusing on $p(T,\mu)$. In contrast, our model covers $p(T,\mu=0)$ and $\Delta p(T,\mu)$ on equal footing. Therefore, we have $p(T,\mu)$ at our disposal.

The quasi-particle model of light quarks (q) and gluons (g) is based on the expression for the pressure

$$p = \sum_{a=q,g} p_a - B(T,\mu),$$

$$p_a = \frac{d_a}{6\pi^2} \int dk \, \frac{k^4}{E_a(k)} \big(f_a^+(k) + f_a^-(k)\big), \tag{2}$$

where $B(T,\mu)$ ensures thermodynamic self-consistency [8], $s = \partial p/\partial T$, $n_q = \partial p/\partial \mu$, together with the stationarity condition $\delta p/\delta m_a^2 = 0$ [12]. The k integrals here and below run from 0 to ∞. Explicitly, the entropy density reads $s = \sum_{a=q,g} s_a$ with[1]

$$s_a = \frac{d_a}{2\pi^2 T} \int dk \, k^2 \bigg(\frac{(\frac{4}{3}k^2 + m_a^2)}{E_a(k)} \big(f_a^+(k) + f_a^-(k)\big) - \mu\big(f_a^+(k) - f_a^-(k)\big) \bigg) \tag{3}$$

and the net quark number density is

$$n_q = \frac{d_q}{2\pi^2} \int dk \, k^2 \big(f_q^+(k) - f_q^-(k)\big) \tag{4}$$

with degeneracies $d_q = 12$ and $d_g = 8$ as for free partons and distribution functions

$$f_a^{\pm}(k) = \big(\exp\big(\big[E_a(k) \mp \mu\big]/T\big) + S\big)^{-1}$$

with $S = +1$ (-1) for quarks (gluons). The chemical potential is μ for light quarks, while for gluons it is zero.

The quasi-particle dispersion relation is approximated by the asymptotic mass shell expression near the light cone

$$E_a^2(k) = k^2 + m_a^2,$$
$$m_a^2(T,\mu) = \Pi_a(k;T,\mu) + (x_a T)^2. \tag{5}$$

The essential part is the self-energy Π_a; the last term accounts for the masses used in the lattice calculation [3], i.e., $x_q = 0.4$ and $x_g = 0$. First direct measurements of the dispersion relation have been reported in [18] and let argue the authors of [19] that additional degrees of freedom are required to saturate the lattice pressure. It should be noticed, however, that for the EoS the excitations at momenta $k \sim T$ matter, for which more accurate measurements are needed. As suitable parametrization of Π_a, we employ here the

[1] In massless φ^4 theory such a structure of the entropy density emerges by resumming the super-daisy diagrams in tadpole topology [13], and [14] argues that such an ansatz is also valid for QCD. [7] point to more complex structures, but we find (2)–(4) flexible enough to accommodate the lattice data. Finite width effects are studied in [15]. In the Φ functional approach the following chain of approximations leads to the given ansatz [16]: (i) two-loop approximation for the Φ functional; (ii) neglect longitudinal gluon modes and the plasmino branch, both being exponentially damped; (iii) restore gauge invariance and ultra-violet finiteness by arming the self-energies with HTL resummed expressions; (iv) neglect imaginary parts in self-energies and Landau damping and approximate suitably the self-energies in the thermodynamically relevant region $k \sim T, \mu$. The pressure follows by an integration.

HTL self-energies with given explicit T and μ dependencies as in [8]. The crucial point is to replace the running coupling in Π_a by an effective coupling, $G^2(T,\mu)$.[2] In doing so, non-perturbative effects are thought to be accommodated in this effective coupling. This assumption needs detailed tests which are presented below. Note that Eqs. (2)–(4) themselves are highly non-perturbative expressions. Expanding them in powers of the coupling strength one recovers the first perturbative terms.

The first expansion coefficients in Eq. (1) follow from (2) as $c_i = \frac{T^{i-4}}{i!}\frac{\partial^i p}{\partial \mu^i}|_{\mu=0}$:

$$c_2 = \frac{3N_f}{\pi^2 T^3}\int dk\, k^2 \frac{e^\omega}{(e^\omega+1)^2}, \tag{6}$$

$$c_4 = \frac{N_f}{4\pi^2 T^3}\int dk\, k^2 \frac{e^\omega}{(e^\omega+1)^4} \times \left(e^{2\omega} - 4e^\omega + 1 - \frac{A_2}{\omega}\left(e^{2\omega}-1\right)\right), \tag{7}$$

