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The CKM matrix from anti-SU(7) unification of GUT families

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We estimate the CKM matrix elements in the recently proposed minimal model, anti-SU(7) GUT for the family unification, $[3]+2[2]+8[\bar{1}]+$(singlets). It is shown that the real angles of the right-handed unitary matrix diagonalizing the mass matrix can be determined to fit the Particle Data Group data. However, the phase in the right-handed unitary matrix is not constrained very much. At present, there are three classes of possible CKM parametrizations, $\delta_{\rm CKM}=\alpha,\beta$, or γ of the unitarity triangle. For the choice of $\delta_{\rm CKM}=\alpha$, it is easy to show that the phase is close to a maximal one, which has a parametrization-independent meaning.

1. Introduction

At present, the unitarity triangle is determined with a very high precision such that any flavor unification models can be tested against it. Therefore, we attempt to see whether the recently proposed unification of grand unification families (UGUTF) based on anti-SU(7) [1] is ruled out or not, from the determination [2] of the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements [3–6]. A simple CKM analysis can be performed in the Kim–Seo (KS) parametrization [7] where only complex phase gives the invariant Jarlskog phase itself [8]. This phase is called the CKM phase $\delta_{\rm CKM}$.

Most family unification models assume a factor group G_f in addition to the standard model (SM) or grand unification (GUT), where for G_f continuous symmetries such as SU(2) [9], SU(3) [10], or U(1)'s [11], and discrete symmetries such as S_3 [12], A_4 [13], Δ_{96} [14], $\mathbf{Z}_{12}$ [15] have been considered. However, a true unification of families in the sense that the couplings of the family symmetry are unified with the three gauge couplings of SM has started with the seminal paper by Georgi [16], starting from an SU(N) GUT [17,18]. Along this line, a UGUTF based on SU(7) $\times$ U(1)^n was suggested [1]. It is derived from string compactification, and contains anti-SU(5) subgroup representations of sixteen chiral fields for one family. These are $\mathbf{10}_{+1/5}\,(d^c,u,d,N^0)$, $\overline{\mathbf{5}}_{-3/5}\,(d^c,\nu_e,e)$, and $\mathbf{1}_{+5/5}\,(e^+)$ [19,20]. It is comforting that a plethora of anti-SU(5) or flipped-SU(5) GUTs can be derived in string compactifications [21,22].

The anti-SU(7) solution of the family problem is to put all fermion representations in

$$\Psi^{[ABC]}+2\,\Psi^{[AB]}+8\,\Psi_{[A]}+\text{singlets}\equiv\mathbf{35}\oplus 2\times\mathbf{21}\oplus 8\times\overline{\mathbf{7}}+\mathbf{1}'\text{s},\tag{1}$$

where the indices $(A,B,C=1,2,\ldots,7)$ inside square brackets imply anti-symmetric combinations, and the bold-faced numbers are the dimensions of representations. Let the SU(5) indices of flipped-SU(5) be $a,b,c=1,2,\ldots,5$, the color indices $\alpha=1,2,3$, and weak indices $i=4,5$. Representations of Eq. (1) are those of SU(7). Breaking anti-SU(7) down to the anti-SU(5), $\Psi^{[ABC]}\to\Psi^{[abc]},\Psi^{[ab6]},\Psi^{[ab7]}$, resulting in one **10**, and $2\,\Psi^{[AB]}\to 2\,\Psi^{[ab]}$, resulting in two **10**'s. Thus, we obtain three **10**'s in the flipped SU(5). For **5**'s of anti-SU(5), they arise from $\Psi^{[a67]}$ from $\Psi^{[ABC]}$, $2\Psi^{[a6]},2\Psi^{[a7]}$ from $2\Psi^{[AB]}$, making up five **5**. For $\overline{\mathbf{5}}$'s of anti-SU(5), we obtain $8\Psi_{[a]}$'s from $8\Psi_{[A]}$'s. Thus, the

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number of remaining $\overline{\mathbf{5}}$ chiral fields are $3\Psi_{[a]}$. These contain the u^c fields of the flipped-SU(5). The flipped-SU(5) singlets contain e^+ fields. Therefore, for the CKM matrix, we can perform the study on Yukawa couplings based on the non-singlet spectra of Eq. (1).

With U(1)'s, it is possible to assign the electromagnetic charge $Q_{\rm em}=0$ for separating the color and weak charges at the location [45], which is the key point for realizing the doublet–triplet splitting in the GUT BEH multiplets [1]. The merits of the UGUTF of Eq. (1) are, (i) it allows the missing partner mechanism naturally based on a suitable μ parameter [23], (ii) it is obtained from string compactification, and (iii) it leads to plausible Yukawa couplings. The first merit has been already discussed in Ref. [1]. The second merit is the following. The R-parity in SUSY and the Peccei–Quinn symmetry are greatly used for proton longevity and toward a solution of the strong CP problem and cold dark matter [24]. Because of the gravity spoil of such symmetries in general [25,26], discrete gauge symmetries were considered in the bottom approach [27,28]. These global symmetries can be discrete subgroups of some gauge group. In the top-down approach, such as in models from string compactification, the resulting approximate discrete and global symmetries are automatically allowed since string theory describes gravity without such problems [29,30].

In this paper, we focus on the third merit by adopting the spectra obtained in Ref. [1], and explicitly check the model by calculating the mixing parameters of the right-handed currents with the measured mass and left-handed CKM parameters as input. Here, we do not use the full description of string theory [31], but use just the supergravity couplings including non-renormalizable terms[1] suppressed by the string scale, M_s. Thus, every nonrenormalizable term introduces an undetermined coefficient of $O(1)$. A great merit of UGUTF is reducing the number of couplings [1].

