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Poincaré invariant gravity with local supersymmetry as a gauge theory for the M-algebra

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Abstract

Here we consider a gravitational action having local Poincaré invariance which is given by the dimensional continuation of the Euler density in ten dimensions. It is shown that the local supersymmetric extension of this action requires the algebra to be the maximal extension of the $\mathcal{N}=1$ super-Poincaré algebra. The resulting action is shown to describe a gauge theory for the M-algebra, and is not the eleven-dimensional supergravity theory of Cremmer–Julia–Scherk. The theory admits a class of vacuum solutions of the form $S^{10-d}\times X_{d+1}$, where X_{d+1} is a warped product of $\mathbb{R}$ with a d-dimensional spacetime. It is shown that a nontrivial propagator for the graviton exists only for $d=4$ and positive cosmological constant. Perturbations of the metric around this solution reproduce linearized General Relativity around four-dimensional de Sitter spacetime.

1. Introduction

A consensus has emerged in the high energy community that a consistent unified theory of all interactions and matter should be formulated in some dimension higher than four. Strong theoretical evidence, both in supergravity and in string theory, leads to conjecture the existence of an underlying fundamental theory in eleven dimensions [1–3]. This is nowadays called M-theory (see, e.g., [4]). The standard procedure to link the higher-dimensional theory with four-dimensional physics has been either to compactify the extra dimensions by the Kaluza–Klein reduction (see, e.g., [5]), or through some more recent alternatives [6].

In these frameworks, however, the physical spacetime dimension is an input rather than a prediction of the theory. In fact, in standard theories whose gravitational sector is described by the Einstein–Hilbert action, there is no obstruction to perform dimensional reductions to spacetimes of dimensions $d\neq 4$. Then the question arises, since eleven-dimensional Minkowski space is a maximally (super)symmetric state, and the

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theory is well-behaved around it, why the theory does not select this configuration as the vacuum, but instead, it chooses a particular compactified space with less symmetry. An ideal situation, instead, would be that the eleven-dimensional theory dynamically predicted a low energy regime which could only be a four-dimensional effective theory. In such a scenario, a background solution with an effective spacetime dimension $d > 4$ should be expected to be a false vacuum where the propagators for the dynamical fields are ill-defined, lest a low energy effective theory could exist in dimensions higher than four.

In this Letter, a new eleven-dimensional theory sharing some of these features is constructed. Indeed, for this theory, eleven-dimensional Minkowski spacetime is a maximally supersymmetric solution that would be a natural candidate for the vacuum. However, propagators around this background are ill-defined and hence it is a sort of false vacuum. On the other hand, the theory admits vacuum geometries of the form $S^{10-d} \times X_{d+1}$, where X_{d+1} is a domain wall whose worldsheet is a d-dimensional constant curvature spacetime M_d. These solutions exist only if M_d has a non-negative cosmological constant, and the graviton can only propagate provided M_d is a four-dimensional de Sitter space. Moreover, the gravitational perturbations reproduce linearized General Relativity in four dimensions. Thus, the resulting four-dimensional effective theory is indistinguishable from gravity with positive cosmological constant in perturbation theory. Our motivation to choose eleven dimensions is to explore new geometrical and dynamical structures that are expected to exist in $d = 11$, and could be regarded as new "cusps" of M-theory (see, e.g., [7]). The theory presented here is not equivalent to the Cremmer–Julia–Scherk supergravity in eleven dimensions [1].

