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Physics Letters B

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Quintom dark energy in the DGP braneworld cosmology

P. Moyassari^a, M.R. Setare^{b,*}^a Department of Physics, Nanjing University 22 Hankou Road, Nanjing 210093, China^b Department of Science, Payame Noor University, Bijar, Iran

ARTICLE INFO

Article history:

Received 18 November 2008

Received in revised form 10 March 2009

Accepted 25 March 2009

Available online 26 March 2009

Editor: S. Dodelson

ABSTRACT

In this Letter we consider a Z_2 symmetrical 3-brane embedded in a 5-dimensional spacetime. We study the effective Einstein equation and acceleration condition in presence of the quintom dark energy fluid as the bulk matter field. It is shown that the time-dependent bulk quintom field induces a time-dependent cosmological constant on the brane. In the framework of the DGP model, the effective Einstein equation is obtained in two different cases: (i) where the quintom field is considered as the bulk matter field and the brane is empty and, (ii) where the quintom dark energy is confined on the brane and the bulk is empty. We show that in both cases one could obtain a self-inflationary solution at late time in positive branch $\epsilon = 1$, and an asymptotically static universe in negative branch $\epsilon = -1$.

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1. Introduction

Recent observations of type Ia supernova (SNIa) and WMAP [1,2] indicate that our universe is currently undergoing an accelerating expansion, which confront the fundamental theories with great challenges and also make the researches on this problem a major endeavor in modern astrophysics and cosmology. Missing energy density—with negative pressure—responsible for this expansion has been dubbed dark energy. Wide range of scenarios have been proposed to explain this acceleration while most of them cannot explain all the features of universe or they have so many parameters that makes them difficult to fit. The models which have been discussed widely in literature are those which consider vacuum energy (cosmological constant) [3] as dark energy, introduce fifth elements and dub it quintessence [4] or scenarios named phantom [5] with $w < -1$, where w is parameter of state. A challenging issue is that the time-dependent dark energy gives a better fitting than a cosmological constant, and in particular the analysis of the properties of dark energy from recent observations mildly favor models with w crossing -1 at redshift $z \approx 0.2$. Neither the quintessence nor the phantom alone can fulfill the transition from $w > -1$ to $w < -1$ and vice versa. Although for k-essence [6] one can have both $w \geq -1$ and $w < -1$, it has been lately considered by Ref. [7] that it is very difficult for k-essence to get w across -1 during evolving. But one can show [8,9] that considering the combination of quintessence and phantom in a joint model, the transition can be fulfilled. This model, dubbed quintom, can produce a better fit to the data than more familiar models with $w \geq -1$. In other words, the quintom model of

dark energy represents a transition of dark energy equation of state from $w > -1$ to $w < -1$, or vice versa, namely from $w < -1$ to $w > -1$ is also one realization of quintom, as can be seen clearly in [10]. Although the models with negative kinetic term are often plagued by instability, there are possibilities that these models might be phenomenologically viable if considered as effective field theories [11,12].

An alternative way of explaining the observed acceleration of the late universe is to modify gravity at large scales. A well-studied model of modified gravity is the Dvali–Gabadadze–Porrati (DGP) braneworld model [13] where the brane is embedded in the flat bulk with infinite extra dimension. In this model gravity leaks of the 4-dimensional brane universe into 5-dimensional bulk spacetime at large scales. The inclusion of a graviton kinetic term on the brane recovers the usual gravitational force law scaling, $1/r^2$, at short distances, but at large distances it asymptotes to the 5-dimension scaling, $1/r^3$. Motivated by string/M theory, the AdS/CFT correspondence, and the hierarchy problem of particle physics, braneworld models were studied actively in recent years [14–17]. In these models, our universe is realized as a boundary of a higher dimensional spacetime. The matter particles cannot freely propagate in those large extra dimensions, but must be constrained to live on a 4-dimensional submanifold. The DGP model has a large scale/low energy effect of causing the expansion rate of the universe to accelerate. In almost all of works on braneworld models, the 5-dimensional bulk spacetime is assumed to be vacuum except for the presence of the cosmological constant, and the matter fields on the brane are regarded as responsible for the dynamics of the brane. However, from the unified theoretic point of view, the gravitational action is not necessarily the Einstein–Hilbert action. In fact, string theory tells us that the dimensionally reduced effective action includes not only higher-order curvature terms but also dilatonic gravitational scalar fields. Thus at the level of the low-

* Corresponding author.