2. Maximal CP violation

It is known that $\delta_{\rm CKM}\approx 90^\circ$ in the KS and KM parametrizations [4,33]. It is very useful if the CKM matrix itself contains the invariant phase $\delta_{\rm CKM}$ in a visible manner. For this purpose, we use the KS parametrization [7,33],

$$V_{\rm KS}=\begin{pmatrix} c_1 & s_1c_3 & s_1s_3\\ -c_2s_1 & e^{-i\delta_{\rm CKM}}s_2s_3+c_1c_2c_3 & -e^{-i\delta_{\rm CKM}}s_2c_3+c_1c_2s_3\\ -e^{i\delta_{\rm CKM}}s_1s_2 & -c_2s_3+c_1s_2c_3e^{i\delta_{\rm CKM}} & c_2c_3+c_1s_2s_3e^{i\delta_{\rm CKM}}\end{pmatrix}. \tag{2}$$

The invariant Jarlskog phase appears in all Jarlskog triangles, not necessarily at the origin. Let us take, as an illustration purpose, $\alpha\simeq 90^{\rm o}=\frac{2\pi}{4}$, $\beta\simeq 22.5^{\rm o}=\frac{2\pi}{16}$, and $\gamma\simeq 67.5^{\rm o}=\frac{2\pi}{16}\times 3$ which are within the experimental bounds. If these phases appear from some $\mathbf{Z}_N$ symmetry, we can choose three kinds of N depending on which angle is used for $\delta_{\rm CKM}$. These discrete values of α,β, and γ can be obtained theoretically, as done in Ref. [34].

Note that J is given as $J_{\rm KS}={\rm Im}\,V^*_{31}V^*_{22}V^*_{13}=c_1c_2c_3s_1^2s_2s_3\sin\alpha=O(\lambda^6\text{–}\lambda^7)$ in the KS parametrization and $J_{\rm CK}={\rm Im}\,V^*_{31}V^*_{22}V^*_{13}=c_{12}c_{13}^2c_{23}s_{12}s_{13}s_{23}\sin\gamma=O(\lambda^6\text{–}\lambda^7)$ in the CK parametrization. If the Cabibbo angle $\theta_C=s_1c_3=s_{12}c_{13}$ is fixed, $J/\sin\theta_C=c_1c_2s_1s_2s_3\times\sin\alpha=c_{12}c_{13}c_{23}s_{13}s_{23}\sin\gamma$. For a numerical study, we can choose a *vertical* Jarlskog triangle of the first and second columns, where two $O(\lambda)$ side lengths are $|c_1c_3s_1|$, $|c_2s_1(c_1c_2c_3+s_2s_3e^{-i\alpha})|$, and an $O(\lambda^5)$ side length is $e^{i\alpha}s_1s_2|(c_1c_3s_2-c_2s_3e^{-i\alpha})|$ with the phase explicitly written for the $O(\lambda^5)$ side to be rotated freely. The corrected area depending on θ_2, θ_3 and α is $J/c_1\sin^2\theta_C=\frac{1}{2}\sin(2\theta_2)\tan(\theta_3)\sin\alpha$. For given $\sin 2\theta_2$ and $\tan\theta_3$, we can rotate α to $90^{\rm o}$ to obtain the largest $\delta_{\rm CKM}$ since in our choice of $\alpha\sim 90^{\rm o}$ is allowed. We cannot give this argument for $\delta_{\rm CKM}=\gamma$, where γ is far from $90^{\rm o}$. However, by varying the real and phase parameters, one should obtain the maximality in the vicinity of $\gamma\simeq 67.5^{\rm o}$, since J must be the same in any CKM parametrizations.

If we consider only the CKM matrix, there are three classes for the Jarlskog phase, α, β, or γ. It is pointed out that if $\delta_{\rm CKM}=\pm\delta_{\rm PMNS}$ is empirically proved then the idea of spontaneous CP violation à la Froggatt and Nielsen with a UGUTF makes sense [35]. In this case, the value $\delta_{\rm PMNS}$ will choose one class of the CKM parametrizations.

3. Yukawa couplings and masses

3.1. U(1) charges in anti-SU(7)

To check the Yukawa couplings, it is useful to have U(1) charges in the anti-SU(7) model. For completeness, therefore, we list them. For the fundamental representation $\mathbf{7}$, the U(1) charges belonging to SU(5) and SU(7) are defined as

$$\begin{aligned} X_5&=\left(\frac{2}{30},\frac{2}{30},\frac{2}{30},\frac{-3}{30},\frac{-3}{30},0,0\right)\\ Z_7&=\left(\frac{-2}{7},\frac{-2}{7},\frac{-2}{7},\frac{-2}{7},\frac{-2}{7},\frac{5}{7},\frac{5}{7}\right)\end{aligned} \tag{3}$$

The extra U(1) charge beyond SU(7) is

$$Z=\left(\frac{-5}{7},\frac{-5}{7},\frac{-5}{7},\frac{-5}{7},\frac{-5}{7},\frac{-5}{7},\frac{-5}{7}\right). \tag{4}$$

For the matter $\mathbf{7}$, therefore, we represent it as $\mathbf{7}_{-5/7}$. The electroweak hypercharge Y of the SM and the U(1) charge X of the flipped-SU(5) are defined as

[1] String calculation of all non-renormalizable couplings are not available at present. See, for an attempt, Ref. [32].

