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Reliability Consideration of a Mobile Communication System with Network Congestion

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Abstract—High quality traffic for networks is urgently needed for mobile communication systems. Mobile stations frequently become unavailable due to communication errors generated by network congestion. Traffic congestion in a network system may occur intermittently and continue for a length of time, sometimes causing communication errors. If congestion happens during communication, communication errors occur and the communication is rejected. This paper considers the problem of reliability in mobile communication systems during congestion by using a recovery scheme. We formulate a stochastic model of a mobile communication system which consists of mobile stations, several base stations and a switching center. When communication errors occur, the system makes a rollback recovery and returns to the recent checkpoint. We derive the mean time to take checkpoint and the expected number of rollback recoveries, handoff, and successful transmissions until communication errors occur. Further, we calculate the expected costs and discuss ways to minimize the costs by analyzing the optimal checkpoint intervals. © 2006 Elsevier Ltd. All rights reserved.

Keywords—Mobile communication, Network congestion, Recovery schemes, Reliability, Checkpoint interval.

1. INTRODUCTION

High quality traffic for networks is required for mobile communication systems. Mobile stations frequently become unavailable due to communication errors generated by network congestion.

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A mobile communication system, which consists of mobile stations, is recovered by a rollback technique. That is, when a communication error occurs, the rollback recovery for the mobile station associated with such an event is executed to the most recent checkpoint, so that the system can restore a consistent state. Recently, three error recovery schemes (LP (logging pessimistic), LL (logging lazy), LT (logging trickle)) in mobile environments have been investigated from various viewpoints [1,2]. We have discussed the mobile communication system with checkpointing and rollback recovery techniques [3]. Control mechanisms for dissolving network congestion also have been researched, e.g., [4,5]. Improved reliability of mobile communication systems involving mobile stations is important for stable network communications even with network congestion.

This paper considers the problem of reliability in mobile communication systems during network congestion by using the LP (logging pessimistic) recovery scheme. We formulate a stochastic model of a mobile communication system which consists of mobile stations, several base stations and a switching center. If congestion happens and is hidden, when either a mobile station or a base station has sent a request for a message, the system interrupts its request transiently and waits a constant time. Then, it sends the request for a message again. When communication errors due to network congestion occur, rollback recovery for a mobile station associated with such an event is executed to the most recent checkpoint, so that the system can recover.

In the above stochastic model, we derive the mean time to take the next checkpoint and the expected number of rollback recoveries, handoff, and successful transmissions until communication errors occur. Using these results, we derive expected costs and optimal checkpoint intervals. Finally, numerical examples are given.

2. MODEL AND ANALYSIS

A mobile communication system consists of mobile stations and several base stations as shown in Figure 1.

Each base station is connected by wired links through a switching center, and one mobile station communicates with the others by wireless links through a base station BS_i ($i = 0, 1, 2, \dots$). A mobile station MS moves from one cell to another and its connection changes from BS_i to BS_{i+1}. Communications between mobile stations can be realized by such control mechanisms.

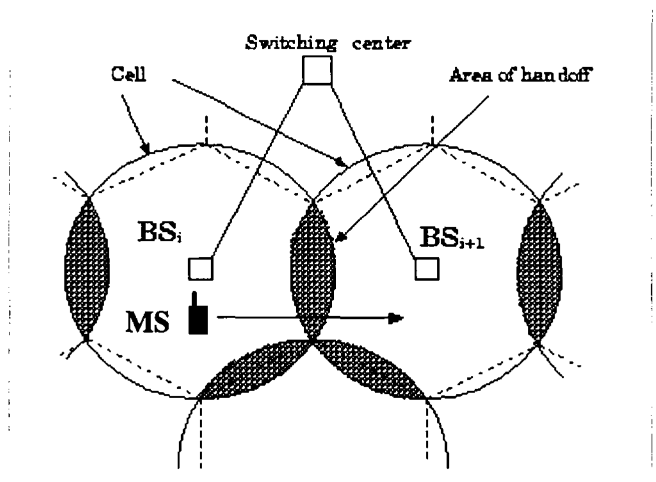

Figure 1. Outline of a mobile network system.

We are concerned only about the communication behaviors of the system with mobile station MS and base stations BS_i ($i = 0, 1, 2, \dots$) and apply an LP (logging pessimistic) recovery scheme to the system [1].

(1) The system begins to operate at time 0 and takes the first checkpoint for BS_0. Next,

it takes the checkpoint for BS_i that manages the operation of mobile station MS, when the transmissions between MS and BS_i have terminated successfully at m ($m = 1, 2, \dots$) times. The processes which are executed at MS are sent to BS_i.