E-mail address: rezakord@ipm.ir (M.R. Setare).

energy 5-dimensional theory, it is naturally expected that there appears a dilaton-like scalar field in addition to the Einstein–Hilbert action [18]. Hence it is of interest to investigate how such a scalar field in the 5-dimensional theory affects the braneworld [19,20].

In this Letter our main motivation is investigating the effects of the bulk quintom field on the evolution of the universe in the braneworld scenario and in the DGP model. We first review the braneworld scenario in presence of the bulk matter field in Section 2. We study the acceleration condition for the universe with quintom dark energy in the bulk and show that the time-dependent bulk quintom field alters the brane as a time-dependent cosmological constant which is related to the quintom potential on the brane. In Section 3 we obtain the generalized Einstein equation in the DGP model in presence of the tension and the bulk matter field. In the two following sections we investigate whether it is possible to have a late time accelerating phase on the brane when there is a quintom dark energy fluid in the bulk and the brane is empty; or inversely, when there is a quintom dark energy fluid on the brane and the bulk is empty.

2. Effective Einstein equation on the braneworld

In this section we briefly review the braneworld scenario in presence of the tension and bulk matter field. In the braneworld scenario, our 4-dimensional world is described by a domain wall (brane) in 5-dimensional spacetime. We consider an ansatz for the 5-dimensional metric of the form

$$ds^2 = -n^2(t, y) dt^2 + a^2(t, y) \gamma_{ij} dx^i dx^j + b^2(t, y) dy^2, \quad (1)$$

where y is the coordinate of the fifth dimension and γ_{ij} is a maximally symmetric 3-dimensional metric. We will use k to parameterize the spatial curvature and assume that the brane is a hypersurface defined by $y = 0$. We shall be interested in the model described by the action

$$S = \int d^5x \sqrt{-g} \left(\frac{1}{2k_5^2} {}^{(5)}R - \Lambda + \mathcal{L}_B^{\text{mat}} \right) + \int d^4x \sqrt{-q} (-\xi + \mathcal{L}_b^{\text{mat}}), \quad (2)$$

where ${}^{(5)}R$ is the scalar curvature of the 5-dimensional metric g_{AB} , Λ is the bulk cosmological constant, ξ is the brane tension, $k_5^2 = 8\pi G_5$, and $q_{AB} = g_{AB} - n_A n_B$ (n_A is the unit vector normal to the brane and $A, B = 0, 1, 2, 3, 5$) is the induced metric on the 3-brane. The 5-dimensional Einstein equation can be written as

$${}^{(5)}R_{AB} - \frac{1}{2} g_{AB} {}^{(5)}R = k_5^2 (-\Lambda g_{AB} + T_{AB} + S_{\mu\nu} \delta_A^\mu \delta_B^\nu \delta(y_b)), \quad (3)$$

here $\delta(y_b) = \frac{\delta(y)}{b}$, T_{AB} is the energy–momentum tensor of the bulk matter and the last term corresponds to the matter content on the brane

$$S_{\mu\nu} = -\xi g_{\mu\nu} + \tau_{\mu\nu}. \quad (4)$$

The non-zero components of the 5-dimensional Einstein equation are

$$3 \left\{ -\frac{\dot{a}}{n^2 a} \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) + \frac{1}{b^2} \left(\frac{a''}{a} + \frac{a'}{a} \left(\frac{a'}{a} - \frac{b'}{b} \right) \right) - \frac{k}{a^2} \right\} = k_5^2 (-\Lambda + T_0^0 + S_0^0 \delta(y_b)), \quad (5)$$

$$\frac{1}{b^2} \delta_j^i \left\{ \frac{a'}{a} \left(\frac{a'}{a} + \frac{n'}{n} \right) - \frac{b'}{ba^2} \left(\frac{n'}{n} + 2 \frac{a'}{a} \right) + \frac{a''}{a} + \frac{n''}{n} \right\} + \frac{1}{n^2} \delta_j^i \left\{ \frac{\dot{a}}{a} \left(-\frac{\dot{a}}{a} + 2 \frac{\dot{n}}{n} \right) - 2 \frac{\ddot{a}}{a} + \frac{\dot{b}}{b} \left(-2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\ddot{b}}{b} \right\} - k \delta_j^i = k_5^2 (-\Lambda + T_j^j + S_j^j \delta(y_b)), \quad (6)$$

