On the dynamics of exotic matter: Towards creation of Perpetuum Mobile of third kind

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ABSTRACT

The one-dimensional dynamics of a classical ideal 'exotic' fluid with equation of state $p = p(\epsilon) < 0$ violating the weak energy condition is discussed. Under certain assumptions it is shown that the well-known Hwa–Bjorken exact solution of one-dimensional relativistic hydrodynamics is confined within the future/past light cone. It is also demonstrated that the total energy of such a solution is equal to zero and that there are regions within the light cone with negative (−) and positive (+) total energies. For certain equations of state there is a continuous energy transfer from the (−)-regions to the (+)-regions resulting in indefinite growth of energy in the (+)-regions with time, which may be interpreted as action of a specific 'Perpetuum Mobile' (Perpetuum Motion). It is speculated that if it is possible to construct a three-dimensional non-stationary flow of an exotic fluid having a finite negative value of energy such a situation would also occur. Such a flow may continuously transfer positive energy to gravitational waves, resulting in a runaway. It is conjectured that theories plagued by such solutions should be discarded as inherently unstable.

1. Introduction

If physical laws do not prohibit the presence of exotic matter violating the weak energy condition[1] and having some other certain properties many exciting possibilities arise. For example, solutions of the Einstein equations coupled to an exotic matter include wormholes, time machines (e.g. [1]) and cosmological models with energy density of the Universe growing with time (see, e.g. [2] for a review and discussion of cosmological consequences) leading to the so-called cosmological Doomsday, see e.g. [3]. Since theories incorporating an exotic matter may lead to counterintuitive and, possibly, physically inconsistent effects it appears to be important to invoke different thought experiments, which could clarify self-consistency of such theories. Here we discuss such an experiment and explicitly show that in a class of models containing an exotic matter of a certain kind there could be ever

expanding with time separated regions of space having positive and negative total energies and that absolute values of the energies in these regions could grow indefinitely with time while the energy of the whole physical system is conserved. This is based on the property of the exotic matter to have negative energy density measured by observers being at rest with respect to some Lorentz frame and, accordingly, given by the (tt)-component of the stress–energy tensor, T^{tt}, provided that there are sufficiently large fluid velocities with respect to this frame. We also speculate that in a more advanced variant of our model there could be an isolated region of space filled by an exotic matter with its total energy indefinitely decreasing with time due to processes of interaction with some other conventional physical fields carrying positive energy. One of such processes could be emission of gravitational waves. If conditions for emission of gravitational waves are always fulfilled in the course of evolution positive energy is continually carried away from the region, which results in a runaway. In this respect it is appropriate to mention that the known results on positiveness of mass in General Relativity are not valid for the matter violating the weak energy condition, see e.g. [4] and the energy of an isolated region could evolve from positive to negative values. Although such a situation resembles the action of a Perpetuum Mobile of second kind, where heat transfer from a

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[1] Let us remind that the weak energy condition is said to be violated for a matter having the stress–energy tensor $T_{\mu\nu}$ if there is such a time-like future directed vector field t^μ that the inequality $T_{\mu\nu}t^\mu t^\nu < 0$ holds in some region of space–time.

colder part of an isolated system to a hotter part occurs, the notion of 'temperature' looks ambiguous in our case and, therefore, because of the lack of notation, we refer to this hypothetical effect as a 'Perpetuum Mobile of third kind'. In many applications a phenomenological description of the exotic matter assumes that the matter dynamics can be described by a hydrodynamical model with the stress–energy tensor of an ideal Pascal fluid, which can be fully specified by its equation of state, $p = p(\epsilon)$, where p is the pressure and ϵ is the comoving energy density, which is assumed to be positive below. In this case, violation of the weak energy condition can be reformulated as a requirement that the pressure is negative with its modulus exceeding the value of the comoving energy density. We deal hereafter only with simplest models of hydrodynamical type and neglect the effects of General Relativity and interactions with other physical fields. Therefore, in the model explicitly considered in the text the transfer of energy from one region of space to another, in particular, between the regions having total energies of different signs, is provided by hydrodynamical effects. However, as we have mentioned above, it seems reasonable to suppose that an analogous runaway effect may happen in a more realistic situation, where e.g. energy is continually carried away from a spacial region having negative total energy by gravitational waves.[2] Taking into account that gravitational interaction is universal, the proposed 'Perpetuum Mobile of third kind' may represent a difficulty in theories, where it emerges. We believe that such theories are inherently unstable and must, therefore, be discarded.