The gravitational action we propose is selected by requiring local Poincaré invariance and is given by the dimensional continuation of the Euler density in ten dimensions. Its local supersymmetric extension requires the algebra to be the maximal extension of the $\mathcal{N} = 1$ super-Poincaré algebra in eleven dimensions, commonly known as the M-algebra. This algebra is spanned by the set $G_A = \{J_{ab}, P_a, Q_\alpha, Z_{ab}, Z_{abcde}\}$, where J_{ab} and P_a are the generators of the Poincaré group and Q_α is a Majorana spinor supercharge with anticommutator [8]

$$\{Q_\alpha, Q_\beta\} = \left(C\Gamma^a\right)_{\alpha\beta} P_a + \left(C\Gamma^{ab}\right)_{\alpha\beta} Z_{ab} + \left(C\Gamma^{abcde}\right)_{\alpha\beta} Z_{abcde}. \tag{1}$$

The charge conjugation matrix C is antisymmetric, and the "central charges" Z_{ab} and Z_{abcde} are tensors under Lorentz rotations but otherwise Abelian generators.[1] As shown below, the algebra fixes the field content to include, apart from the graviton e^a_μ, the spin connection ω^{ab}_μ and the gravitino ψ_μ, two one-form fields b^{ab}_μ, b^{abcde}_μ, which are rank two and five antisymmetric tensors under the Lorentz group, respectively. The local supersymmetry transformations close off-shell without requiring auxiliary fields. As will be seen below, the supersymmetric Lagrangian can be explicitly written as a Chern–Simons form. It is known that for Chern–Simons theories bosonic and fermionic degrees of freedom do not necessarily match, since there exists an alternative to the introduction of auxiliary fields (see, e.g., [9]). Indeed, the matching may not occur when the dynamical fields are assumed to belong to a connection instead of a multiplet for the supergroup [10].

2. Gravitational sector

In dimensions higher than four, under the same assumptions of General Relativity in four dimensions (i.e., general covariance, second order field equations for the metric), the so-called Lovelock actions are obtained [11], which include the Einstein–Hilbert Lagrangian as a particular case. In general, these Lagrangians are linear combinations of the dimensional continuations of the Euler densities from all lower dimensions [12] and therefore contain higher powers of the curvature. Since the action can be expressed in terms of differential forms without using the Hodge dual, it is easy to see why these theories do not yield higher derivative field equations. In the first order formalism (analogous to Palatini's) the field equations

[1] In standard eleven-dimensional supergravity, these generators correspond to the "electric" and "magnetic" charges of the M2 and M5 branes, respectively. Note that, contrary to the case in standard supergravity, the generators of diffeomorphisms ($\mathcal{H}_\mu$) are absent from the right-hand side of (1).

can only involve first order derivatives of the dynamical fields (for more on this, see [13]). Furthermore, if one then imposes the torsion to vanish the field equations become at most second order. In the vanishing torsion sector, the theory has the same degrees of freedom as General Relativity [14].

Without imposing the torsion constraint, the field equations remain first order, even if one couples this theory to other p-form fields without involving the Hodge. In fact, in this way it is impossible to generate higher derivative terms in this theory.

An action containing (1) as a local symmetry must be, in particular, invariant under local translations,

$$\delta e^a = D\lambda^a = d\lambda^a + \omega^a_b \lambda^b, \qquad \delta\omega^{ab} = 0. \tag{2}$$

The only gravitational action in eleven dimensions constructed out of the vielbein e^a and the spin connection ω^{ab}, leading to second order field equations for the metric, invariant under diffeomorphisms and local Poincaré transformations is given by [13,15]

$$I_G[e,\omega] = \int_{M_{11}} \epsilon_{a_1\ldots a_{11}} R^{a_1a_2}\cdots R^{a_9a_{10}} e^{a_{11}}. \tag{3}$$

Here $R^{ab} = d\omega^{ab} + \omega^a{}_c\omega^{cb}$ is the curvature two-form, and wedge product between forms is understood.[2] For the reason given above, we take I_G as the gravitational sector of our theory rather than the Einstein–Hilbert action which, is not invariant under (2).[3] The Lagrangian in (3) is the ten-dimensional Euler density continued to eleven dimensions and contains the degrees of freedom of eleven-dimensional gravity [14].

A local Poincaré transformation acting on the dynamical fields is a gauge transformation $\delta_\lambda A = d\lambda + [A,\lambda]$, with parameter $\lambda = \lambda^a P_a + \frac{1}{2}\lambda^{ab} J_{ab}$, provided e^a and ω^{ab} are the components of a single connection for the Poincaré group, $A = e^a P_a + \frac{1}{2}\omega^{ab} J_{ab}$. This observation will be the guiding principle for the construction of a locally supersymmetric extension of I_G.

