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## Calibration of Dynamic Building Energy Models with Multiple Responses Using Bayesian Inference and Linear Regression Models

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### Abstract

This paper proposes a lightweight Bayesian calibration of dynamic models that accounts for model parameter uncertainties. A regression model was built to represent the dynamic model using parameter screening and multiple linear regression. Given this regression model and prior probability distributions of its input parameters, a Bayesian calibration method is developed to provide their posterior distributions. A case study is presented and the result shows considerable alignment between model prediction and measurement after calibration. This indicates its capability to perform fast risk-conscious calibration for most current retrofit practice where only monthly consumption and demand data are available.

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### 1. Introduction

Dynamic building energy models are widely implemented in building energy modeling for design, operational management, life-cycle assessment, and retrofit analysis. They are physics-based and can well capture the heat transfer and energy balance of the building. Inputs for dynamic models usually correspond to real physical properties, so building performance can be estimated in detail and the impact of changes to a building can be assessed. However, due to uncertainties in manufacturing, construction and actual operation, as well as the assumptions, simplifications and approximations inherent in modeling, considerable discrepancies exist between model outputs and field measurements of building energy use. Calibration of building energy models can help alleviate these discrepancies by adjusting uncertain model inputs through comparison between predictions and measurements, such that the model outputs are close enough to reality while model inputs remain realistic. In retrofit analysis, calibrated models can then be used to predict potential savings of energy conservation measures (ECMs) and thereby support ECM selection decisions.

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Currently used calibration methods and procedures are well summarized in [1,2]. Automated calibration, as one major category, uses mathematical and statistical techniques to find the optimal set of inputs such that the discrepancy between model prediction and measured data is minimized. A variety of automated calibration techniques have been proposed in the literature [3-7]. Bayesian calibration, as proposed by Kennedy and O'Hagan [8], can systematically handle input uncertainties and combine information from different sources into an estimation of model inputs using Bayesian inference. This technique was applied by Heo et al. [9] to retrofit analysis in which a reduced order, yet still physics-based, energy model featuring a quasi-steady-state formulation of heat balance equations and aggregated building parameters is used as the simulation engine. The energy model's parameters were calibrated by fitting a Gaussian process model (GPM) to the energy model and probabilistically inferring the parameters via Bayes' rule using the GPM as a substitute for the reduced-order model. Booth et al. [10] adopted a hierarchical Bayesian framework into calibration of micro-level reduced order energy models with macro-level measured data.

This study proposes a modified Bayesian calibration method based on [8] to meet the particular demand of dynamic building energy simulation, such that analysis of hundreds of inputs involved in dynamic models can be performed with manageable computational effort. In addition, multiple types of measurements, including those obtained by detailed on-site visits, sub-metering, etc., can be used systematically to improve the result. This method will be demonstrated by a case study, and its comparison with the current Bayesian calibration method in terms of accuracy and efficiency will also be provided.

## 2. Methodology

### 2.1. Description of dynamic building energy model

A building on the Georgia Tech campus was chosen for the case study. It is a three-story building with offices along the perimeter and biological laboratories in the core. The building system consumes steam and chilled water from a central (district) plant as well as electricity from a utility. This building uses a variable air volume (VAV) with reheat system, with six air handling units (AHUs) that supply conditioned air to the whole building except mechanical rooms. Domestic hot water comes from district heating through two heat exchangers. A dynamic building energy model was built in OpenStudio 1.2 based on building design specifications (Fig. 1). An on-site building audit was also performed to adjust the model. Because four of the six AHUs operate to similar specifications, they were modeled as a single AHU for simplicity. The model then was exported into an EnergyPlus 7.0 model for later use.