$$3 \left\{ \frac{n'}{n} \frac{\dot{a}}{a} + \frac{a'}{a} \frac{\dot{b}}{b} - \frac{\dot{a}'}{a} \right\} = k_5^2 T_{05}, \quad (7)$$

$$3 \left\{ \frac{a'}{ab^2} \left(\frac{a'}{a} + \frac{n'}{n} \right) - \frac{1}{n^2} \left(\frac{\dot{a}}{a} \left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right) - \frac{k}{a^2} \right\} = k_5^2 (-\Lambda + T_5^5), \quad (8)$$

where primes indicate derivatives with respect to y , while dots derivatives with respect to t . Assuming a perfect fluid on the brane

$$\tau_\nu^\mu = \text{diag}(-\rho_b, p_b, p_b, p_b), \quad (9)$$

and a quintom field in the bulk space containing the normal scalar field $\phi(t, y)$ and negative kinetic scalar field $\sigma(t, y)$, with the Lagrangian expressed as the following form

$$\mathcal{L}_B^{\text{mat}} = \frac{1}{2} g^{AB} (\phi_{,A} \phi_{,B} - \sigma_{,A} \sigma_{,B}) + V(\phi, \sigma). \quad (10)$$

According to this action, the energy–momentum tensor of the bulk quintom field is given by

$$T_{AB} = \phi_{,A} \phi_{,B} - \sigma_{,A} \sigma_{,B} - g_{AB} \left(\frac{1}{2} g^{CD} (\phi_{,C} \phi_{,D} - \sigma_{,C} \sigma_{,D}) + V(\phi, \sigma) \right). \quad (11)$$

In order to focus on the cosmological evolution on the brane we use the Gaussian normal coordinates ($b(y, t) = 1$) [21]. Thus the equations of motion of the scalar field ϕ and σ in bulk space are

$$\begin{aligned} -\ddot{\phi} - \left(3 \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \dot{\phi} + n^2 \left[\left(\frac{n'}{n} + 3 \frac{a'}{a} \right) \phi' + \phi'' \right] - n^2 \frac{\partial V(\phi, \sigma)}{\partial \phi} \\ = \frac{\delta \mathcal{L}_b^{\text{mat}}}{\delta \phi} \delta(y_b), \\ -\ddot{\sigma} - \left(3 \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) \dot{\sigma} + n^2 \left[\left(\frac{n'}{n} + 3 \frac{a'}{a} \right) \sigma' + \sigma'' \right] + n^2 \frac{\partial V(\phi, \sigma)}{\partial \sigma} \\ = -\frac{\delta \mathcal{L}_b^{\text{mat}}}{\delta \sigma} \delta(y_b). \end{aligned} \quad (12)$$

We are interested in studying the Einstein equation in presence of a quintom field in the bulk at the location of the brane. Without losing generality we choose $n(t, 0) = 1$ which can be achieved by scaling the time coordinate. As is well known, the presence of the brane leads to a singular term proportional to δ -function in y on the right-hand sides of the Einstein equations (5) and (6) and the equation of motions (12), which have to be matched by singularity in the second derivatives in y on the left-hand side. Since all fields under consideration are symmetric under the orbifold symmetry Z_2 , these jumps in the first derivatives in y fix these first derivatives completely at $y = 0$. Here, these junction conditions read

$$\begin{aligned} \frac{a'}{a} \Big|_{y=0} &= -\frac{k_5^2}{6} (\rho_b + \xi), \\ n' \Big|_{y=0} &= \frac{k_5^2}{6} (3p_b + 2\rho_b - \xi), \end{aligned} \quad (13)$$

and

$$\begin{aligned} \phi' \Big|_{y=0} &= \frac{1}{2} \frac{\delta \mathcal{L}_b^{\text{mat}}}{\delta \phi}, \\ \sigma' \Big|_{y=0} &= -\frac{1}{2} \frac{\delta \mathcal{L}_b^{\text{mat}}}{\delta \sigma}. \end{aligned} \quad (14)$$