$$\begin{aligned} c_6 = \frac{3N_f}{385\pi^2 T^3}\int dk\, k^2 \frac{e^\omega}{(e^\omega+1)^6} &\times \bigg\{e^{4\omega} - 26e^{3\omega} + 66e^{2\omega} - 26e^\omega + 1 \\ &- \frac{10}{3}\frac{A_2}{\omega}\left(e^{4\omega} - 10e^{3\omega} + 10e^\omega - 1\right) \\ &+ \frac{4}{3}\frac{A_2^2}{\omega^2}\left(e^{4\omega} - 2e^{3\omega} - 6e^{2\omega} - 2e^\omega + 1\right) \\ &+ \left(\frac{5}{3}\frac{A_2^2}{\omega^3} - 10\frac{T^2 A_4}{\omega}\right) \\ &\times \left(e^{4\omega} + 2e^{3\omega} - 2e^\omega - 1\right)\bigg\}, \end{aligned} \tag{8}$$

where $\omega = (k^2 + \frac{1}{3}T^2 G^2|_{\mu=0})^{1/2}/T$, $A_2 = (G^2/\pi^2 + \frac{1}{2}T^2\partial^2 G^2/\partial\mu^2)|_{\mu=0}$, $A_4 = (\frac{1}{\pi^2}\partial^2 G^2/\partial\mu^2 + \frac{T^2}{12}\partial^4 G^2/\partial\mu^4)|_{\mu=0}$. (We have not displayed the terms $\propto x_q$ stemming from the lattice masses; in the calculations presented below, however, these terms are included to make the model as analog as possible to the lattice performance.) c_j with odd j vanish. In deriving these equations we have used the flow equation [8]

$$a_\mu \frac{\partial G^2}{\partial \mu} + a_T \frac{\partial G^2}{\partial T} = a_{\mu T}, \tag{9}$$

where the lengthy coefficients $a_{\mu,T,\mu T}(T,\mu)$ [16] obey $a_T(T,\mu=0)=0$ and $a_{\mu T}(T,\mu=0)=0$. This flow equation follows from a thermodynamic consistency condition. The meaning of Eq. (9) is to map G^2, given on some curve $T(\mu)$, e.g., on $T(\mu=0)$, into the μ plane to get $G^2(T,\mu)$ which is needed to calculate p, s, n from Eqs. (2)–(4) at non-vanishing values of μ. The terms needed in Eqs. (7), (8) follow from the flow equation and its derivatives yielding

$$\left.\frac{\partial^2 G^2}{\partial\mu^2}\right|_{\mu=0} = \frac{1}{a_\mu}\left(\frac{\partial a_{\mu T}}{\partial\mu} - \frac{\partial a_T}{\partial\mu}\frac{\partial G^2}{\partial T}\right)\bigg|_{\mu=0}, \tag{10}$$

$$\begin{aligned} \left.\frac{\partial^4 G^2}{\partial\mu^4}\right|_{\mu=0} = \frac{1}{a_\mu}\bigg(&\frac{\partial^3 a_{\mu T}}{\partial\mu^3} - \frac{\partial^3 a_T}{\partial\mu^3}\frac{\partial G^2}{\partial T} \\ &- 3\frac{\partial^2 a_\mu}{\partial\mu^2}\frac{\partial^2 G^2}{\partial\mu^2} \\ &- \frac{3}{a_\mu}\frac{\partial a_T}{\partial\mu}\bigg[\frac{\partial^2 a_{\mu T}}{\partial\mu\partial T} - \frac{\partial^2 a_T}{\partial\mu\partial T}\frac{\partial G^2}{\partial T} \\ &- \frac{\partial a_T}{\partial\mu}\frac{\partial^2 G^2}{\partial T^2} - \frac{\partial a_\mu}{\partial T}\frac{\partial^2 G^2}{\partial\mu^2}\bigg]\bigg)\bigg|_{\mu=0}. \end{aligned} \tag{11}$$

We adjust $G^2(T)$ through Eq. (6) to $c_2(T)$ from [3] for $N_f = 2$. We find as convenient parametrization

$$G^2(T) = \begin{cases} G^2_{\text{2-loop}}(T), & T \geqslant T_c, \\ G^2_{\text{2-loop}}(T_c) + b(1 - T/T_c), & T < T_c, \end{cases} \tag{12}$$

where $G^2_{\text{2-loop}}$ is the relevant part of the 2-loop coupling

$$G^2_{\text{2-loop}}(T) = \frac{16\pi^2}{\beta_0 \log\xi^2}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\log(\log\xi^2)}{\log\xi^2}\right] \tag{13}$$

with $\beta_0 = (11N_c - 2N_f)/3$, $\beta_1 = (34N_c^2 - 13N_f N_c + 3N_f/N_c)/6$, and the argument $\xi = \lambda(T - T_s)/T_c$. T_s acts as regulator at T_c, and λ sets the scale. The parameters for $N_c = 3$ are $\lambda = 12$, $T_s = 0.87T_c$, and $b = 426.1$. Fig. 1 exhibits the comparison of Δp and n calculated via Eqs. (2), (4) (dashed curves) or by using the expansion coefficients Eqs. (6), (7), (solid curves) with the lattice QCD data [3] based on the