3. Supersymmetric extension

A natural way to construct a locally supersymmetric extension of (3) without breaking local Poincaré invariance is that the extra fields required by supersymmetry enter on a similar footing with the original fields. In other words, all dynamical fields will be assumed to belong to a connection for a supersymmetric extension of the Poincaré group. This approach strongly deviates from the standard assumption in supergravity, where the fields are assumed to belong to a multiplet. As we shall see now, the M-algebra emerges naturally from our approach. The simplest tentative option would be to consider the $\mathcal{N} = 1$ super-Poincaré algebra without central extensions. However, this possibility must be ruled out. Indeed, in this case, the connection would be extended by the addition of a gravitino as $A \to A + \psi Q/\sqrt{2}$, and the gauge generator would change as $\lambda \to \lambda + \epsilon Q/\sqrt{2}$, where ϵ is a zero-form Majorana spinor. This would fix the supersymmetric transformations to be $\delta e^a = \bar{\epsilon}\Gamma^a\psi/2$, $\delta\psi = D\epsilon$ and $\delta\omega^{ab} = 0$. Then the variation of (3) under supersymmetry can be cancelled by a kinetic term for the gravitino of the form

$$I_\psi = -\frac{1}{6}\int_{M_{11}} R_{abc}\bar{\psi}\Gamma^{abc} D\psi, \tag{4}$$

where $R_{abc} := \epsilon_{abca_1\ldots a_8} R^{a_1a_2}\cdots R^{a_7a_8}$. However, the variation of I_ψ produces, in turn, an extra piece which cannot be cancelled by a local Lagrangian for e^a, ω^{ab}, and ψ, and hence the super-Poincaré algebra is not rich enough to ensure the off-shell supersymmetry of the action. Nevertheless, following the Noether procedure, it can be seen that supersymmetry can be achieved introducing additional bosonic fields. These fields can only be either a second-rank or a fifth-rank tensor one-forms b^{ab}, and b^{abcde}, that transform like $\bar{\epsilon}\Gamma^{ab}\psi$ and $\bar{\epsilon}\Gamma^{abcde}\psi$, respectively. Assuming that the dynamical fields belong to a single connection for a supersymmetric extension of the Poincaré group, the only option that brings in these extra bosonic fields is to consider the M-algebra (1), which also prescribes their supersymmetry transformations in the expected

[2] We do not consider Lorentz–Chern–Simons forms which are also invariant under (2), because they lead to third order field equations for the metric in the second order formalism, as it occurs in three dimensions. See, e.g., Ref. [16].

[3] Under the transformations (2), the Einstein–Hilbert action $I_{EH} = \int \epsilon_{a_1\ldots a_{11}} R^{a_1a_2} e^{a_3}\cdots e^{a_{11}}$ changes by a term proportional to $\int \epsilon_{a_1\ldots a_{11}} R^{a_1a_2} T^{a_3} e^{a_4}\cdots e^{a_{10}}\lambda^{a_{11}}$, which vanishes only if the torsion $T^a = De^a$ does. However, this last condition is incompatible with the transformations because ω^{ab} would be a function of e^a and hence, its variation could not vanish. See Ref. [17].

form. This means that the field content is given by the components of a single fundamental field, the M-algebra connection,

$$A = \frac{1}{2}\omega^{ab}J_{ab} + e^a P_a + \frac{1}{\sqrt{2}}\psi^\alpha Q_\alpha + b^{ab}Z_{ab} + b^{abcde}Z_{abcde}, \tag{5}$$

and hence, the required local supersymmetry transformations are obtained from a gauge transformation of the M-connection (5) with parameter $\lambda = 1/\sqrt{2}\epsilon^\alpha Q_\alpha$,

$$\delta_\varepsilon e^a = \frac{1}{2}\bar{\epsilon}\Gamma^a\psi, \qquad \delta_\varepsilon\psi = D\epsilon, \qquad \delta_\varepsilon\omega^{ab} = 0,$$
$$\delta_\varepsilon b^{ab} = \frac{1}{2}\bar{\epsilon}\Gamma^{ab}\psi, \qquad \delta_\varepsilon b^{abcde} = \frac{1}{2}\bar{\epsilon}\Gamma^{abcde}\psi. \tag{6}$$