Available measurements include chilled water, steam and electricity consumption recorded at 15-minute time intervals from January 2011 to November 2013. However due to the poor quality of steam consumption data, only chilled water and electricity were considered in this case study. Electricity usage comes mainly from lighting and plug loads with fairly constant schedules during the year, so their usage density and schedules were calibrated with respect to electricity consumption using manual calibration approaches. These remain fixed in the later automatic calibration process. The 15-min chilled water consumption data was converted into daily average consumption, i.e. the total monthly consumption divided by the number of days of each month, and peak demand for each month to represent utility bills. A few data gaps due to sensor dysfunction or system maintenance are filled by taking the average of readings of the same time for six neighboring days, three forwards and three afterwards. Measurements from 2011 to 2012 were used to calibrate the model and the remaining were used for validation.

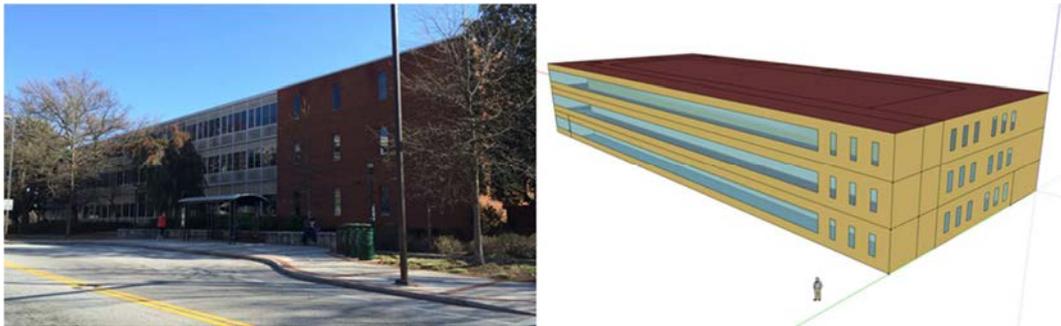


Fig. 1. Left: Campus building; Right: OpenStudio model.

2.2. Meta-model fitting

In automated calibration methods where uncertainties in model inputs are considered, the energy model may need to be evaluated thousands of times in order to quantify the impact of uncertainties on model output, such that alignment between model prediction and measurement can be expected by tuning input values. To reduce the computational load associated with dynamic energy model evaluation, in current Bayesian calibration a GPM was used as the meta-model, in which all of the simulation outputs form a multivariate Gaussian distribution. However, as will be explained later, this meta-model is still computationally expensive when the number of simulations is large. Therefore in this study a multiple linear regression model was used instead of a GPM as the meta-model that “models” the dynamic model:

$$\eta(d) = \beta_0 + \beta_1 d_1 + \dots + \beta_p d_p + \epsilon_s = \beta_0 + \sum_{i=1}^p \beta_i d_i + \epsilon_s \tag{1}$$

where  $\eta(d)$  is the dynamic model output,  $d_i$  and  $\beta_j$  are model inputs and regression coefficients respectively, and  $\epsilon_s$  is the random error that assumed to be identically and independently distributed, following a normal distribution with zero mean and constant variance, i.e.  $\epsilon_s \sim N(0, \sigma_s^2)$ . In this way, given a few important inputs, the linear model can provide similar results to the dynamic model with much reduced computational demands.

Training data is needed both to select important inputs for inclusion in the linear model and to fit the linear model, i.e. estimate all of the coefficients. This training data was generated by first performing a Latin Hypercube (LHS) generation [11] of 200 samples of uncertain model inputs. Uncertainties of these inputs were assumed to follow triangle distributions and the distribution parameters, i.e. mode, minimum and maximum values, were derived from an integrated standard uncertainty quantification (UQ) repository [12,13] which include uncertainty information for building envelope and materials, system components, and usage scenarios and operation. In addition, uncertainties in case-specific inputs, including cooling set point temperature and actual outdoor air rate for each air system, were also quantified and sampled. Then for each sample, i.e. a set of values for all of the uncertain model inputs, EnergyPlus simulation was performed in a program developed by Lee, et al. [14], using Actual Meteorology Year (AMY) data from Hartsfield-Jackson Atlanta International Airport. The simulated monthly consumption are regarded as the corresponding output of the sample. Both daily average consumption and peak demand for each month were normalized into a range of 0 to 1 by their minimum and maximum values of the simulation samples respectively, so as to remove the effect of units. A discrete dummy input named “Type” was introduced and denoted as  $d_1$ , such that  $d_1 = 0$  or 1 enables the single meta-model to generate two different outputs  $\eta_1(d)$  and  $\eta_2(d)$  given all other inputs are the same, when all coefficients,  $\beta_0, \beta_1, \dots, \beta_p$  are known from estimation:

$$\eta_1(d) = \beta_0 + \beta_1 \times 0 + \sum_{i=2}^p \beta_i d_i + \epsilon_s \quad \eta_2(d) = \beta_0 + \beta_1 \times 1 + \sum_{i=2}^p \beta_i d_i + \epsilon_s \tag{2}$$

in which  $\beta_1$ , the regression coefficient of the dummy variable, represent the average difference between two types of output given the same remaining inputs. Weather conditions, including dry bulb temperature, relative humidity, solar radiation intensity, etc. were also considered as model inputs in the form of monthly average values. Therefore a single EnergyPlus simulation of two years (2011-2012) generated 48 outputs, half of them daily average consumption and the other half being peak demand. At last the complete training data includes 9600 samples in total, each containing uncertain model inputs, weather inputs, the dummy input and the corresponding normalized output.

Given the training data, the lasso method [15] is used to select significant inputs by estimating the coefficients  $\beta_j$ s in Eq. 2

$$\hat{\beta}_{lasso} = \arg \min \left\{ \sum_n \left( \eta(d) - \beta_0 - \sum_{j=1}^p \beta_j d_j \right)^2 + \lambda \sum_{j=1}^p |\beta_j| \right\} \tag{3}$$

where  $\lambda$  is a tuning parameter that controls how many inputs are selected, and  $n$  is the total number of data points. The resulting significant inputs are shown in Table 1. Six variable inputs refer to weather and the seventh is the dummy input “Type”; these variable inputs are denoted by  $x$ . Six calibration inputs refer to the uncertain model inputs to be calibrated; these are denoted as  $\theta$ , and therefore  $d = (x, \theta)$ . A multiple linear regression model was then fitted using stepwise regression with Akaike information criterion (AIC) [16] to optimize estimation capability. The resultant model included all main effects and 60 two-factor interaction effects, and obtained an  $R^2$  of 0.973. This indicates that uncertainties of these inputs can explain most of the observed output variations.

Table 1. Significant model inputs

Category	Name
Variable inputs	Dry bulb temperature
	Relative humidity
	Global horizontal radiation
	Direct normal radiation
	Wind direction
	Wind speed
Calibration inputs	Type
	Cooling Set-point at occupied hours
	Occupancy density: lab & office
	Outdoor air rate per person: lab & office
	Outdoor air rate per system for VAV 1, 2 and 3

2.3. Automated calibration

From a statistical perspective, the calibration problem can be represented by the following formulation:

$$y = \eta(d) + \delta(x) + \epsilon_m \tag{4}$$

where  $y$  is normalized field measurement, e.g. monthly energy consumption,  $\eta(d)$  is the (regressed) meta-model,  $\delta(x)$  is the error due to model inadequacy, and  $\epsilon_m$  is random measurement error. Model inadequacy  $\delta(x)$  is assumed to be a GPM, where all  $\delta$  s in the measurements form a joint multivariate Gaussian distribution. It assumes that outputs are highly correlated when inputs are close enough under a certain measure of dissimilarity, and the Gaussian distribution is specified by mean and covariance matrix. For each element  $c_{ij}$  that represents the covariance between  $\delta_i$  and  $\delta_j$ , the logarithm of a weighted Euclidean distance as the dissimilarity function is employed:

$$c_{ij} = \exp\left(-\sum_s \omega_s (x_{is} - x_{js})^2\right) \tag{5}$$

where  $x_{is}, x_{js}$  is the  $s$  th dimension of variable inputs,  $\omega_s$  is the corresponding weight factor. Hence by specifying the structure of the covariance function, a Gaussian process model can flexibly represent the model behavior and obtain an exact fit on given samples.