[2] As shown in [17], it is the introduced $G^2(T,\mu)$ which allows to describe lattice QCD data near T_c, while the use of the pure 1-loop or 2-loop perturbative coupling together with a more complete description of the plasmon term and Landau damping restricts the approach to $T > 2T_c$.

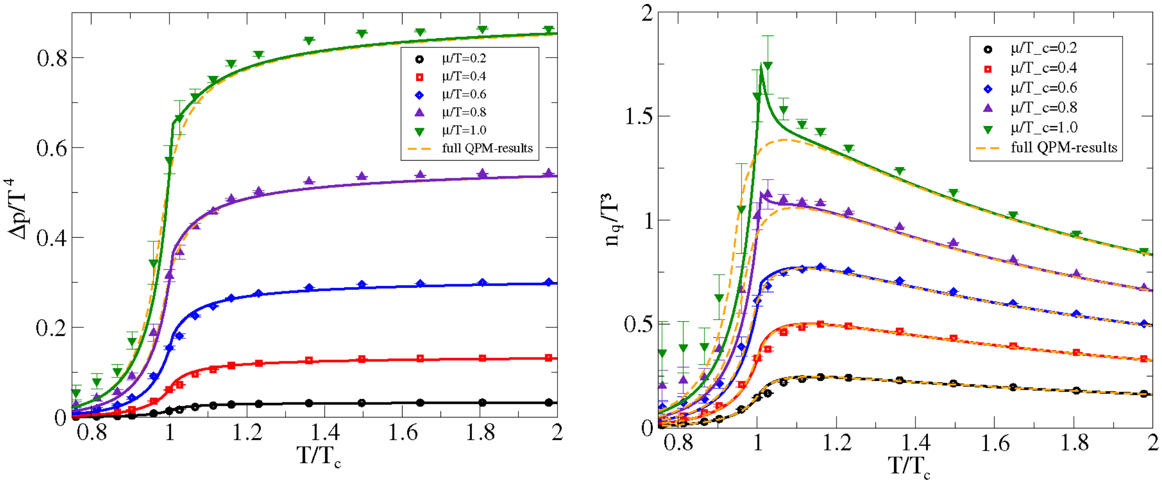

Fig. 1. Comparison of the quasi-particle model with lattice QCD results [3] for the excess pressure (left panel, for constant μ/T) and net quark number density (right panel, for constant μ/T_c). As for the lattice QCD data (symbols) the quasi-particle model results (solid curves) are based on the expansion coefficients $c_{2,4}$, i.e., $\Delta p/T^4 = c_2(T)(\mu/T)^2 + c_4(T)(\mu/T)^4$ and $n_q/T^3 = 2c_2(T)(\mu/T) + 4c_4(T)(\mu/T)^3$. For comparison, the full quasi-particle model results (dashed curves) are exhibited.

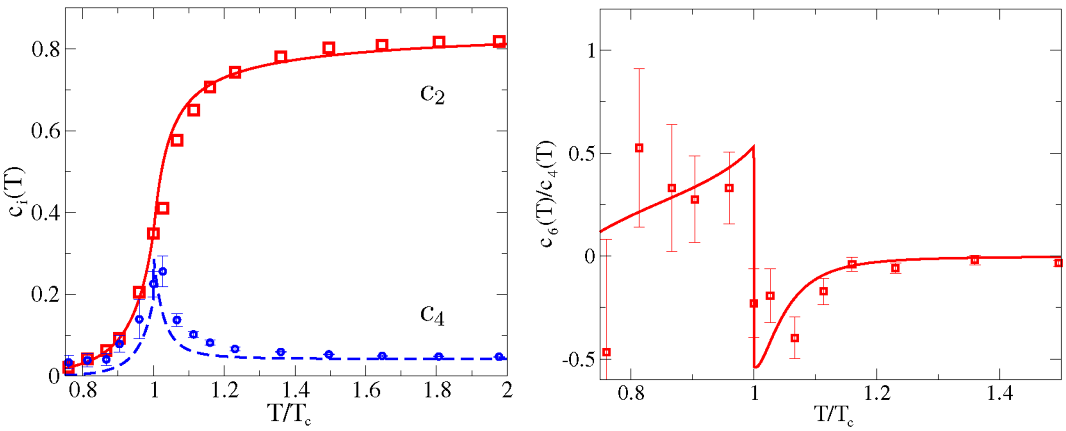

Fig. 2. The expansion coefficients $c_{2,4}$ (left panel, data from [3]) and the ratio c_6/c_4 (right panel, data from [10]) as a function of the temperature.