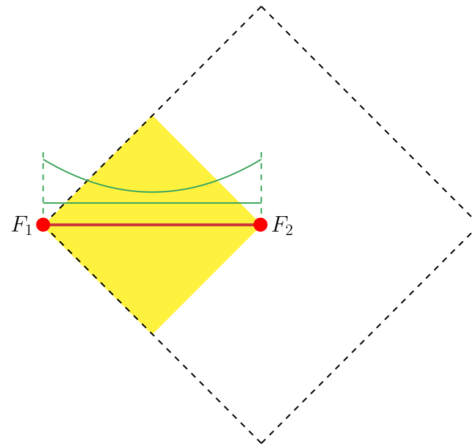

Fig. 1. The wave function in the fundamental domain (in the bulk). With $\mathbf{Z}_4$ symmetry, there are two fixed points in the two-dimensional torus. The yellow region is the fundamental domain and red bullets are two fixed points. The bulk wave function must be symmetric on the line connecting the fixed points, for which two examples of green lines are shown.

$$
\begin{aligned}
Y &= \left(\frac{-1}{3}, \frac{-1}{3}, \frac{-1}{3}, \frac{1}{2}, \frac{1}{2}, 0, 0\right) = X_5 + X, \\
X &= \left(\frac{18}{30}, \frac{18}{30}, \frac{18}{30}, \frac{18}{30}, \frac{18}{30}, 0, 0\right) = -\frac{3}{5}(Z_7 + Z).
\end{aligned} \tag{5}
$$

When $\mathbf{21}$ branches to SU(5) representations $\mathbf{10}$, $2 \cdot \mathbf{5}$, and $\mathbf{1}$, the SM U(1) charges are required to be the familiar ones, determining subscripts a, b, c in the following,

$$
(\mathbf{10}_a;\, \mathbf{5}_b,\, \mathbf{5}_b;\, \mathbf{1}_c) = \left(\frac{1}{3}(d^c),\, \frac{1}{6}(q),\, 0(N);\, \mathbf{5}_a,\, \mathbf{5}_a,\, \mathbf{1}_b\right) \to a = \frac{1}{5}\,(\sim \frac{6}{5}),\; b = \frac{3}{5},\; c = 0, \tag{6}
$$

where we used Eqs. (3) and (5) and used quantum numbers of $\mathbf{21} = \Psi^{[AB]}$. When $\mathbf{35}$ branches to SU(5) representations as $\overline{\mathbf{10}}$, $2 \cdot \mathbf{10}$, and $\mathbf{5}$, similarly subscripts d, e, f in the following are determined as

$$
\left(\overline{\mathbf{10}}_d;\, \mathbf{10}_e,\, \mathbf{10}_e;\, \mathbf{5}_f\right) = \left(\frac{-1}{3},\, \frac{-1}{6},\, 0(\overline{N})\,;\, 2\left[\frac{1}{3}(d^c),\, \frac{1}{6}(q),\, 0(N)\right];\, \mathbf{5}_g\right) \to d = \frac{-1}{5}\,(\sim \frac{9}{5}),\; e = \frac{1}{5}\,(\sim \frac{6}{5}),\; f = \frac{3}{5}, \tag{7}
$$

where we used Eqs. (3) and (5) and used quantum numbers of $\mathbf{35} = \Psi^{[ABC]}$. Because of the compact group nature, the naive U(1) charge calculation given in the bracket just by the tensor representation components is not exact. We use the $|X| \le 1$ for the fundamental representation Eq. (5). With two SU(5) indices, the $|X|$ charge are redundantly added, and we subtract ± 1. With one more indices in addition to the two indices, again we subtract ± 1 once more. The rule to use in Eqs. (6) and (7) is to subtract $(N-1)$ from X for N SU(5) indices. Because $d = -\frac{1}{5}$ and $e = \frac{1}{5}$, one vectorlike pair of $\mathbf{10}$ and $\overline{\mathbf{10}}$ are removed at the GUT scale and we obtain two $\mathbf{10}_{1/5}$'s from two $\mathbf{21}$ of Eq. (6) and one $\mathbf{10}_{1/5}$ from $\mathbf{35}$ of Eq. (7). In particular, note that $\overline{\mathbf{10}}_{-1/5}$ of Eq. (7) contains $\overline{N}$ which can develop a VEV. Thus, there result three SM families. Therefore, for the chiral representations we treat the anti-SU(5) representations as usual. For the BEH scalars, we need U(1) charges of the anti-SU(5) as $\mathbf{5}_{-2/5}$ which houses H_d and $\overline{\mathbf{5}}_{+2/5}$ which houses H_u.

Now we can calculate the Yukawa coupling matrices for the quark sector. Here, we attempt to calculate $V_{\rm CKM}$, and comment on $U_{\rm PMNS}$ in the end. For charged leptons including e^+, μ^+, τ^+, which appear as SU(7) singlets, we must obtain all SU(7) singlet spectra. These singlets are not available at present. Thus, we try to calculate $V_{\rm CKM}$ and $U_{\rm PMNS}$ without the knowledge on the singlets. The CKM matrix is obtained if we know the $Q_{\rm em} = \frac{2}{3}$ and $\frac{-1}{3}$ quark mass matrices,

$$
\begin{aligned}
&\overline{u}_L M^{(2/3)} u_R = \mathbf{10}_{1/5}\, \overline{\mathbf{5}}_{-3/5}\, \langle \overline{\Phi}_{\rm BEH, 2/5} \cdot (\cdots) \rangle \\
&\overline{d}_L M^{(-1/3)} d_R = \mathbf{10}_{1/5}\, \mathbf{10}_{1/5}\, \langle \Phi_{\rm BEH, -2/5} \cdot (\cdots) \rangle
\end{aligned} \tag{8}
$$

where we used the anti-SU(5) notation. For the PMNS matrix [36], we need information on the flipped-SU(5) singlets, in particular those corresponding to e^+. Since the anti-SU(7) singlets are not available now, we cannot discuss the PMNS matrix.