Assume that the random errors are also identically and independently distributed, i.e.  $\epsilon_m \sim N(0, \sigma_m^2)$ , then by using matrix form the following result is obtained:

$$\mathbf{y} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \boldsymbol{\mu} = f(\mathbf{d}), \boldsymbol{\Sigma} = C(\mathbf{x}) + (\sigma_s^2 + \sigma_m^2) \mathbf{I} \tag{6}$$

in which  $\sigma_s^2$  was estimated during the meta-model fitting. As a result, the calibration problem becomes finding a set of distributions for  $\theta, \omega$  and  $\sigma_m$  such that the possibility of obtained measurement data,  $\mathbf{y}_{me}$ , becomes maximal, and this is where Bayesian inference is introduced. Following Bayes' rule:

$$p(\theta, \omega, \sigma_m | \mathbf{y} = \mathbf{y}_{me}) \propto p(\mathbf{y} = \mathbf{y}_{me} | \theta, \omega, \sigma_m) p(\theta) p(\omega) p(\sigma_m) \tag{7}$$

Therefore the posterior distributions  $p(\theta, \omega, \sigma_m | \mathbf{y} = \mathbf{y}_{me})$  are obtained from the product of their prior distributions  $p(\theta), p(\omega)$  and  $p(\sigma_m)$ , multiplied by the likelihood of measurements that derived from the probability density function (PDF) of  $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ . Prior distributions of calibration inputs  $\theta$  were defined as the same triangular distributions used in the generation of training data, and prior distributions of  $\omega$  and  $\sigma_m$  were adopted from [17] to assume a 20% model inadequacy and 5% random error. A full Bayesian calibration approach was adopted to estimate all the parameters at the same time, which uses random walk Markov Chain Monte Carlo (MCMC) simulation with the Metropolis algorithm [18] and uniform distributions as the proposal distribution. 30000 MCMC simulations were performed, with the first 3000 samples thrown away as burn-in. The total computation time used in this automatic calibration process

was less than 3 minutes. In current Bayesian calibration when a GPM is also used as the meta-model, calculating the likelihood requires taking the inverse of the covariance matrix with a size of  $(m+n) \times (m+n)$ , in which  $m$  equals to the total number of simulation outputs and  $n$  to the field measurement. This would become computationally prohibitive when  $m$  gets large. In this study, by using a linear model instead, the size of the covariance matrix becomes  $n \times n$ , and the computation effort can be significantly reduced since  $m \gg n$  in most of time.

### 3. Result and discussion

#### 3.1. Estimates of calibration inputs

PDFs of posterior distributions for the six calibration inputs, as compared with the prior distributions, can be found in Fig. 2. The cooling set point for occupied hours was higher than expected, which may suggest a higher occupant setting. Occupant density in laboratories and offices was slightly lower than expected. Outdoor air flow rates for all the AHUs were all close to the minimum requirement (30%) of design specification, which may suggest dysfunction of the control of the outdoor air dampers. However, interpretations of the calibration result may require further on-site visit and detailed investigations.

