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A Specular Shape from Shading by Fast Marching Method

Guohui Wang*, Wei Su, Fen Gao

School of Optoelectronic Engineering, Xi'an Technological University, No.4 Jinhua North Road, Xi'an 710032, China

Abstract

In order to reconstruct a surface which has specular reflection from the grayscale information contained in a single two-dimensional (2D) image, a specular shape from shading (SFS) using the fast marching method is presented. Under assumptions that the camera performs an orthographic projection whose direction is equal to the direction of the light source and that the Phong model is used to depict the reflectance property of the surface, the image irradiance equation for specular surface is derived and it is a first-order nonlinear partial differential equation (PDE). By transforming it into a standard Eiknoal PDE, the fast marching method is applied to compute a solution of the resultant Eiknoal equation. Experiments are tested on both a synthetic vase image and a sphere image and the results demonstrate the effectiveness of the presented specular SFS method.

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Keywords: Specular shape from shading; fast marching method; Eiknoal equation

1. Introduction

Three-dimensional (3D) shape reconstruction is a fundamental topic in the domain of computer vision. Shape from shading (SFS), pioneered by Horn [1], is one approach to obtain the shape form the grayscale information contained in a single two-dimensional (2D) image. It is a simple technique to recover the 3D shape of a surface with the certain assumptions on its reflectance property and on the light source and camera projection model. In recent years, more and more work on SFS has drastically come out [2, 3]. At the same time, the fast marching method which is presented by Sethian [4] is widely applied to research the SFS problem. Kimmel and Sethian [5] formulated the image irradiance equation under Lambertian reflectance model as an Eikonal partial differential equation (PDE). They computed its solution by using the fast marching method firstly. Tankus et al. [6] proposed a perspective SFS via a modification of the approach of Kimmel and Sethian [5]. They derived the image irradiance equation under the perspective projection and a distant light source and solved it with the fast marching method. Prados and Soatto [7]

* Corresponding author. Tel.: +86-29-8320-8164 ; fax: +86-29-8320-8072 .
E-mail address: booler@126.com .

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generalized the problem of SFS. They associated the image irradiance equation with a Hamiltonian and computed its solution using optimal control strategy and the fast marching method. Yuen et al. [8] also gave a SFS by adapting the fast marching method to the perspective problem under frontal illumination. Based on the work of Tankus et al. [6], Zhang and Tan [9] presented a de-warping approach that first reconstructs the surface by a perspective SFS technique using fast marching method.

The above work, however, pays more attention to the perfectly diffuse surfaces and the Lambertian model is used to depict their reflectance property. For non-Lambertian surfaces, the work has high error. In order to reconstruct these surfaces, the authors [10, 11] proposed two non-Lambertian SFS methods based on Oren-Nayar model [12] or Phong model [13]. In this paper, we present a specular SFS by using the fast marching method based on our preceding work [10, 11]. Under assumptions that the camera performs an orthographic projection whose direction is equal to the direction of the light source and that the Phong model is used to characterize the reflectance property of the surface, the image irradiance equation for specular surface is derived a standard Eiknoal PDE, and the fast marching method is applied to compute a solution of the resultant Eiknoal equation.

2. A specular shape from shading

With the basis that the image plane is x–y plane and the optical axis of the camera aligned with z– axis, the shape from shading problem can be regarded as that of reconstructing a surface, z, satisfying the following image irradiance equation [1, 11]:

$$E(x,y) = R(p(x,y), q(x,y)) = R\left(\frac{\partial z(x,y)}{\partial x}, \frac{\partial z(x,y)}{\partial y}\right), \tag{1}$$

where $E(x,y)$ is the image irradiance which is equal to the image brightness. R is the reflectance map established by the reflectance model.

Assuming that the surface has a Phong reflectance, so the surface reflected radiance is given as

$$L_r = I_0\left(\frac{\mathbf{n}}{\|\mathbf{n}\|} \cdot \frac{\mathbf{h}}{\|\mathbf{h}\|}\right)^M = I_0(\cos\delta)^M, \tag{2}$$

where I_0 is the intensity of the light source, M is a power modeling the specular reflected light, and δ is the angle between the surface normal $\mathbf{n}$ and $\mathbf{h}$ as shown in Fig. 1.

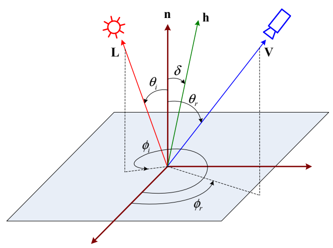

Fig. 1. Reflection geometry model. $\mathbf{h}$ is the angular bisector of reflected direction $\mathbf{V}$ (θ_r, ϕ_r) and incident direction $\mathbf{L}$ (θ_i, ϕ_i).