Using the components 00 and 55 of the Einstein equation in bulk space one can obtain

$$F' = \frac{2k_5^2}{3} (\Lambda - T_0^0) a^3 a' - \frac{2k_5^2}{3} T_5^0 a^3 \dot{a}, \quad (15)$$

$$\dot{F} = \frac{2k_5^2}{3} (\Lambda - T_5^5) a^3 \dot{a} - \frac{2k_5^2}{3} n^2 T_5^0 a^3 a', \quad (16)$$

where F is a function of t and y defined by

$$F(t, y) = \frac{(\dot{a}a)^2}{n^2} - (a')^2 + ka^2. \quad (17)$$

Since the quintom field does not appear in the matter field Lagrangian on the brane ($\mathcal{L}_b^{\text{mat}}$), Eq. (14) implies that the quintom field is independent of y on the brane, namely $\phi'|_{y=0} = \sigma'|_{y=0} = 0$ [19,20]. Therefor the non-vanishing components of the quintom energy-momentum tensor at the location of the brane are

$$\begin{aligned} T_0^0 &= -\rho_B = -\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\sigma}^2 - V(\phi, \sigma), \\ T_i^i &= p_B = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\sigma}^2 - V(\phi, \sigma), \\ T_{05} &= \dot{\phi}\phi' - \dot{\sigma}\sigma' = 0, \\ T_5^5 &= \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\dot{\sigma}^2 - V(\phi, \sigma). \end{aligned} \quad (18)$$

As one can see, in the case of y independent bulk quintom field T_{05} vanishes. It means that there is no flow of matter along the fifth dimension. Using (18) one can solve Eq. (15) which leads to the first integral of the 00 component of Einstein equation as

$$\frac{k_5^2}{6}(\Lambda + \rho_B) + \frac{C}{a^4} - \frac{\dot{a}^2}{a^2 n^2} + \frac{a'^2}{a^2} - \frac{k}{a^2} = 0, \quad (19)$$

where C is a constant of integration which is usually referred to dark radiation [22]. Substituting the junction conditions (13) into above equation, we arrive at the generalized Friedmann equation on the brane as

$$H^2 + \frac{k}{a^2} = \frac{k_5^2}{6} \left(\Lambda + \frac{k_5^2}{6} \xi^2 \right) + \frac{k_5^4}{18} \xi \rho_b + \frac{k_5^2}{6} \rho_B + \frac{k_5^4}{36} \rho_b^2 + \frac{C}{a^4}, \quad (20)$$

here $H = \frac{\dot{a}}{a}$ is the Hubble parameter. As one can see from Eq. (20), in the absence of the bulk matter field, the cosmological constant and the brane tension, the equation gives rise to a Friedmann equation of the form $H \propto \rho_b$ instead of $H \propto \sqrt{\rho_b}$ which is inconsistent with cosmological observation. This problem can be solved by either considering the cosmological constant and tension on the brane or considering a matter field in the bulk [23,24]. Recalling the junction conditions (13), the 05 component of Einstein equation and field equations (12) on the brane take the following form respectively

$$\dot{\rho}_b + 3H(\rho_b + p_b) = 0, \quad (21)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V(\phi, \sigma)}{\partial \phi} = 0, \quad (22)$$

$$\ddot{\sigma} + 3\frac{\dot{a}}{a}\dot{\sigma} - \frac{\partial V(\phi, \sigma)}{\partial \sigma} = 0. \quad (23)$$

It should be noted that if scalar field ϕ and σ satisfy the field equations (22) and (23) respectively, the bulk energy-momentum tensor is automatically conserved and we have

$$\dot{\rho}_B + 3H(\rho_B + p_B) = 0. \quad (24)$$

We are interested in studying the acceleration condition for a universe with the quintom field in the bulk. The condition for acceleration can be obtained from (20) by using the conservation equation of the brane and bulk matter field (21) and (24)

$$\begin{aligned} \frac{\ddot{a}}{a} &= \frac{k_5^2}{6} \left(\Lambda + \frac{k_5^2}{6} \xi^2 \right) - \frac{k_5^4}{36} \xi (\rho_b + 3p_b) \\ &\quad - \frac{k_5^2}{12} (\rho_B + 3p_B) - \frac{k_5^4}{36} (2\rho_b^2 + 3\rho_b p_b) - \frac{C}{a^4}. \end{aligned} \quad (25)$$

To study the role of the bulk quintom field in the late time acceleration phase on the brane, we ignore the effect of tension, brane matter, cosmological constant and dark radiation

$$\frac{\ddot{a}}{a} = -\frac{k_5^2}{12} (\rho_B + 3p_B). \quad (26)$$

Thus the acceleration condition for a universe with quintom dark energy in bulk is

$$p_B < -\frac{\rho_B}{3}, \quad \text{or} \quad \dot{\phi}^2 - \dot{\sigma}^2 < V(\phi, \sigma). \quad (27)$$