As commented in Ref. [1], the b-quark mass is expected to be much smaller than the t-quark mass, $O(\langle T^{\mathbf{21}}_{3,\rm BEH}\rangle \langle T^{\mathbf{7}}_{3,\rm BEH}\rangle / M_s \langle T^{\mathbf{7}}_{6,\rm BEH}\rangle)$, where $\langle T^{\mathbf{21}}_{3,\rm BEH}\rangle$ is the SU(5) splitting VEV $\langle \Phi^{[67]}\rangle$. Thus, we expect $m_b/m_t \sim \frac{\langle \Phi^{[67]}\rangle}{M_s \tan\beta}$. Even if $\tan\beta = O(1)$, we can fit m_b/m_t to the observed value by appropriately tuning $\langle \Phi^{[67]}\rangle$. A similar suppression occurs for the second family members.

3.2. A democratic submatrix of $M_{\rm weak}$

The multiplicity 2 [1] of the fields from T_3 leads to a democratic form for the submatrix of the mass matrix. We interpret this in the field theory language. In the two-dimensional torus, we depict the situation for $\mathbf{Z}_4$ in Fig. 1. Let us simplify the fundamental region to the one-dimensional line shown as red line in Fig. 1, whose coordinate is y. The massless fields sitting at the fixed points have the wave function $\propto \delta(y - F_1)$ and $\propto \delta(y - F_2)$. The Yukawa couplings of fermions $\int dy_1 dy_2 dy_3 \Psi_a(y_1)\Psi_b(y_2)H_c(y_3)$ must be of the democratic form due to the symmetric wave function of $H_c(y_3)$, *i.e.* those of the green lines of Fig. 1. In the string computation this feature is summarized as allowed Yukawa couplings of the twisted sector fields sitting at y_1, y_2, and y_3 [31,37]. In our case, the second and third family (quark and antiquark) fields are at T_3, and the Higgs fields are at T_6 for H_u and at T_3 for H_d. The Higgs field at T_6 has multiplicity 1 [1]. Also, the Higgs field at T_3 has multiplicity 1 due to the anomaly cancellation condition, and the singlet is a symmetric combination $\frac{1}{\sqrt{2}}(F_1+F_2)$. Thus, the area of the triangle between the quark, antiquark, and Higgs fields is zero because they sit at the same point or at the ends of the red line. The area rule of the coupling between three fixed point fields $e^{-\text{Area}}$ is 1. This statement is the case after integrating out over the 1st and 3rd tori internal coordinates, and Fig. 1 is the 2nd torus. Thus, the multiplicity 2 of the T_3 fields must have the democratic form for mass matrix if all the other (in particular the gauge) quantum numbers are the same. Thus, we consider

$$\begin{pmatrix} \frac12 & \frac12 \\ \frac12 & \frac12 \end{pmatrix} \to \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{9}$$

which can be diagonalized to give the eigenvalues 0 and 1. The democratic form can be extended to have a permutation symmetric form S_2 which has only singlet representations. Introducing two small numbers x and y (for the two independent singlets) for breaking the S_2 symmetry, it can be diagonalized to

$$M = \begin{pmatrix} \frac12+\frac{y}{2}, & \frac12+\frac{x}{2} \\ \frac12+\frac{x}{2}, & \frac12+\frac{y}{2} \end{pmatrix} \to \begin{pmatrix} \frac{-x+y}{2}, & 0 \\ 0, & 1+\frac{x+y}{2} \end{pmatrix} \to \begin{pmatrix} -\epsilon+\epsilon', & 0 \\ 0, & 1+\epsilon \end{pmatrix}, \text{ with } \epsilon = \frac{x+y}{2},\ \epsilon' = y \tag{10}$$

by

$$U^\dagger_{2\times2} M U_{2\times2}, \ \text{ with } U_{2\times2} = \begin{pmatrix} \frac{1}{\sqrt2}, & \frac{1}{\sqrt2} \\ -\frac{1}{\sqrt2}, & \frac{1}{\sqrt2} \end{pmatrix}. \tag{11}$$

A 3×3 mass matrix is changed, using a $U_{3\times3}$ matrix,

$$U^\dagger_{3\times3} \begin{pmatrix} u_1, & u_2, & u_2 \\ u_3^*, & \frac12+\frac{y}{2}, & \frac12+\frac{x}{2} \\ u_3^*, & \frac12+\frac{x}{2}, & \frac12+\frac{y}{2} \end{pmatrix} U_{3\times3} \to \begin{pmatrix} u_1, & 0, & \epsilon_2 \\ 0, & -\frac{x}{2}, & 0 \\ \epsilon_3^*, & 0, & 1+\frac{x}{2} \end{pmatrix} \tag{12}$$

where $U_{3\times3}$ contains the $U_{2\times2}$ submatrix. Here, u's denote small parameters, breaking S_2 spontaneously by the GUT scale VEVs of some SM singlet fields: $u = O(\langle\Phi\rangle/M_s)$. In view of the worry on the gravity spoil of discrete symmetries [26–29], two singlet fields are better to be two components of a doublet representation Φ of a hypothetical gauge group SU(2) in the bottom-up scenario.[2] The VEV $\langle\Phi\rangle$ breaks the S_2 symmetry spontaneously [29]. Then, the trace of Φ quantum number is zero. Thus, trace of Eq. (10) is 1, leading to $\epsilon' = 0$. Thus, for the gravity-safe correction, which is our case arising from string compactification, let us diagonalize the democratic form to

$$\begin{pmatrix} -\frac{x}{2}, & 0 \\ 0, & 1+\frac{x}{2} \end{pmatrix}. \tag{13}$$