Under assumptions that the camera performs an orthographic projection whose direction is equal to the direction of the light source, $\mathbf{n} = [p, q, -1]^T$ is used to describe the vector $\mathbf{n}$ at the point $(x, y, z(x,y))$ and we have $\theta_i = \theta_r$, $\phi_r = \phi_i$. Now, for the angle δ, it is equal to the angle θ_i and Eq. (2) is turned into

$$L_r = I_0(\cos\theta_i)^M = I_0\left(\frac{\mathbf{n}}{\|\mathbf{n}\|} \cdot \frac{\mathbf{L}}{\|\mathbf{L}\|}\right)^M. \tag{3}$$

If the camera's direction is set to $[0,0,-1]^T$ and based on our preceding work [11], we get the reflectance map of specular shape from shading

$$R(p,q)=\left(\frac{1}{\sqrt{1+p^2+q^2}}\right)^M=\left(\frac{1}{\sqrt{1+\|\nabla z\|^2}}\right)^M. \tag{4}$$

Also, the image irradiance equation derived by specular shape from shading is

$$\|\nabla z(x,y)\|=\sqrt{E(x,y)^{-2/M}-1}. \tag{5}$$

Now, with the change of variable $F(x,y)\equiv\sqrt{E(x,y)^{-2/M}-1}$ the specular SFS problem (5) can be further formulated as a standard Eiknoal PDE

$$\begin{cases}\|\nabla z(x,y)\|=\sqrt{E(x,y)^{-2/M}-1}=F(x,y) & \forall x\in\Omega,\\ z(x,y)=\varphi(x,y) & \forall x\in\partial\Omega,\end{cases} \tag{6}$$

where Ω is the image domain and $\varphi(\mathbf{x})$ is a real continuous function defined on $\partial\Omega$.

3. Fast marching method for the resultant fiknoal PDE

The fast marching method [5] is a non-iterative algorithm and has a time complexity of $O(N\log N)$, where N is the total number of grid points. The basic idea of the method is to systematically construct the solution at every grid point in an order that is consistent with the way wave fronts propagate. It uses a fast sorting technique to find the solution in one-pass algorithm. Now we use the fast marching method to compute a solution of the resultant Eiknoal PDE.

Consider a uniform discretization $\{(x_i,y_j)=(ih,\ jh),\ i=1,2,\cdots,m,\ j=1,2,\cdots,n\}$ of the domain Ω with grid size (h,h). Now our goal is to obtain a discrete solution $z_{i,j}=z(x_i,y_j)$ of the unknown $z(x,y)$. The following equation is the numerical approximation of the Eikonal PDE (6) [10]:

$$\max\left[\frac{z_{i,j}-z_{i-1,j}}{h},-\frac{z_{i+1,j}-z_{i,j}}{h},0\right]^2+\max\left[\frac{z_{i,j}-z_{i,j-1}}{h},-\frac{z_{i,j+1}-z_{i,j}}{h},0\right]^2=F_{i,j}^2. \tag{7}$$

Define $z_1=\min[z_{i-1,j},z_{i+1,j}]$ and $z_2=\min[z_{i,j-1},z_{i,j+1}]$. Now, the solution of the Eikonal PDE (6) is:

$$z_{i,j}=\begin{cases}\dfrac{z_1+z_2+\sqrt{2h^2F_{i,j}^2-(z_1-z_2)^2}}{2} & |z_1-z_2|<hF_{i,j},\\ \min[z_1,z_2]+hF_{i,j} & |z_1-z_2|\geq hF_{i,j}.\end{cases} \tag{8}$$

The fast marching method for specular shape from shading is summarized in the following:

1) Initialization: All grid points are assigned to one of three sets, *known*, *trial*, *far*, as follows:

- *Known* is the set of the initial grid points corresponding to the extremums of z. *Known* points are the seeded points and their values will not be changed.
- *Trial* is the set of the neighbours of *known* points, not in *known*. In our algorithm, *trial* points are the four nearest neighbours of the seeded points with z values computed as $\varphi(ih,\ jh)+hF(ih,\ jh)$ and their values will be changed later.
- *Far* is the set of all other grid points with $z=\infty$, where there is not yet a computation.