Thus, the supersymmetric extension of (3), invariant under (6) is found to be

$$I_\alpha = I_G + I_\psi - \frac{\alpha}{6}\int_{M_{11}} R_{abc}R_{de}b^{abcde} + 8(1-\alpha) \times \int_{M_{11}} \left[R^2 R_{ab} - 6\left(R^3\right)_{ab}\right] \times R_{cd}\left(\bar{\psi}\Gamma^{abcd}D\psi - 12R^{[ab}b^{cd]}\right), \tag{7}$$

where $R^2 := R^{ab}R_{ba}$ and $(R^3)^{ab} := R^{ac}R_{cd}R^{db}$. Here α is a dimensionless constant whose meaning will be discussed below.

This action is invariant under (2), (6), local Lorentz rotations, and also under the local Abelian transformations

$$\delta b^{ab} = D\theta^{ab}, \qquad \delta b^{abcde} = D\theta^{abcde}. \tag{8}$$

Invariance under general coordinate transformations is guaranteed by the use of forms. It is simple to see that the local invariances of the action, including Poincaré transformations, supersymmetry (6) together with (8), are a gauge transformation for the M-connection (5) with parameter $\lambda = \lambda^a P_a + \frac{1}{2}\lambda^{ab}J_{ab} + \theta^{ab}Z_{ab} + \theta^{abcde}Z_{abcde} + 1/\sqrt{2}\epsilon^\alpha Q_\alpha$. As a consequence, the invariance of the action under the supersymmetry algebra is ensured by construction without invoking field equations or requiring auxiliary fields.

3.1. Manifest M-covariance

The action (7) describes a gauge theory for the M-algebra with fiber bundle structure, which can be seen explicitly by writing the Lagrangian as a Chern–Simons form [18] for the M-connection (5). Indeed, the Lagrangian satisfies $dL = \langle F^6\rangle$, where the curvature $F = dA + A^2$ is given by

$$F = \frac{1}{2}R^{ab}J_{ab} + \tilde{T}^a P_a + 1/\sqrt{2}D\psi^\alpha Q_\alpha + \tilde{F}^{[2]}Z_{[2]} + \tilde{F}^{[5]}Z_{[5]},$$

with $\tilde{T}^a = De^a - (1/4)\bar{\psi}\Gamma^a\psi$ and $\tilde{F}^{[k]} = Db^{[k]} - (1/4)\bar{\psi}\Gamma^{[k]}\psi$ for $k = 2, 5$. The bracket $\langle\cdots\rangle$ stands for an invariant multilinear form of the M-algebra generators G_A whose only non-vanishing components are given by

$$\langle J_{a_1a_2}, \ldots, J_{a_9a_{10}}, P_{a_{11}}\rangle = \frac{16}{3}\epsilon_{a_1\ldots a_{11}},$$
$$\left\langle J_{a_1a_2}, \ldots, J_{a_9a_{10}}, Z_{abcde}\right\rangle = -\alpha\frac{4}{9}\epsilon_{a_1\ldots a_8abc}\eta_{[a_9a_{10}][de]},$$
$$\left\langle J_{a_1a_2}, J_{a_3a_4}, J_{a_5a_6}, J^{a_7a_8}, J^{a_9a_{10}}, Z^{ab}\right\rangle = (1-\alpha)\frac{16}{3}\left[\delta^{a_7\ldots a_{10}ab}_{a_1\ldots\ldots a_6} - \delta^{a_9a_{10}ab}_{a_1\ldots a_4}\delta^{a_7a_8}_{a_5a_6}\right],$$
$$\left\langle Q, J_{a_1a_2}, J^{a_3a_4}, J^{a_5a_6}, J^{a_7a_8}, Q\right\rangle = \frac{32}{15}\left[C\Gamma_{a_1a_2}{}^{a_3\ldots a_8} + (1-\alpha)\left(3\delta^{a_3\ldots a_6}_{a_1a_2ab}C\Gamma^{a_7a_8ab} + 2C\Gamma^{a_3\ldots a_6}\delta^{a_7a_8}_{a_1a_2}\right)\right],$$