It is important to note, however, that in a general scenario the region emitting gravitational waves could have a non-linear three-dimensional dynamics. Explicit solutions of this kind may be quite difficult to obtain due to severe technical problems.

2. Basic definitions and equations

Let us discuss a one-dimensional planar relativistic flow of an ideal fluid with a baratropic equation of state

$$p = p(\epsilon), \tag{1}$$

where p is the pressure and ϵ is the comoving energy density. As has been mentioned in Introduction, we are going to consider later in the text the case of an exotic fluid, where the pressure is negative and the weak energy condition is violated. For a baratropic fluid this leads to:

$$\sigma \equiv -p > \epsilon, \tag{2}$$

where we introduce the negative of pressure p, $\sigma = -p$. Since only one-dimensional flows will be considered, we can also apply our analysis to a situation, where a fluid has an anisotropic stress tensor. Say, we can assume only one of its components to be non-negative and proportional to $\delta(y, z)$, where y and z are the

Minkowski spacial coordinates corresponding to directions perpendicular to the direction of motion. In this case, equations of motion will describe dynamics of a straight string consisting of exotic matter.

Equations of motion may be written in a divergent form reflecting the laws of conservation of energy and momentum

$$T^{tt}_{,t} + T^{tx}_{,x} = 0, \qquad T^{tx}_{,t} + T^{xx}_{,x} = 0, \tag{3}$$

where (t, x) are the standard Minkowski coordinates, comma stands for differentiation, and T^{ij} are the corresponding components of the stress–energy tensor:

$$T^{tt} = \gamma^2(\epsilon + v^2 p), \qquad T^{tx} = \gamma^2 v(\epsilon + p),$$
$$T^{xx} = \gamma^2(p + v^2 \epsilon), \tag{4}$$

where v is the three-velocity and $\gamma = \frac{1}{\sqrt{1-v^2}}$.

3. The Hwa–Bjorken solution and the Milne coordinates

As has been first shown by Hwa [8] and later by Bjorken [9], the set of Eqs. (3) has an especially simple 'acceleration-free' solution valid for a fluid having an arbitrary equation of state. In this solution velocity of any fluid element conserves along the path of the fluid element and the velocity field has a very simple form

$$v = x/t \equiv \xi. \tag{5}$$

For a baratropic fluid the distribution of energy is given by another simple implicit relation

$$\tau = \exp\left\{ -\int_{\epsilon_*}^{\epsilon} \frac{d\epsilon'}{\epsilon' + p(\epsilon')} \right\} = \exp\left\{ -\int_{\epsilon}^{\epsilon_*} \frac{d\epsilon'}{\sigma(\epsilon') - \epsilon'} \right\}, \tag{6}$$

where

$$\tau = \sqrt{t^2 - x^2}, \tag{7}$$

and ϵ_* is a constant of integration.

Obviously, Eqs. (6)–(7) are defined only inside the future/past light cone, $|t| > |x|$, in an effective two-dimensional Minkowski space described by the metric

$$ds^2 = dt^2 - dx^2. \tag{8}$$

The analytic continuation of the solution on the right/left Rindler wedge $|t| < |x|$ is straightforward.

Although the two-dimensional Minkowski space appears naturally due to one-dimensional character of the problem let us remind that the problem is defined in four-dimensional Minkowski space. Therefore, for the problem with Pascal pressure, where it is assumed that all variables do not depend on the coordinates (y, z) perpendicular to x, it is better to say that from the four-dimensional point of view the condition $|t| = |x|$ determines four-dimensional "light wedges" since it does not depend on directions perpendicular to the direction of motion.