Now we consider the 55 component of Einstein equation at the position of the brane which leads to the Raychaudhuri equation

$$\begin{aligned} \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} &= \frac{k_5^2}{3} \left(\Lambda + \frac{k_5^2}{6} \xi^2 \right) - \frac{k_5^2}{36} (\xi (3p_b - \rho_b) \\ &\quad + \rho_b (3p_b + \rho_b)) - \frac{k_5^2}{3} T_5^5. \end{aligned} \quad (28)$$

Using Eq. (20) one can rewrite the above equation as

$$\begin{aligned} \frac{\ddot{a}}{a} &= \frac{k_5^2}{6} \left(\Lambda + \frac{k_5^2}{6} \xi^2 \right) - \frac{k_5^4}{36} \xi (\rho_b + 3p_b) - \frac{k_5^2}{6} \rho_B \\ &\quad - \frac{k_5^4}{36} (2\rho_b^2 + 3\rho_b p_b) - \frac{C}{a^4} - \frac{k_5^4}{3} T_5^5. \end{aligned} \quad (29)$$

Comparing Eq. (25) with (29) provide a constraint on the bulk energy-momentum tensor

$$(3p_B - \rho_B) = 4T_5^5, \quad (30)$$

which for the quintom field with the energy-momentum tensor (18) leads to

$$\dot{\phi}^2 - \dot{\sigma}^2 = 0. \quad (31)$$

It means that the time-dependent bulk quintom field influences the brane like a time-dependent cosmological constant which can be written in terms of the quintom potential energy, $p_B = -\rho_B = -V(\phi)$ [25]. For a particular solution of (31) in which ϕ and σ are both constant on the brane, we arrive at the natural cosmological constant induced by the time-dependent bulk quintom field.

3. Generalized Einstein equation in the DGP braneworld

In the DGP model, which provide a simple mechanism to modify gravity at large distances, it is supposed that a 3-dimensional brane is embedded in a flat 5-dimensional bulk. This model predicts that 4-dimensional Einstein gravity is a short-distance phenomenon with deviations showing up at large distances. The transition between four- and higher dimensional gravitational potentials in the DGP model arises as a consequence of the presence of both brane and bulk Einstein terms in the action. The DGP model includes a length scale below which the potential has usual Newtonian form and above which the gravity becomes 5-dimensional. The cross over scale between the 4-dimensional and 5-dimensional gravity is $r_c = \frac{k_5^2}{2\mu^2}$ in which $\mu^2 = 8\pi G_4$. In this framework, existence of a higher dimensional embedding space allows for the existence of bulk or brane matter which can certainly influence the cosmological evolution on the brane. Now we proceed to obtain the generalized DGP model in which both bulk cosmological constant Λ and brane tension ξ are non-zero. We consider the model described by the gravitational bulk-brane action

$$\begin{aligned} S &= \int d^5x \sqrt{-g} \left(\frac{1}{2k_5^2} {}^{(5)}R - \Lambda + \mathcal{L}_B^{\text{mat}} \right) \\ &\quad + \int d^4x \sqrt{-q} \left(\frac{1}{2\mu^2} {}^{(4)}R - \xi + \mathcal{L}_b^{\text{mat}} \right), \end{aligned} \quad (32)$$

where ${}^{(4)}R$ is the Ricci scalar of the induced metric $q_{\mu\nu}$. The 5-dimensional Einstein equation takes the form

$${}^{(5)}R_{AB} - \frac{1}{2}g_{AB}{}^{(5)}R = k_5^2(-\Lambda g_{AB} + T_{AB} + S_{\mu\nu}\delta_A^\mu\delta_B^\nu\delta(y_b)). \quad (33)$$

Here T_{AB} is the energy–momentum tensor of the bulk matter, Λ is the cosmological constant of the bulk spacetime and the energy–momentum tensor on the brane is given by

$$S_{\mu\nu} = -\xi q_{\mu\nu} + \tau_{\mu\nu} - \mu^{-2}U_{\mu\nu}, \quad (34)$$

the last term is the contribution coming from the scalar curvature of the brane with the non-vanishing components given by

$$U_{00} = 3\left(H^2 + k\frac{n^2}{a^2}\right),$$

$$U_{ij} = \left(\frac{a^2}{n^2}\left(-H^2 + 2H\frac{\dot{n}}{n} - 2\frac{\ddot{a}}{a}\right) - k\right)\gamma_{ij}. \quad (35)$$

From 00 and ij components of the Einstein equation (33) we find the following junction conditions which simply relate the jumps of derivatives of the metric across the brane to the stress tensor inside the brane

$$\frac{a'}{a}\Big|_{y=0} = -\frac{k_5^2}{6}(\rho_b + \xi) + r_c\left(H^2 + \frac{k}{a^2}\right),$$

$$n'\Big|_{y=0} = \frac{k_5^2}{6}(3p_b + 2\rho_b - \xi) + r_c\left(-H^2 + 2\frac{\ddot{a}}{a} - \frac{k}{a^2}\right). \quad (36)$$

In the following two sections we study the Einstein equation (33) when the quintom field is considered as the bulk matter field, and a quintom dark energy confined on the brane, respectively.