Therefore, from the information on the origin of families in the untwisted and twisted sectors (U_1, T_3, T_5^+) [1], we can write the up- and down-type mass matrices as

$$\frac{M^{(u)}}{m_t} \approx \begin{pmatrix} & | & \Psi_{[A]}(T_5^+) & \Psi_{[A]}(T_3) & \Psi_{[A]}(T_3) \\ ----- & | & ------ & ------ & ------ \\ \Psi^{[ABC]}(U_1) & | & \epsilon_u & 0 & \epsilon_2 \\ \Psi^{[AB]}(T_3) & | & 0 & x_c & 0 \\ \Psi^{[AB]}(T_3) & | & \epsilon_3^* & 0 & 1 \end{pmatrix} \tag{14}$$

$$\frac{M^{(d)}}{m_b} \approx \begin{pmatrix} & | & \Psi^{[ABC]}(U_1) & \Psi^{[AB]}(T_3) & \Psi^{[AB]}(T_3) \\ ----- & | & ------ & ------ & ------ \\ \Psi^{[ABC]}(U_1) & | & \epsilon_d & 0 & \epsilon_1 \\ \Psi^{[AB]}(T_3) & | & 0 & x_s & 0 \\ \Psi^{[AB]}(T_3) & | & \epsilon_1 & 0 & 1 \end{pmatrix} \tag{15}$$

where the parameters in Eqs. (14), (15) can be complex in general. The zero entries in (23) and (32) elements in Eqs. (14), (15) are approximate, as commented above, since S_2 is broken by the GUT scale VEVs of some SM singlet fields: $u = O(\langle\Phi\rangle/M_s)$. However, its effect is small $x_{c,s}\cdot O(\langle\Phi\rangle/M_s)$, not significantly changing the subsequent numerical study. Note that $M^{(u)}$ is not a Hermitian matrix and $M^{(d)}$ is a symmetric matrix. In the bases where Eqs. (14), (15) are written, we proceed to calculate the CKM and PMNS matrices. Parameter ϵ_1 is given in the democratic form of the 2×2 matrix. But Eq. (15) is written in the bases where the democratic form is broken. Thus, we expect two parameters $\epsilon_1(1\pm O(x_s))$. Since x_s is small, we neglect this S_2 breaking correction as commented above. Similar comments apply to ϵ_2 and ϵ_3.

[2] In the top-down scenario, there will be no gravity spoil problem, presumably satisfying the above condition automatically.

3.3. The CKM matrix

Since $M^{(d)}$ is symmetric, let us absorb two phases ϵ_1 and ϵ_d in $\Psi^{[AB]}(T_3)$ and $\Psi^{[ABC]}(T_3)$. So, the d-quark Yukawa couplings can be considered real. And we allow a real VEV for H_d^0. If it were complex, its phase can be absorbed to right-handed d quarks. Then the real symmetric matrix $M^{(d)}$ is diagonalized by an orthogonal matrix $O = O_L = O_R$,

$$M^{(d)}_{\rm weak} = O \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} O^T \tag{16}$$

where

$$M^{(d)}_{\rm weak} = \begin{pmatrix} \begin{array}{l} m_d c_1^2 + m_s c_2^2 s_1^2 \\ +m_b s_1^2 s_2^2 \end{array} & \begin{array}{l} m_d c_1 c_3 s_1 \\ -m_s c_2 s_1[c_1c_2c_3 + s_2s_3] \\ -m_b s_1 s_2[-c_2s_3 + c_1c_3s_2] \end{array} & \begin{array}{l} m_d c_1 s_1 s_3 \\ -m_s c_2 s_1[c_1c_2s_3 - s_2c_3] \\ -m_b s_1 s_2[c_2c_3 + c_1s_2s_3] \end{array} \\[2em] \begin{array}{l} m_d c_1 c_3 s_1 \\ -m_s c_2 s_1[c_1c_2c_3 + s_2s_3] \\ -m_b s_1 s_2[-c_2s_3 + c_1s_2c_3] \end{array} & \begin{array}{l} m_d c_3^2 s_1^2 \\ +m_s[c_1c_2c_3 + s_2s_3]^2 \\ +m_b[-c_2s_3 + c_1s_2c_3]^2 \end{array} & \begin{array}{l} m_d c_3 s_1^2 s_3 \\ +m_s[c_1c_2c_3 + s_2s_3] \\ \cdot[c_1c_2s_3 - s_2c_3] \\ +m_b[-c_2s_3 + c_1s_2c_3] \\ \cdot[c_2c_3 + c_1s_2s_3] \end{array} \\[2em] \begin{array}{l} m_d c_1 s_1 s_3 \\ -m_s c_2 s_1[c_1c_2s_3 - s_2c_3] \\ -m_b s_1 s_2[c_2c_3 + c_1s_2s_3] \end{array} & \begin{array}{l} m_d c_3 s_1^2 s_3 \\ +m_s[c_1c_2s_3 - s_2c_3] \\ \cdot[c_1c_2c_3 + s_2s_3] \\ +m_b[c_2c_3 + c_1s_2s_3] \\ \cdot[-c_2s_3 + c_1s_2c_3] \end{array} & \begin{array}{l} m_d s_1^2 s_3^2 \\ +m_s[c_1c_2s_3 - s_2c_3]^2 \\ +m_b[c_2c_3 + c_1s_2s_3]^2 \end{array} \end{pmatrix} \tag{17}$$

where θ_i represent the orthogonal matrix angles $\theta_{{\rm O},i}$. Here, O is taken as a real KS parametrization,