(2) A mobile station MS begins to move from BS_0. The request time for transmissions between MS and BS_i has a general distribution $A(t)$ with finite mean α. Then, MS connects with BS_i as follows.

(i) Congestion in a network system occurs intermittently and is hidden. Therefore, if congestion happens in the network, the system interrupts its request transiently and waits a constant time w, that is,

$$W(t) \equiv \begin{cases} 1 & : \quad t \geq w, \\ 0 & : \quad t < w. \end{cases}$$

(ii) The time required for transmission of one message including the time to save message logs at BS_i has an exponential distribution $(1 - e^{-at})$ $(0 < a < \infty)$.

(3) Congestion happens in the network according to an exponential distribution $(1 - e^{-\lambda t})$ $(0 < \lambda < \infty)$ and continues according to an exponential distribution $(1 - e^{-\beta t})$ $(0 < \beta < \infty)$. We define the following states of a network system.

State 0: No congestion occurs and the network system is in a normal condition.

State 1: Congestion occurs.

The network system states defined above form a two-state Markov process [6]. Thus, we have the following probabilities under the initial condition that $P_{00}(0) = P_{11}(0) = 1$, $P_{01}(0) = P_{10}(0) = 0$,

$$\begin{aligned} P_{00}(t) &\equiv \frac{\beta}{\lambda + \beta} + \frac{\lambda}{\lambda + \beta} e^{-(\lambda+\beta)t}, \\ P_{11}(t) &\equiv \frac{\lambda}{\lambda + \beta} + \frac{\beta}{\lambda + \beta} e^{-(\lambda+\beta)t}, \\ P_{01}(t) &= 1 - P_{00}(t), \\ P_{10}(t) &= 1 - P_{11}(t), \end{aligned}$$

where $P_{i,j}(t)$ are probabilities that the network system is in State i ($i = 0, 1$) at time 0 and State j ($j = 0, 1$) at time $t (> 0)$.

Communication errors occur as follows.

(i) When congestion happens in the network and either request for transmissions between MS and BS_i or handoff occurs, the rollback recovery for MS associated with such an event is executed from that time to the most recent checkpoint. The state of processes and message logs are sent from BS_i to MS.

(ii) The system is regenerated by rollback recovery.

(iii) The time required for rollback recovery has a general distribution V(t) with finite mean v.

(4) When the operation of MS moves from BS_i to BS_{i+1}, i.e., when MS goes into the area of handoff, the system interrupts its operation transiently.

(i) Handoff occurs according to a general distribution $U(t)$ with finite mean $1/u$.

(ii) The time required for handoff including the time to transmit the most recent checkpoint and message logs, has a general distribution $G(t)$ with finite mean $1/\mu$.

Under the above assumptions, we define the following states of the system.

State F: Network congestion happens.

State 2: The system begins to operate or restart.

State 3: Request for transmissions between MS and BS_i occurs.

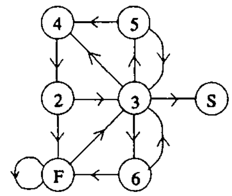

Figure 2. Transition diagram between system states.

State 4: Communication errors occur.
State 5: Handoff occurs.
State 6: Transmission of one message succeeds.
State S: Transmissions of m messages have succeeded and the system takes the checkpoint for BS_i.

The system states defined above form a Markov renewal process [3], where S is an absorbing state, and States 2–6 and F are regeneration points. A transition diagram between the system states is shown in Figure 2.

By a method similar to [7], Laplace-Stieltjes (LS) transforms $\tilde{Q}_{i,j}(s)$ of transition probabilities $Q_{i,j}(t)$ from State i ($i = 2, F, 3, 5$) to State j ($j = F, 3, 4, 5, 6$) are given by the following equations,

$$\tilde{Q}_{2,F}(s) = \int_0^\infty e^{-st} P_{01}(t)\, dA(t), \tag{1}$$

$$\tilde{Q}_{2,3}(s) = \int_0^\infty e^{-st} P_{00}(t)\, dA(t), \tag{2}$$

$$\tilde{Q}_{F,F}(s) = \int_0^\infty e^{-st} P_{11}(t)\, dW(t) = P_{11}(w)\, e^{-sw}, \tag{3}$$

$$\tilde{Q}_{F,3}(s) = \int_0^\infty e^{-st} P_{10}(t)\, dW(t) = P_{10}(w)\, e^{-sw}, \tag{4}$$