3.1. Quintom field in bulk space

We consider the quintom field as the bulk matter field with Lagrangian expression in Eq. (10). Integrating the equation of 00 component of (33) around $y = 0$ and using junction conditions (36), we arrive at the generalized (first) Friedmann equation

$$\left(1 + \frac{\xi k_5^2}{6\mu^2}\right)\left(H^2 + \frac{k}{a^2}\right) - \frac{k_5^2}{6}\left(\Lambda + \frac{k_5^2}{6}\xi^2\right) - \frac{k_5^4}{18}\xi\rho_b - \frac{k_5^2}{6}\rho_B + \frac{C}{a^4}$$

$$= \frac{k_5^4}{36\mu^4}\left(-\mu^2\rho_b + 3\left(H^2 + \frac{k}{a^2}\right)\right)^2. \quad (37)$$

The brane-Friedmann equation (20) can be derived from above equation by letting μ go to infinity. We are interested to study the effect of the quintom field on the brane. Ignoring the cosmological constant Λ , the brane tension and the matter field on the brane, Eq. (37) can straightforwardly be rewritten as

$$H^2 + \frac{k}{a^2} = \frac{1}{2r_c^2}\left(1 + \epsilon\sqrt{1 - \frac{2k_5^2}{3}\rho_B r_c^2}\right), \quad (38)$$

here ρ_B is the energy density of the bulk quintom field on the brane derived in Eq. (18). The two different possible ϵ namely $\epsilon = \pm 1$, correspond to two different embeddings of the brane into the bulk spacetime [26]. Since the bulk quintom field satisfies the usual energy–momentum conservation law on the brane (24), we have $\rho_B \propto a^{-3(\omega+1)}$ (ω is the state parameter). Integrating Eq. (38) for $k = 0$ and $\omega \geq -1$ where $\dot{\phi} > \dot{\sigma}$, shows that the scale factor a diverges at late time¹ (see Fig. 1). Thus the energy density of the

¹ The integration of Eq. (38) leads to $l = \int \frac{da}{a\sqrt{1+\epsilon\sqrt{1-\frac{\beta}{a^m}}}} = \frac{1}{\sqrt{2}r_c} \int dt$ in which $m > 0$ when $\omega > -1$ and $m < 0$ when $\omega < -1$ and $\beta = \frac{2k_5^2 r_c^2}{3}$. The variation of l against a in different value of m and ϵ are plotted in Fig. 1.

bulk matter goes to zero at late time and reaches a regime where it is small in comparison with $1/r_c^2$. In the case $\omega < -1$ where $\dot{\phi} < \dot{\sigma}$, integrating Eq. (38) indicates a vanishing scale factor a at late time, so the matter density goes to zero and we could use the assumption $k_5^2\rho_B \ll 1/r_c^2$. Therefore, in the DGP model with a quintom dark energy fluid in the bulk space, one can expand the Einstein equation (38) under the condition that $k_5^2\rho_B \ll 1/r_c^2$ for all range of ω . At zero order and for spatially flat metric, two different results depend on the value of ϵ can be derived. Considering the case $\epsilon = -1$ yields

$$H^2 = 0, \quad (39)$$

which describes an asymptotically static universe. In the other case we take $\epsilon = 1$ which leads to

$$H^2 = \frac{1}{r_c^2}, \quad \text{or} \quad a(t) \propto \exp\left(\frac{t}{r_c}\right). \quad (40)$$

This provides the self-inflationary solution at late time which is the most important aspect of the DGP model. Therefore, the late time behavior of the universe does not alter even if we ignore the matter field on the brane and consider a model of the universe filled with the bulk quintom dark energy.