where (anti-)symmetrization under permutations of each pair of generators is understood when all the indices are lowered. The existence of this bracket allows writing the field equations in a manifestly covariant form as

$$\left\langle F^5 G_A\right\rangle = 0. \tag{9}$$

In addition, if the eleven-dimensional spacetime is the boundary of a twelve-dimensional manifold, $\partial\Omega_{12} = M_{11}$, the action (7) can also be written as $I = \int_{\Omega_{12}}\langle F^6\rangle$, which describes a topological theory in twelve dimensions. In spite of its topological origin, the action does possess propagating degrees of freedom in eleven dimensions and hence it should not be thought of as a topological field theory.

4. Gravitons and four-dimensional spacetime

We now turn to the problem of identifying the true vacuum of the theory. Obviously, a configuration with a locally flat connection, $F = 0$, solves the field equations and would be a natural candidate for vacuum in a standard field theory. However, no local degrees of freedom can propagate on such background because all perturbations around it are zero modes. Note that eleven-dimensional Minkowski spacetime is maximally supersymmetric by virtue of (6), however as it obeys $F = 0$, the propagators on it are ill-defined, and hence it is a sort of false vacuum.

In a matter-free configuration, Eq. (9) is a set of quintic polynomials for the Riemann two-form R^{ab}. The dynamical field equations take the form

$$\epsilon^{0ij_1\ldots j_9}\langle F_{j_1j_2}\cdots F_{j_7j_8}(\partial_t A_{j_9} - \nabla_{j_9}A_0)G_A\rangle = 0. \quad (10)$$

So, in order to have propagation for A_j, the spatial components F_{ij} cannot be small. Hence, a deviation around $F = 0$ that propagates cannot be infinitesimal and is therefore non-perturbative and non-local. A necessary condition to have well-defined perturbations is that the background solution be a simple zero of at least one of the polynomials. In particular, this requires the curvature to be non-vanishing on a submanifold of a large enough dimension.

Let us consider a torsionless spacetime with a product geometry of the form $X_{d+1} \times S^{10-d}$, where X_{d+1} is a domain wall whose worldsheet is a d-dimensional constant curvature spacetime M_d. The line element is given by

$$ds^2 = \exp(-2a|z|)(dz^2 + \tilde{g}^{(d)}_{\mu\nu}(x)\,dx^\mu\,dx^\nu) + \gamma^{(10-d)}_{mn}(y)\,dy^m\,dy^n, \quad (11)$$

where $\tilde{g}^{(d)}_{\mu\nu}$ stands for the worldsheet metric with $\mu, \nu = 0, \ldots, d-1$; $\gamma^{(10-d)}_{mn}$ is the metric of S^{10-d} of radius r_0 and a is a constant.

This Ansatz solves the vacuum field equations provided the projection of the Riemann tensor along the worldsheet,

$$R^{ij} = \tilde{R}^{ij} - a^2\tilde{e}^i \wedge \tilde{e}^j,$$

vanishes (here $\tilde{e}^i$ and $\tilde{R}^{ij}$ stand for the vielbein and the Riemann curvature of the worldsheet, respectively). This means that M_d is either locally de Sitter spacetime of radius a^{-1}, or locally Minkowski for $a = 0$.