#### 3.2. Estimates of model outputs

Two criteria from ASHRAE Guideline 14 [19], net mean bias error (NMBE) and coefficient of variance of root mean squared error (CVRMSE) were calculated to evaluate the performance of calibrated models,

$$NMBE = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)}{(n-p)\bar{y}}, CVRMSE = \frac{1}{\bar{y}} \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-p}} \quad (8)$$

which compares simulation-predicted data  $\hat{y}_i$  that comes from a single EnergyPlus simulation using the mean value of the posterior distribution for each calibration input, to the utility data  $y_i$ , with  $n$  equals the number of data points and  $p=1$ . According to the guideline, a NMBE smaller than 0.05 and CVRMSE smaller than 0.15 for monthly calibration indicate good fit of the calibrated model. The result of calculation in Table 2 clearly shows a significant improvement and acceptable accuracy.

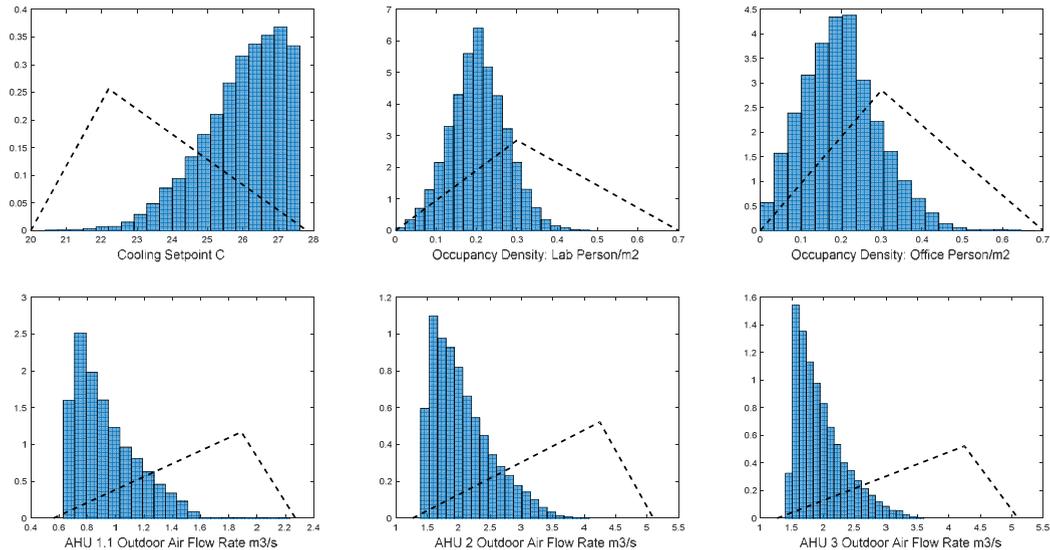


Fig. 2. Prior (dashed lines) and posterior (in blue) distributions of calibration inputs

Table 2. Calibration result compared with measurement

Output Type	Calibration	Calibration period: 2011-2012		Validation period: 2013	
		NMBE	CVRMSE	NMBE	CVRMSE
Daily average consumption	Before	0.03	0.33	0.18	0.45
	After	0.04	0.13	0.01	0.12
Peak demand	Before	0.40	0.48	0.36	0.45
	After	0.06	0.13	0.06	0.14

#### 4. Conclusion

This study has proposed a new method that calibrates dynamic building energy models with multiple types of field measurements. This method fitted a multiple linear regression model, rather than a GPM, to replace dynamic model in the calibration and thus reduce computational time, and obtained posterior distributions of calibration inputs by using Bayesian inference. A case study was performed that demonstrated acceptable results in terms of both input values and model performance with maintainable computation effort.

This lightweight, flexible calibration method applies to most building operation and retrofit analysis when only utility bills are available, but additional measurements can also be considered to incorporate more valuable information. This method is also capable of handling high dimensional inputs of dynamic models and large quantity of simulation samples. All these preferable properties provide it with great potential in future applications, such as real-time building optimal control with integrated sensor system, large scale retrofit and urban energy management with aggregated consumption data. Nevertheless, this method can only facilitate the automated input-tuning process while incorporating modeler's experience and uncertainties. Interpretations of the results should be substantiated by a deeper analysis of the model and corresponding building operation in reality.

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