3.2. Quintom field on the brane

In this section we ignore the bulk matter and consider a quintom dark energy confined on the brane in the DGP model with the Lagrangian expression as

$$\mathcal{L}_b^{\text{mat}} = \frac{1}{2}q^{\mu\nu}(\phi_{,\mu}\phi_{,\nu} - \sigma_{,\mu}\sigma_{,\nu}) + \tilde{V}(\phi, \sigma). \quad (41)$$

The energy–momentum tensor of the quintom field on the brane is given by

$$\tau_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} - \sigma_{,\mu}\sigma_{,\nu}$$

$$- q_{\mu\nu}\left(\frac{1}{2}q^{\alpha\beta}(\phi_{,\alpha}\phi_{,\beta} - \sigma_{,\alpha}\sigma_{,\beta}) + \tilde{V}(\phi, \sigma)\right). \quad (42)$$

In absence of the bulk matter field and the brane tension, the generalized Friedmann equation (37) leads to [27]

$$\sqrt{H^2 + \frac{k}{a^2}} = \frac{1}{2r_c}\left(\epsilon + \sqrt{1 + \frac{4\mu^2}{3}\rho_b r_c^2}\right), \quad (43)$$

in which ρ_b is quintom energy density obtained by (42). Since the energy–momentum tensor on the brane is conserved, we could apply the strategy used in the previous section to study the late time cosmology on the brane. Integrating Eq. (43) for a spatially flat spacetime yields the same result as the previous section. It indicates that for $\omega \geq -1$ the scale factor a diverges at late time [27] and for $\omega < -1$ the scale factor a vanishes at late time² (Fig. 2). Thus the energy density of quintom dark energy goes to zero for late time and reaches a regime where it is small in comparison with $1/r_c^2$. Expanding Eq. (43) under the condition $\mu^2\rho_b \ll 1/r_c^2$ provides an asymptotically static universe, $H = 0$, in the case $\epsilon = -1$ and a self-accelerating phase, $H = \frac{1}{r_c}$, in the case $\epsilon = 1$. Therefore the presence of quintom dark energy on the brane or in the bulk does not change the late time behavior of the universe. In both cases for all range of ω ($\omega < -1$ or $\omega \geq -1$), one can derive the self-accelerating universe at late time.

² The integration of Eq. (43) leads to $l' = \int \frac{da}{a(1+\epsilon\sqrt{1+\frac{\beta}{a^m}})} = \frac{1}{2r_c} \int dt$ in which $m > 0$ when $\omega > -1$ and $m < 0$ when $\omega < -1$ and $\beta = \frac{4\mu^2 r_c^2}{3}$.

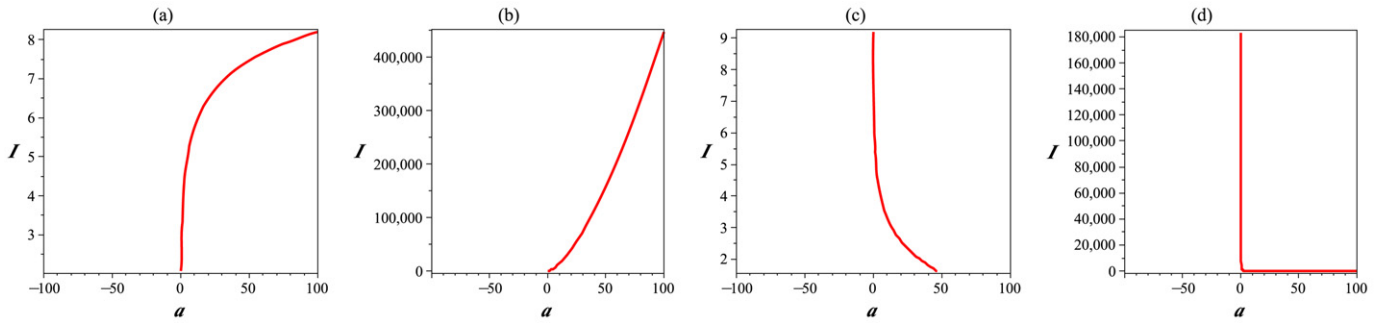


Fig. 1. These figures show the evolution of I as a function of scale factor.