$$V^{\rm KS}_{\rm real} = \begin{pmatrix} c_{{\rm O},1} & s_{{\rm O},1}c_{{\rm O},3} & s_{{\rm O},1}s_{{\rm O},3} \\ -c_{{\rm O},2}s_{{\rm O},1} & s_{{\rm O},2}s_{{\rm O},3} + c_{{\rm O},1}c_{{\rm O},2}c_{{\rm O},3} & -s_{{\rm O},2}c_{{\rm O},3} + c_{{\rm O},1}c_{{\rm O},2}s_{{\rm O},3} \\ -s_{{\rm O},1}s_{{\rm O},2} & -c_{{\rm O},2}s_{{\rm O},3} + c_{{\rm O},1}s_{{\rm O},2}c_{{\rm O},3} & c_{{\rm O},2}c_{{\rm O},3} + c_{{\rm O},1}s_{{\rm O},2}s_{{\rm O},3} \end{pmatrix}. \tag{18}$$

We consider $m_d = O(\lambda^4) \times m_b$. In Eq. (17), the (23) and (32) elements are vanishing up to O(λ^9) for

$$s_{{\rm O},1} = 0, \;\; t_{{\rm O},2} = t_{{\rm O},3}, \tag{19}$$

where the angles are in the 1st quadrant. Angles given in (19) matches to Eq. (15). Thus, there is one angle parameter in $V^{\rm KS}_{\rm real}$, which is taken as $\theta_{\rm O} = \theta_{{\rm O},2} = \theta_{{\rm O},2}$. So, the orthogonal matrix diagonalizing $M^{(d)}$ is

$$V^{(d)}_L = V^{(d)}_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{20}$$

However, because of the S_2 breaking effect as commented above, $V^{(d)}_{L,R}$ contains small parameters of $O(\epsilon_1 x_s)$. For simplicity, we neglect the $O(\epsilon_1 x_s)$ correction. In the model of Ref. [1], $\epsilon_1 = O(V_{\rm GUT}/M_s)$. This is because one may consider the following for ϵ_1

$$\frac{1}{M_s^2}\epsilon_{ABCDEFG}\Psi^{[ABC]}_{U_1}\Psi^{[DE]}_{T_3}\Phi^F_{T_3,{\rm BEH}}\langle\Phi^G_{T_3,{\rm BEH}}\rangle\langle\mathbf{1}_{T_{11},{\rm BEH}}\rangle,$$

and $m_b = O(V_{\rm GUT}/M_s)$. Thus, $\epsilon_1 x_s$ is estimated to be $O(\lambda^4)$. Then, the determination of the CKM matrix depends approximately on the diagonalization of $M^{(u)}$.

3.4. The CKM and PMNS matrices from anti-SU(7) UGUTF

Now the CKM matrix is determined from the diagonalization of $M^{(u)}$ by bi-unitary matrices: $V^{(u)}_L$ and $V^{(u)}_R$ with $V^{(u)}_L \neq V^{(u)}_R$,

$$V_{\rm CKM} = V^{(u)}_L O^{(d)\,T}_L \simeq V^{(u)}_L \tag{21}$$

which does not depend on $V^{(u)}_R$. The matrix elements and $Q_{\rm em} = \frac{2}{3}$ quark masses have the following relations

$$\begin{aligned} &u_L^{({\rm mass}\,i)} = V_L^{ia} u_L^a, \;\; u_R^{({\rm mass}\,i)} = V_R^{ia} u_R^a, \\ &\bar u_R^b M^{ba}_{{\rm weak},u} u_L^a = \bar u_R^{({\rm mass}\,j)} (V_R)^{jb} M^{ba}_{{\rm weak},u} (V_L^\dagger)^{ai} u_L^{({\rm mass}\,i)}. \end{aligned} \tag{22}$$

Thus, the mass matrix elements in the weak basis are

$$M^{ba}_{\text{weak},u} = (V_R^\dagger)^{bj} M^{ji}_{\text{diag},u} (V_L)^{ia} = m_u (V_R^\dagger)^{b1}(V_L)^{1a} + m_c (V_R^\dagger)^{b2}(V_L)^{2a} + m_t (V_R^\dagger)^{b3}(V_L)^{3a}, \tag{23}$$

or

$$\begin{pmatrix}
\begin{array}{l} m_u c'_1 c_1 \\ +m_c c'_2 s'_1 c_2 s_1 \\ +m_t s'_1 s'_2 s_1 s_2 e^{i\delta_{\text{CKM}} - i\delta'_{\text{CKM}}} \end{array} &
\begin{array}{l} m_u c'_1 s_1 c_3 \\ -m_c c'_2 s'_1 [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] \\ -m_t s'_1 s'_2 e^{-i\delta'_{\text{CKM}}} \\ \cdot[-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] \end{array} &
\begin{array}{l} m_u c'_1 s_1 s_3 \\ -m_c c'_2 s'_1 [c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] \\ -m_t s'_1 s'_2 e^{-i\delta'_{\text{CKM}}} \\ \cdot[c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] \end{array} \\[2em]
\begin{array}{l} m_u c'_3 s'_1 c_1 \\ -m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] c_2 s_1 \\ -m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \\ \cdot s_1 s_2 e^{i\delta_{\text{CKM}}} \end{array} &
\begin{array}{l} m_u c'_3 s'_1 s_1 c_3 \\ +m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] \\ \cdot[c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] \\ +m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \\ \cdot[-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] \end{array} &
\begin{array}{l} m_u c'_3 s'_1 s_1 s_3 \\ +m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] \\ \cdot[c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] \\ +m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \\ \cdot[c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] \end{array} \\[2em]
\begin{array}{l} m_u s'_1 s'_3 c_1 \\ -m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta_{\text{CKM}}}] c_2 s_1 \\ -m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \\ \cdot s_1 s_2 e^{i\delta_{\text{CKM}}} \end{array} &
\begin{array}{l} m_u s'_1 s'_3 s_1 c_3 \\ +m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta'_{\text{CKM}}}] \\ \cdot[c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] \\ +m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \\ \cdot[-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] \end{array} &
\begin{array}{l} m_u s'_1 s'_3 s_1 s_3 \\ +m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta_{\text{CKM}}}] \\ \cdot[c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] \\ +m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \\ \cdot[c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] \end{array}
\end{pmatrix} \tag{24}$$

where angles in $V_L^{(u)}$ are θ_i, δ and angles in $V_R^{(u)}$ are θ'_i, δ'.