coefficients $c_{2,4}$ (symbols). One observes an astonishingly good description of the data, even slightly below T_c, where the resonance gas model [20] is appropriate.[3] Interesting is the deviation of the full model from the results based on the truncated expansion in a small interval around T_c. It should be noted that conceptionally different models [21] reproduce fairly well the lattice data for Δp and n above T_c, however, since for small values of μ the higher order coefficients c_4 and in particular c_6 are less important for Δp and n, a more stringent test of the model is accomplished by a direct comparison of the individual Taylor expansion coefficients c_i with the corresponding lattice QCD results.

Straightforward evaluation of Eqs. (6)–(8) delivers the results exhibited in Fig. 2. Since $G^2(T)$ was already adjusted to $c_2(T)$ the agreement is good. It should be emphasized that all coefficients $c_i(T)$ are determined by $G^2(T)$. That means the same $G^2(T)$ describes also the features of c_4 and c_6. Particularly

[3] Some reasoning why the model may be applicable also slightly below T_c emerges from duality [22], similar to the application of a hadronic model slightly above T_c [23].

interesting are the peak of c_4 (left panel of Fig. 2) and the double-peak of c_6/c_4 (right panel of Fig. 2) or c_6 (not exhibited) at T_c. Numerically, these pronounced structures stem from the change of the curvature behavior of $G^2(T)$ at T_c which determines the terms $\partial^2 G^2/\partial\mu^2|_{\mu=0}$ and $\partial^4 G^2/\partial\mu^4|_{\mu=0}$ via Eqs. (9)–(11). Neglecting these terms would completely alter the shape of $c_{4,6}$. That means, via $c_{4,6}$ the flow equation (9) is probed, which is the key for extrapolating to large values of μ. Similar to [3], we interpret the peak in c_4 as indicator of some critical behavior, while the pressure itself is smoothly but rapidly varying at T_c. Note that the results exhibited in Fig. 2 are robust with respect to the chosen form of the effective coupling (12). Testing the 1-loop coupling of G^2 above T_c or a quadratic function in T/T_c below T_c or both, e.g., the higher order coefficients and in particular their pronounced behavior about T_c are quantitatively reproduced when adjusting G^2 to describe c_2.

In summary we present a quasi-particle model which describes the recent lattice QCD data for nonvanishing chemical potential remarkably well. Besides the excess pressure $\Delta p(T,\mu)$ and density n above and even slightly below T_c at small values of the chemical potential, the individual Taylor expansion coefficients agree well with the data and turn out to depend on each other. Once $G^2(T)$ is adjusted, also $p(T,\mu=0)$ follows up to an integration constant. We find a small deviation (maximum 15%) from an optimized description of the data [11] which might be attributed to differences in calculating $p(T,\mu=0)$ and the Taylor coefficients of $\Delta p(T,\mu)$ on the lattice. Consequently, adjusting $G^2(T)$ directly to $p(T,\mu=0)$ a mean quadratic deviation of 0.0027 between c_2 data [3] and our model is observed [16], while our direct fit to c_2 delivers 0.0010. Nevertheless, the shape and in particular the structures of the higher order Taylor coefficients are well reproduced. We find $\chi^2/\text{d.o.f.}=8.8$ for c_4 and $\chi^2/\text{d.o.f.}=0.24$ for c_6/c_4, while the above adjustment to c_2 delivered 8.7 and 0.39, respectively (the small values of $\chi^2/\text{d.o.f.}$ for c_6/c_4 are due to the large error bars).

Having tested these details of the quasi-particle model, we can directly apply the found parametrization and calculate the total pressure at arbitrary baryon densities, while lattice QCD calculations are yet constraint to small baryon densities. This application is of interest for the CBM project at FAIR and for studying hot proto-neutron stars and cold neutron stars with quark cores and will be reported elsewhere. Another application to cosmic confinement dynamics is reported in [24]. These applications need a controlled chiral extrapolation which must base on improved lattice QCD data.

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