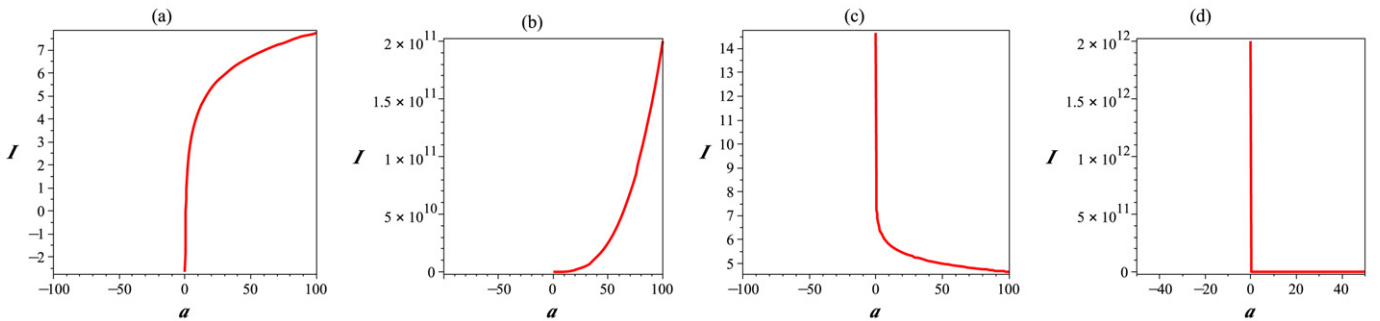


Fig. 2. These figures show the evolution of I' as a function of scale factor.

4. Conclusion

We have studied the cosmology of a Z_2 symmetrical 3-brane embedded in a 5-dimensional spacetime including a quintom dark energy fluid in bulk space. In the braneworld scenario, we derived the acceleration condition due to the bulk quintom field. It was indicated that the time-dependent bulk quintom field alters the dynamics on the brane as a time-dependent cosmological constant which can be derived in terms of the quintom field potential. It means that to have an accelerated expansion phase on the brane, the potential energy due to the bulk quintom field must be a positive function of time.

In the DGP model, when an intrinsic curvature term is added on the brane, we have obtained the generalized Einstein equation, where both bulk and brane matter field are non-zero. Cosmology on 3-brane have been studied in two different cases. In first case, we have considered the quintom field as the bulk matter field and ignored the brane matter. It was shown that two different solutions are obtained for two different embeddings of the brane. In negative branch where $\epsilon = -1$, the generalized Friedmann equation describes an asymptotically static universe, and in positive branch where $\epsilon = 1$, we obtained $H = \frac{1}{r_c}$, which provides a self-inflationary solution at late time. In second case, we have considered a quintom dark energy confined on the brane in the DGP model, and studied the solution of the generalized Friedmann equation, when the bulk matter field and the brane tension were ignored. Similar to the first case, here also two different results can occur, $H = 0$ for $\epsilon = -1$ and a self-inflationary solution at late time, $H = \frac{1}{r_c}$, for $\epsilon = 1$. Therefore, the late time behavior of the universe does not alter even if we ignore the matter field on the brane and consider a model of the universe filled with the bulk quintom dark energy, or vice versa, we ignore the bulk matter field and consider only the quintom dark energy on the brane.

Finally we should stress on the ghost instabilities present in the self-accelerating branch of this DGP-inspired model. The self-accelerating branch of the DGP model contains a ghost at the linearized level [28]. Since the ghost carries negative energy den-

sity, it leads to the instability of the spacetime. The presence of the ghost can be attributed to the infinite volume of the extra-dimension in DGP setup. When there are ghosts instabilities in self-accelerating branch, it is natural to ask what are the results of solutions decay. As a possible answer we can state that since the normal branch solutions are ghost-free, one can think that the self-accelerating solutions may decay into the normal branch solutions. In fact for a given brane tension, the Hubble parameter in the self-accelerating universe is larger than that of the normal branch solutions. Then it is possible to have nucleation of bubbles of the normal branch in the environment of the self-accelerating branch solution. This is similar to the false vacuum decay in de Sitter space. However, there are arguments against this kind of reasoning which suggest that the self-accelerating branch does not decay into the normal branch by forming normal branch bubbles (see [28] for more details). It was also shown that the introduction of Gauss–Bonnet term in the bulk does not help to overcome this problem [29]. In fact, it is still unclear what is the end state of the ghost instability in self-accelerated branch of DGP inspired setups. On the other hand, quintom scalar fields and induced gravity in our setup provides a new degree of freedom which requires special fine tuning and this may provide a suitable basis to treat ghost instability. It seems that in our model this additional degree of freedom has the capability to provide the background for a more reliable solution to ghost instability due to wider parameter space.

Acknowledgement

We would like to thank an anonymous referee for helpful comments on the Letter.

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