The energy density ϵ is equal to zero on the light cone $|t| = |x|$ if and only if the integrals in the exponents in Eq. (6) are positive and diverge when $\epsilon \to 0$. Accordingly, in this case, the condition (2) must be satisfied. Additionally, in order to make the integrals divergent we must have

$$\sigma - \epsilon \leqslant |O(\epsilon)| \tag{9}$$

when $\epsilon \to 0$. Provided that condition (9) is valid the solution may be considered as confined within the future/past light cone with no flows of energy and momentum through the cone boundary. For

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[2] Note that one should distinguish between the standard hydrodynamical instabilities e.g. the ones operating in a fluid having a negative square of speed of sound and the instability related to violation of the weak energy condition. While the former could lead to a highly irregular non-linear dynamics of the system they cannot themselves result in formation of spacial regions having a negative total energy, and, a runaway of the kind discussed in this Letter, see also the footnote in the next section.

Let us also stress that the runaway effect is different from the well-known instabilities of linear modes of stationary hydrodynamical flows having a negative energy difference with respect to a stationary configuration like the Chandrasekhar–Friedman–Schutz instability [5,6] or the instability operating in shear flows, see e.g. [7] and references therein. Although the linear instabilities can lead to a decrease of energy of the main flow they cannot bring a system non-violating the weak energy condition to a state with a negative total energy. Therefore, in the latter case the runaway effect of the type we consider in this Letter is not possible.

simplicity we are going to consider only this case in our analytical calculations later on.[3]

The solution (5)–(6) has a self-evident form in the Milne coordinates (τ, y), where the time τ is defined by Eq. (7) and we introduce the rapidity

$$y = \ln \sqrt{\frac{1+\xi}{1-\xi}} \qquad (10)$$

as a new spacial coordinate. In these coordinates the metric (8) has the form

$$ds^2 = d\tau^2 - \tau^2\, dy^2. \qquad (11)$$

Although the metric (11) may be understood as describing an expanding one-dimensional spatially-uniform universe with the scale factor $a(\tau) = \tau$, obviously it corresponds to the same flat Minkowski space since it is obtained from the metric (8) by the coordinate transformation (7), (10), see e.g. [10] for an additional discussion of the Milne coordinates. Taking into account that the transformation law between the velocity v defined with respect to Minkowski coordinates (t, x) and the peculiar velocity $\bar{v}$ defined with respect to the orthonormal frame associated with the Milne coordinates is determined by the relation

$$v = \frac{\xi + \bar{v}}{1 + \xi \bar{v}}, \qquad (12)$$

it is clear that the solution (5)–(6) is simply the spatially-uniform solution in the Milne coordinates with the peculiar velocity $\bar{v} = 0$. In particular, Eq. (6) immediately follows from the first law of thermodynamics written for an adiabatic expansion of a fluid having distribution of its thermodynamical variables uniform with respect to the coordinate y:

$$\frac{dV}{V} = -\frac{d\epsilon}{\epsilon + p}, \qquad (13)$$

where $V \propto \tau$ is a comoving volume.

4. Properties of the solution

4.1. The energy integral

Provided that the condition (9) is valid we can assume that the energy density and, accordingly, the components of the stress–energy tensor are different from zero only within the future/past light cone. For simplicity, let us consider only the future light cone implying that $t > 0$ from now on. In this case, Eqs. (3) yield that the total energy of the flow

$$E = \int_{-t}^{t} dx\, T^{tt} = t \int_{-1}^{1} d\xi\, T^{tt} \qquad (14)$$

does not depend on time t.

Let us show that this integral is precisely equal to zero for an exotic baratropic fluid satisfying the condition (9). Taking into account that the distribution of energy density and pressure are even functions and the velocity distribution is an odd function of the

spacial coordinate x, respectively, it suffices to prove that the quantity

$$\mathcal{E} = \int_0^1 d\xi\, T^{tt} = \int_0^1 \frac{d\xi}{(1-\xi^2)}\left(\epsilon - \xi^2\sigma\right) \qquad (15)$$

is equal to zero. From Eq. (6) it follows that

$$\sigma = \tau \frac{d\epsilon}{d\tau} + \epsilon \qquad (16)$$

and, therefore, we have

$$\mathcal{E} = \int_0^1 d\xi \left(\epsilon - \frac{\xi^2\tau}{(1-\xi^2)}\frac{d\epsilon}{d\tau}\right). \qquad (17)$$