Comparing Eqs. (14) and (24), we have 9 constraints. The order of magnitudes of the elements are such that the determinant of mass matrix is $O(\lambda^8 m_t^3)$ with $m_c = O(\lambda^2)m_t$ and $m_u = O(\lambda^6)m_t$. Thus, the product of (11), (23), and (32) elements is $O(\lambda^8 m_t^3)$, and the product of (11), (22), and (33) elements is also $O(\lambda^8 m_t^3)$. So, let the (22) element is $O(\lambda m_t)$ or $O(\lambda^2 m_t)$. For $M^{(u)}_{(11)} = O(\lambda^5 m_t)$, we require $M^{(u)}_{(23)} = O(\lambda^{3/2} m_t)$ and $M^{(u)}_{(32)} = O(\lambda^{3/2} m_t)$. For $M^{(u)}_{(11)} = O(\lambda^4 m_t)$, we require $M^{(u)}_{(23)} = O(\lambda^2 m_t)$ and $M^{(u)}_{(32)} = O(\lambda^2 m_t)$. So, whether the (22) element is $O(\lambda m_t)$ or $O(\lambda^2 m_t)$, we consider $M^{(u)}_{(23)} = O(\lambda^2 m_t)$ and $M^{(u)}_{(32)} = O(\lambda^2 m_t)$. Because the (33) element is $O(1)$, we require both (12) and (21) elements to be $O(\lambda^4)$. By the same argument, we require both (13) and (31) elements to be $O(\lambda^3)$. Thus, we require

$$(11) \lesssim O(\lambda^4) m_t, \tag{25}$$
$$(12) \lesssim O(\lambda^4) m_t, \tag{26}$$
$$(13) = O(\lambda^3) m_t, \tag{27}$$
$$(21) \lesssim O(\lambda^4) m_t, \tag{28}$$
$$(31) = O(\lambda^3) m_t, \tag{29}$$
$$(22) \lesssim O(\lambda^2 \text{ or } \lambda) m_t, \tag{30}$$
$$(23) = O(\lambda^2) m_t, \tag{31}$$
$$(32) = O(\lambda^2) m_t, \tag{32}$$
$$(33) = O(1) m_t, \tag{33}$$

where we used $m_t = 173.21$ GeV, $m_c = 1.275$ GeV, and $\lambda = \sin\theta_1 \cos\theta_3 = 0.2253$. The determinant can be $m_u m_c m_t$ with (31), (22), and (13) elements for the orders given above. So, we take (11), (12), and (21) elements with inequality signs.

Before presenting a numerical study, let us check that solutions suggested in Eqs. (25)–(33) are possible. From the (23) element, we restrict s'_2 and s'_3 at order λ^2,

$$\begin{aligned}
(23):\quad & m_u c'_3 s'_1 s_1 s_3 + m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\text{CKM}}}] \\
& + m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\text{CKM}}}] \simeq 0 \\
& \to s'_2 = O(\lambda^2), s'_3 = O(\lambda^2).
\end{aligned} \tag{34}$$

Then, we satisfy (32) and (22) elements,

$$\begin{aligned}
(32):\quad & m_u s'_1 s'_3 s_1 c_3 + m_c [c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] \\
& + m_t [c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] = O(\lambda^2),
\end{aligned} \tag{35}$$

$$\begin{aligned}
(22):\quad & m_u c'_3 s'_1 s_1 c_3 + m_c [c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\text{CKM}}}] \cdot [c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\text{CKM}}}] \\
& + m_t [-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\text{CKM}}}] \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\text{CKM}}}] = O(\lambda^2).
\end{aligned} \tag{36}$$

Now, the (12) element restricts s'_1 at order λ^2,

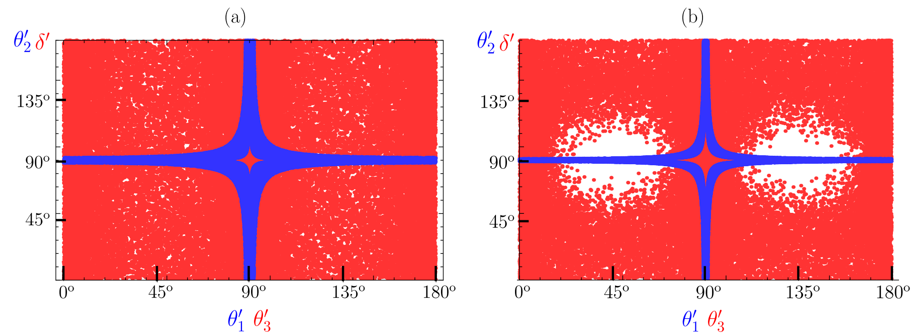

Fig. 2. The bounds on the angles of the right-handed unitary matrix $V_R^{(u)}$ diagonalizing $M^{(u)}$, (a) $a = \frac{1}{1.5}\sin\theta_c$, $b = 1.5\sin\theta_c$, and (b) $a = \frac{1}{1.2}\sin\theta_C$, $b = 1.2\sin\theta_c$. The white regions are not allowed.