2) Marching forwards:

- Begin loop: Let $(i_{\min},j_{\min})$ be the point among all *trial* points with the smallest value for z.
- Add the point $(i_{\min},j_{\min})$ to *known* and remove it from *trial*.
- Tag as neighbours any points $(i_{\min}-1,j_{\min})$, $(i_{\min},j_{\min}-1)$, $(i_{\min}+1,j_{\min})$, $(i_{\min},j_{\min}+1)$ that are either in *trial* or *far*. If the neighbour is in *far*, remove it from that set and add it to *trial*.
- Calculate the value of z at the four nearest neighbours of $(i_{\min},j_{\min})$ which belong to *trial*

using the scheme (8).

- If all grid points are *known* then exit, otherwise return to top of loop.

4. Experimental results

The experiments are performed on a PC with an Athlon 64 X2 5000+ CPU and 2GB memory. All algorithms are implemented using Matlab code with C mex functions for the core operations.

One synthetic vase image which is generated by the following formula is tested:

$$z(x,y)=\sqrt{f(x)^2-y^2}, \tag{9}$$

where $f(x)=0.15-0.025(2x-1)(3x-2)^2(2x+1)^2(6x+1)$ and $(x,y)\in[-0.5,0.5]\times[-0.5,0.5]$. We map the x and y ranges to $[-49,50]$ and scale $z(x,y)$ by a factor of 100. This yields a maximum depth value of approximately 28.55.

Another synthetic sphere image which is generated by the following formula is also tested:

$$z(x,y)=\sqrt{R^2-x^2-y^2}, \tag{10}$$

where $(x,y)\in[-49,50]\times[-49,50]$ and $R=40$ is the radius.

The synthetic specular vase and sphere images generated by the Eq. (5) are illustrated in Fig. 2(a) and Fig. 3(a), respectively. The power parameter M is set as 8. Fig. 2(b) and Fig. 3(b) show the ground truth of the vase and sphere. The recovered surfaces using the presented approach are shown in Fig. 2(c) and Fig. 3(c), while the error surfaces with the ground truth are shown in Fig. 2(d) and Fig. 3(d), respectively. The mean absolute error (MAE) and the root mean square error (RMSE) are 0.5014 pixels and 0.6027 pixels for vase and 0.6533 pixels and 0.7957 pixels for sphere, respectively. From the reconstructed results illustrated in Fig. 2 and Fig. 3 and quantitative MAE and RMSE, we can see that the presented approach is more accurate and effective.

5. Conclusion

This paper describes a specular shape from shading by using the fast marching method to reconstruct a surface which has specular reflection from the grayscale information. With assumptions that the camera performs an orthographic projection whose direction is equal to the direction of the light source and that the Phong model is used to depict the reflectance property of the surface, the image irradiance equation for specular surface is derived and transformed into a standard Eiknoal PDE. Then, the fast marching method is applied to compute the solution of the resultant Eiknoal equation. Experiments are tested on two images and the results demonstrate the effectiveness of the presented approach.

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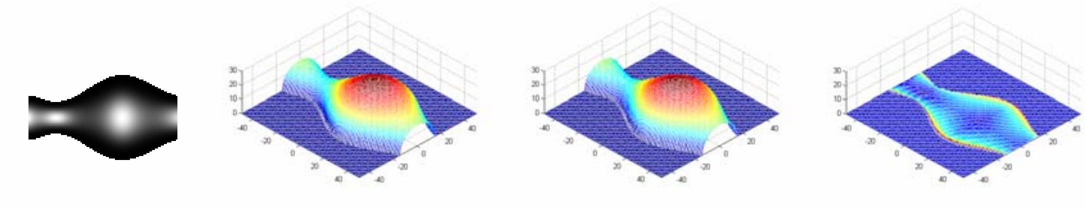
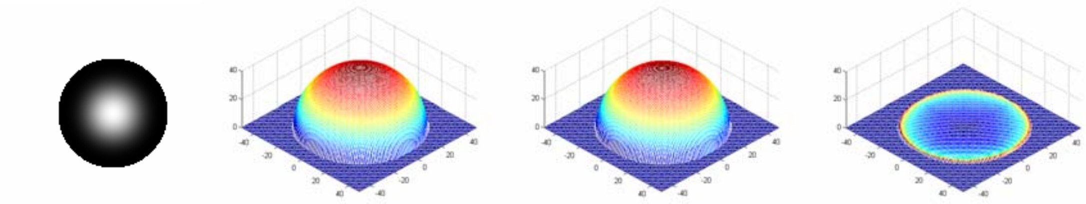


(a) (b) (c) (d)

Fig. 2. Experimental results for the vase.



(a) (b) (c) (d)

Fig. 3. Experimental results for the sphere.

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