Now we change the integration variable from ξ to τ keeping the value of t fixed. Taking into account that

$$d\xi = -\frac{\tau}{t\sqrt{t^2-\tau^2}}d\tau, \qquad (18)$$

$$\xi = \frac{1}{t}\sqrt{t^2-\tau^2}, \qquad \sqrt{1-\xi^2} = \frac{\tau}{t}, \qquad (19)$$

we obtain

$$\mathcal{E} = \frac{1}{t}\int \sqrt{t^2-\tau^2}\, d\epsilon - \frac{1}{t}\int \frac{\epsilon\tau\, d\tau}{\sqrt{t^2-\tau^2}}, \qquad (20)$$

where the values of τ and ϵ corresponding to $\xi = 0, 1$ are omitted. Integrating by parts the first integral in (20) and taking into account that the boundary terms are equal to zero for the solution (5)–(6) satisfying the condition (9), we obtain

$$\mathcal{E} = -\frac{1}{t}\int \left(\frac{\tau}{\sqrt{t^2-\tau^2}} + \frac{d}{d\tau}\sqrt{t^2-\tau^2}\right)\epsilon\, d\tau = 0. \qquad (21)$$

4.2. Lorentz invariance and vacuum-like nature

It is easy to see that solution (5)–(6) has the same form in all coordinate systems connected by the Lorentz transformations: $(t, x) \rightarrow (t', x')$. Indeed, as follows from Eq. (7) the time τ is invariant under the Lorentz transformations. Therefore, Eq. (6) contains only invariant quantities and is the same in all Lorentz frames. It is also evident that when the Lorentz transformations are considered the quantity $\xi = x/t$ and the three-velocity v are transformed in the same way. Therefore, from Eq. (5) it follows that the same equation is valid for the transformed quantities.

It is clear that the total energy and momentum of the flow are equal to zero in all Lorentz frames. This is frequently considered as being the definition of vacuum solutions in different theoretical schemes. Thus, one may state that solution (5)–(6) plays a role of a non-trivial vacuum solution for the exotic fluids satisfying (9).

4.3. A hypothetical model of Perpetuum Mobile

As has been mentioned in Introduction, the very possibility of existence of solutions having negative/zero total energy is determined by the fact that for equations of state violating the weak energy condition the energy density determined with respect to a fixed Lorentz frame

$$T^{tt} \propto \epsilon - v^2\sigma \qquad (22)$$

can be negative provided that the fluid velocity is sufficiently large,

$$v > v_{\text{crit}} = \sqrt{\frac{\epsilon}{\sigma}}. \qquad (23)$$

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[3] In this case, the corresponding hydrodynamical models are unstable with respect to growth of small perturbations. One may proceed, however, either assuming that these models are valid only for the considered types of hydrodynamical flows or considering them as effective models invalid for sufficiently large perturbation wavenumbers. In any case, we expect that our main conclusions do not depend on whether the considered models are hydrodynamically unstable or not.

In the case of our solution the space bounded by the light cone condition $|x| < t$ is divided into a set of regions having opposite signs of T^{tt} and, accordingly, different signs of the total energy. In what follows let us refer to the regions with $T^{tt} > 0$ ($T^{tt} < 0$) as $(+)$-regions $((-)$-regions). The coordinates of boundaries between the $(+)$ and $(-)$-regions, x_{crit}, can be found from the condition $v = v_{\text{crit}}$ and, respectively, from the implicit equation

$$x_{\text{crit}} = t \sqrt{\frac{\epsilon(\tau_{\text{crit}})}{\sigma(\tau_{\text{crit}})}}, \tag{24}$$

where $\tau_{\text{crit}} = \sqrt{t^2 - x_{\text{crit}}^2}$. In general, Eq. (24) could have several roots on the interval $0 < x < t$ with values depending on equation of state. However, for a reasonable equation of state with sufficiently smooth dependence of σ on ϵ there must be a $(-)$-region adjacent to the light cone boundary $x = t$ and a $(+)$-region close to the point $x = 0$. The total energy of the region adjacent to the light cone, E_-, can be easily calculated from Eqs. (14), (15), and (20), where we take into account that after integration by parts of Eq. (20) only the boundary term at $x = x_{\text{crit}}$ contributes to the integral:

$$E_- = \int_{x_{\text{crit}}}^{t} T^{tt} \, dx = -x_{\text{crit}} \epsilon, \tag{25}$$

where x_{crit} denotes the largest root of Eq. (24) in the interval $0 < x < t$ from now on. Taking into account the fact that the total energy is zero and using the symmetry of the problem, we see that the energy in the interval $0 < x < x_{\text{crit}}$, $E_+ = -E_-$, and therefore

$$E_+ = \int_0^{x_{\text{crit}}} T^{tt} \, dx = \frac{t \epsilon^{3/2}}{\sigma^{1/2}} = \frac{\epsilon^{3/2}}{\sqrt{\sigma - \epsilon}} \exp\left(\int^{\epsilon} \frac{d\epsilon'}{\sigma' - \epsilon'}\right), \tag{26}$$

where we use Eqs. (6), (24), (25) and all quantities are assumed to be evaluated along the world line determined by Eq. (24). It is instructive to calculate the time derivative of E_+ differentiating the integral in (26) on time and using Eqs. (3) and (23) to obtain:

$$\dot{E}_+ = \sigma v = \sqrt{\sigma \epsilon}. \tag{27}$$

From Eq. (27) it follows that the energy of the region $0 < x < x_{\text{crit}}$ constantly grows with time due to energy flow from the $(-)$-region $x_{\text{crit}} < x < t$. In principal, there could be two possibilities for the asymptotic behaviour of E_+ in the limit $t \to \infty$ depending on form of the equation of state: (1) there could be a finite asymptotic value of E_+, and (2) a finite asymptotic value could be absent and the energy E_+ could infinitely grow with time. The latter case represents a specific instability, where there is infinite growth of energy in one region of space and infinite decrease of energy in the other region. Let us suppose that physical laws do not prohibit existence of the hydrodynamical systems violating the weak energy condition and having solutions of this type. Assuming that such a system could be made by an advanced civilization, which could also ensure that utilization of energy released in the region $0 < x < x_{\text{crit}}$ does not significantly perturb the solution this could provide an infinite source of energy. Therefore, this type of solution may be classified as a hypothetical 'Perpetuum Mobile'. Such a solution is discussed below.

4.4. An explicit example

Let us specify the equation of state and consider the simplest possible case of linear dependence of σ on ϵ:

$$\sigma = (1 + \alpha)\epsilon, \tag{28}$$

where the parameter $\alpha > 0$.[4] In this case integration of Eq. (6) gives

$$\epsilon = C \tau^{\alpha}, \tag{29}$$

where the parameter $C > 0$. The critical velocity $v_{\text{crit}} = \frac{1}{\sqrt{1+\alpha}}$ and, accordingly,[5]

$$x_{\text{crit}} = \frac{t}{\sqrt{1 + \alpha}}. \tag{30}$$

From Eqs. (7), (24), (27), (28) and (29) we obtain

$$E_+ = \frac{C \alpha^{\alpha/2}}{(1 + \alpha)^{(1+\alpha)/2}} t^{1+\alpha}. \tag{31}$$

From Eq. (31) it follows that in the case of the linear equation of state there is one $(+)$-region and one $(-)$-region in the range $0 < x < t$. The energy of the $(+)$-region $((-)$-region) grows (decreases) indefinitely. Therefore, the simplest linear equation of state determines solution, which may be classified as the 'Perpetuum Mobile'.