$$\begin{aligned}(12):\quad & m_u c'_1 s_1 c_3 - m_c c'_2 s'_1[c_1 c_2 c_3 + s_2 s_3 e^{-i\delta_{\rm CKM}}] - m_t s'_1 s'_2 e^{-i\delta'_{\rm CKM}} \cdot [-c_2 s_3 + c_1 s_2 c_3 e^{i\delta_{\rm CKM}}] = O(\lambda^4) \\ & \to s'_1 = O(\lambda^2)\end{aligned} \tag{37}$$

Then, the (11) element is very small, $O(\lambda^5)$. The remaining (21), (13), and (31) elements are

$$(21):\quad m_u c'_3 s'_1 c_1 - m_c[c'_1 c'_2 c'_3 + s'_2 s'_3 e^{i\delta'_{\rm CKM}}]c_2 s_1 - m_t[-c'_2 s'_3 + c'_1 s'_2 c'_3 e^{-i\delta'_{\rm CKM}}]\cdot s_1 s_2 e^{i\delta_{\rm CKM}} = O(\lambda^3), \tag{38}$$

$$(13):\quad m_u c'_1 s_1 s_3 - m_c c'_2 s'_1[c_1 c_2 s_3 - s_2 c_3 e^{-i\delta_{\rm CKM}}] - m_t s'_1 s'_2 e^{-i\delta'_{\rm CKM}}\cdot[c_2 c_3 + c_1 s_2 s_3 e^{i\delta_{\rm CKM}}] = O(\lambda^4), \tag{39}$$

$$(31):\quad m_u s'_1 s'_3 c_1 - m_c[c'_1 c'_2 s'_3 - s'_2 c'_3 e^{i\delta_{\rm CKM}}]c_2 s_1 - m_t[c'_2 c'_3 + c'_1 s'_2 s'_3 e^{-i\delta'_{\rm CKM}}]\cdot s_1 s_2 e^{i\delta_{\rm CKM}} = O(\lambda^3) \tag{40}$$

where we considered $m_c = O(\lambda^2)m_t$. Here the rough bound of Eqs. (25)–(33) are satisfied except in Eq. (38). But, m_c is between $O(\lambda^2)m_t$ and $O(\lambda^3)m_t$ and Eqs. (38) is acceptable in our rough estimation. In our order of estimation, $\delta'_{\rm CKM}$ is not restricted.[3]

Therefore, the mass matrices Eqs. (14) and (15) obtained from anti-SU(7) UGUTF leads to a reasonable CKM matrix. Similarly, one can consider the lepton mixing angles which however need singlet contributions. Since there will appear additional parameters for the unknown heavy neutral lepton masses, there will be more freedom fitting for a reasonable PMNS matrix [36].

4. Bounds on the parameters of right-handed unitary matrix $V_R^{(u)}$

In Fig. 2, we present the allowed angles of $V_R^{(u)}$. The color code is: the projection on θ'_2 versus θ'_1 for all allowed θ'_3 and $\delta'_{\rm CKM}$ as blue, and $\delta'_{\rm CKM}$ versus θ'_3 for all allowed θ'_1 and θ'_2 as red. We allowed the 1σ for $\theta_1, \theta_2, \theta_3$ and $\delta_{\rm CKM}$ in $V_L^{(u)}$. We choose the V_L angles as $\theta_1 = 13.025^{\circ}{}^{+0.039^\circ}_{-0.038^\circ}$, $\theta_2 = 2.292^{\circ}{}^{+2.625^\circ}_{-2.217^\circ}$,[4] $\theta_3 = 0.8923^{\circ}{}^{+0.0382^\circ}_{-0.0357^\circ}$, and $\delta_{\rm CKM} = 85.4^{\circ}{}^{+3.9^\circ}_{-3.8^\circ}$ [2]. For the right-hand sides (of equality or inequalities) in Eqs. (25)–(33) the expansion parameter λ^n is varied in the region $a^n \le \lambda^n \le b^n$. In Fig. 2(a), we choose $a = \frac{2}{3}\sin\theta_C$ and $b = \frac{3}{2}\sin\theta_C$. In Fig. 2(b), we choose $a = \frac{1}{1.2}\sin\theta_C$ and $b = 1.2\sin\theta_C$. From Fig. 2, we conclude that the mass matrices Eqs. (14) and (15), suggested from the UGUTF anti-SU(7), are phenomenologically allowed.

5. Conclusion

We presented bounds on the mixing angles of the right-handed currents, diagonalizing the quark mass matrices, suggested from a recently proposed families unified GUT model based on anti-SU(7) [1]. The investigation suggests that quark mass matrices suggested in [1] are phenomenologically allowable, and a numerical search is presented in figures on four mixing angles of $V_R^{(u)}$ within the 1σ bounds of the CKM parameters, θ_1, θ_2, θ_3, and $\delta_{\rm CKM}$. The currently allowed CKM parametrization falls into three classes by choosing $\delta_{\rm CKM} = \alpha, \beta$, or γ of the PDG book. The Kobayashi–Maskawa and Kim–Seo parametrization choose $\delta_{\rm CKM} = \alpha$ and Chau–Keung–Maiani parametrization chooses $\delta_{\rm CKM} = \gamma$. It suggests that with three real CKM angles fixed, the area of the Jarlskog triangle is close to the maximum.

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[3] In the numerical study below, θ'_3 is not bounded also.

[4] The lower limit is given from the measured value of $J \simeq 3 \times 10^{-5}$.

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