The requirement that the curvature of (11) be a simple zero, implies, after a straightforward computation, that d cannot be greater than four. Then, the condition of having well-defined propagators singles out the dimension of the worldsheet to be $d = 4$, and $a^2 > 0$. Indeed, for $d = 4$, the only relevant equation for the perturbations is the one that arises from the variation with respect to $\tilde{e}^i$,

$$a\delta(z)\epsilon_{ijkl}\delta(\tilde{R}^{jk} - a^2\tilde{e}^j\tilde{e}^k)\tilde{e}^l = 0. \quad (12)$$

Since for $a = 0$ this equation becomes empty, Minkowski spacetime must be ruled out. Thus, the existence of the propagator requires the four-dimensional cosmological constant to be strictly positive and given by $\Lambda_4 = 3a^2$.

Note that Eq. (12) has support only on the $z = 0$ plane. Perturbations along the worldsheet, $\delta\tilde{g}_{\mu\nu} = h_{\mu\nu}(x)$ reproduce the linearized Einstein equations in four-dimensional de Sitter spacetime. The modes that depend on the coordinates transverse to the worldsheet fall into two classes. Those of the form $\delta\tilde{g}_{\mu\nu} = h_{\mu\nu}(x, y)$ are massive Kaluza–Klein modes with a discrete spectrum, while $\delta\tilde{g}_{\mu\nu} = h_{\mu\nu}(x, z)$ correspond to Randall–Sundrum-like massive modes whose spectrum is continuous and has a mass gap. The perturbations of the remaining metric components are zero modes, which is related to the fact that the equations are not deterministic for the compact space. A detailed analysis of this, as well as of the perturbations of matter fields will be presented elsewhere [19].

5. Discussion

We have presented a framework in which the spacetime dimension is dynamically selected to be four. The mechanism is based on a new eleven-dimensional action of the Chern–Simons type, which is a gauge theory for the M-algebra. The possibility of dynamical dimensional reduction arises because the theory has radically different spectra around backgrounds of different effective spacetime dimensions. Thus, in a family of product spaces of the form $X_{d+1} \times S^{10-d}$, the only option that yields a well-defined low energy propagator for the graviton is $d = 4$ and $\Lambda_4 > 0$. It should be stressed that for all gravity theories of the type discussed here, possessing local Poincaré invariance in dimensions $D = 2n+1 \geqslant 5$, four-dimensional de Sitter spacetime is also uniquely selected by the same mechanism as the background for the low energy effective theory.

The action discussed in this Letter has a free parameter α, which reflects the fact that the theory contains two natural limits which correspond to different subalgebras of (1). For $\alpha = 0$, the action I_0 in Eq. (7) does not depend on $b^{[5]}$ and corresponds to a gauge theory for the supermembrane algebra, while for $\alpha = 1$, the bosonic field $b^{[2]}$ decouples, and I_1 is a gauge theory for the super five-brane algebra as discussed in [20]. It is interesting to note that the linear combination of both limits, $I_\alpha = I_0 + \alpha(I_1 - I_0)$, is not only invariant under the intersection of both algebras, but under the entire M-algebra. As the term $I_1 - I_0$ does not couple to the vielbein and is invariant under supersymmetry by itself, α is an independent coupling constant. A similar situation occurs in nine dimensions where, in one limit, the theory corresponds to the super five-brane algebra, while for the other it is a gauge theory for the super-Poincaré algebra with a central extension [21].

In the presence of negative cosmological constant, the eleven-dimensional AdS supergravity presented in Ref. [10] can be written as a Chern–Simons theory for $osp(32|1)$, which is the supersymmetric extension of AdS_{11}. It is natural to ask whether there is a link between that theory in the vanishing cosmological constant limit, and the one discussed here. Since the M-algebra has 55 bosonic generators more than $osp(32|1)$, these theories cannot be related through a Inönü–Wigner contraction for a generic value of α. However, it has been recently pointed out in [22], generalizing the procedure of [23], that it is possible to obtain the M-algebra from an expansion of $osp(32|1)$. In this light, applying this procedure to the eleven-dimensional AdS supergravity theory, it should be expected that the action presented here will be recovered up to some additional terms decoupled from the vielbein, that are supersymmetric by themselves.

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