5. Discussion

When the weak energy condition is violated the Hwa–Bjorken solution is likely to be not the unique one having the total energy E equal to zero. Say, in the model with the linear equation of state (28) it is easy to find a family of self-similar solutions, where the velocity v is a function of the self-similar variable ξ, $v = f_1(\xi)$, and the energy density ϵ has the form:

$$\epsilon = t^{\beta} f_2(\xi). \tag{32}$$

Substituting these expressions in Eqs. (3) and assuming that the equation of state is given by (28) one can easily get two ordinary differential equations for the functions f_1 and f_2. In this case we have

$$E = \int_{-\infty}^{+\infty} dx \, T^{tt} = t^{\beta+1} \int_{-\infty}^{+\infty} d\xi \, f_3(\xi), \tag{33}$$

where f_3 is expressed through f_1 and f_2. It is assumed that the energy density is either equal to zero when ξ is sufficiently large or tends to zero with increase of ξ fast enough to make the integral convergent. When this condition is fulfilled and $\beta \neq -1$ the energy E is equal to zero. Indeed, the energy must not depend on time. On the other hand it is seen from (33) that the energy is proportional to $t^{\beta+1}$. This means that the integral $\int_{-\infty}^{+\infty} d\xi \, f_3(\xi)$ must be equal to zero provided that $\beta \neq -1$.

A more difficult and interesting problem would be to construct an explicit solution having a negative value of the total energy. In this case one should invoke more sophisticated methods of one-dimensional relativistic hydrodynamics such as e.g. the hodograph method introduced by Khalatnikov [11] (see also [12,13]). In the case of the linear equation of state (28) it would also be interesting to exploit the formalism developed in Ref. [14], where the set of Eqs. (3) is reduced to a single equation, which can be analysed for new solutions.

Another approach to the problem consists in use of numerical methods. It seems that numerical methods are more suitable for hydrodynamically stable models e.g. based on the 'Chaplygin type'

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[4] Let us stress that there is no well defined limit to the case of the 'standard' vacuum equation of state $\alpha = 0$ (i.e. $p = -\epsilon$) and that all our conclusions are not applicable to this case.

[5] Note that in this case it is obvious that the hypersurfaces separating the $(+)$ and $(-)$-regions are time-like.

equation of state $p = -\epsilon_*^2/\epsilon$, where ϵ_* is a constant. In framework of numerical methods one can consider a hydrodynamical motion with fixed boundary conditions on a fixed spacial interval, e.g. $v = 0$ when $x = 0$ and when $x = x_1$, where x_1 is fixed. The total energy of the motion determined by initial conditions can, in principal, be either negative or zero for fluids violating the weak energy condition.

An interesting development of these studies would be to consider a model, where the exotic matter having anisotropic tension is concentrated on a straight one-dimensional line in three-dimensional space. One-dimensional motion excited on the line could produce gravitational waves carrying away positive energy from the system. Accordingly, the energy of the motion could decrease indefinitely provided that generation of gravitational waves persists in the course of evolution of the system. Being extrapolated on realistic three-dimensional motions this effect could be dangerous for models of exotic matter, which can be effectively described in the hydrodynamical approximation. The energy of such motions could decrease indefinitely, resulting in a runaway. In this case, the corresponding models should be discarded.

It would be also interesting to look for a quantum model having the properties discussed in the Letter. Note that in quantum case some exotic properties of behaviour of field systems may be expected even when they have a "normal" classical limit. Say, as was discussed e.g. in Ref. [15], in the Milne universe vacuum expectation value of the comoving energy density be negative even for the simplest model of a non-interacting scalar field with sufficiently small mass, in a certain vacuum state.

At the end we would like to point out that although the runaway process related to emission of gravitational waves may be technically difficult to construct for the exotic matter with positive comoving energy density, it can be easily constructed for even more exotic 'ghost' matter having a negative value of the energy density in all frames implemented by physical bodies and clocks. For example, we can make use of the model of rotating relativistic string with two monopoles at its ends emitting weak gravitational waves, see Ref. [16]. It is sufficient to change the sign of the Lagrangians describing the string and the monopoles while keeping the sign of the gravitation part of the action fixed to convert this model to a model of a 'ghost' matter interacting with gravity. It is clear that neither the string dynamics nor conditions of emission of gravitational waves are significantly affected by this procedure. Such a model describes a finite length string with its length ever increasing with time thus making the total energy of the string-monopoles system ever decreasing. The positive energy carried away by gravitational waves may be exploited by an advanced civilization able to construct such a device.

Acknowledgements

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