$$\tilde{Q}_{3,4}(s) = \int_0^\infty \lambda e^{-(s+\lambda+a)t} \bar{U}(t)\, dt = \frac{\lambda}{s+\lambda+a}\left[1 - \tilde{U}(s+\lambda+a)\right], \tag{5}$$

$$\tilde{Q}_{3,5}(s) = \int_0^\infty e^{-(s+\lambda+a)t}\, dU(t) = \tilde{U}(s+\lambda+a), \tag{6}$$

$$\tilde{Q}_{3,6}(s) = \int_0^\infty a e^{-(s+\lambda+a)t} \bar{U}(t)\, dt = \frac{a}{s+\lambda+a}\left[1 - \tilde{U}(s+\lambda+a)\right], \tag{7}$$

$$\tilde{Q}_{5,3}(s) = \int_0^\infty e^{-(s+\lambda)t}\, dG(t) = \tilde{G}(s+\lambda), \tag{8}$$

$$\tilde{Q}_{5,4}(s) = \int_0^\infty \lambda e^{-(s+\lambda)t} \bar{G}(t)\, dt = \frac{\lambda}{s+\lambda}\left[1 - \tilde{G}(s+\lambda)\right], \tag{9}$$

where $\tilde{Q}_{2,F}(s) \equiv \tilde{Q}_{6,F}(s)$, $\tilde{Q}_{2,3}(s) \equiv \tilde{Q}_{6,3}(s)$, and $\bar{\Phi}(t) \equiv 1 - \Phi(t)$ represent a survival function of any function $\Phi(t)$.

We derive the mean time $\ell_{2,S}(m)$ from the beginning of the system operation to the next checkpoint. Let $H_{2,3}(t)$ be the time distribution from State 2 to State 3 and $H_{6,3}(t)$ be the time distribution from State 6 to State 3. Then, we have

$$H_{2,3}(t) = Q_{2,3}(t) + Q_{2,F} * \sum_{i=1}^{\infty} Q_{F,F}^{(i-1)} * Q_{F,3}(t), \tag{10}$$

where $H_{2,3}(t) \equiv H_{6,3}(t)$ and the asterisk mark denotes the Stieltjes convolution.

Thus, the LS transform $\tilde{H}_{2,S}(s)$ of the time distribution from the beginning of operation to the next checkpoint is

$$\tilde{H}_{2,S}(s) \equiv \frac{\left[\tilde{H}_{2,3}(s)\tilde{M}(s)\right]^m}{1-\tilde{Z}(s)\tilde{V}(s)}, \tag{11}$$

where

$$M(t) \equiv \sum_{i=1}^{\infty}\left[Q_{2,5} * Q_{5,3}(t)\right]^{(i-1)} * Q_{3,6}(t),$$
$$X(t) \equiv \sum_{i=1}^{\infty}\left[Q_{3,5} * Q_{5,3}(t)\right]^{(i-1)} * \left[Q_{3,4}(t) + Q_{3,5} * Q_{5,4}(t)\right],$$
$$Z(t) \equiv \sum_{j=1}^{m}\left[H_{2,3} * M(t)\right]^{(j-1)} * \left[H_{2,3} * X(t)\right],$$

and $\Phi^{(i)}(t)$ is the i-fold convolution of $\Phi(t)$, $\tilde{\Phi}(s) \equiv \int_0^\infty e^{-st}d\Phi(t)$, for $s > 0$. Note that $M(t)$ is a probability distribution that one transmission succeeds and $X(t)$ is a probability distribution that communication errors occur. Note that $X(0) = M(0) = 0$ and $\lim_{t\to\infty}\{X(t) + M(t)\} = 1$. Therefore, the mean time $\ell_{2,S}(m)$ is

$$\begin{aligned}\ell_{2,S}(m) &\equiv \lim_{s\to 0}\frac{-d\tilde{H}_{2,S}(s)}{ds}\\ &= \frac{1-M^m}{(1-M)M^m}\left[\alpha + \frac{w}{P_{10}(w)}\int_0^\infty P_{01}(t)\,dA(t) + \frac{1}{\lambda}D_a\left[1-\tilde{G}(\lambda)\right]\right.\\ &\quad\left.+\frac{1}{\lambda+a}D_b + v(1-M)\right] \qquad (m = 1, 2, \dots),\end{aligned} \tag{12}$$

where

$$D_a \equiv \frac{\tilde{U}(\lambda+a)}{1-\tilde{U}(\lambda+a)\tilde{G}(\lambda)},$$
$$D_b \equiv \frac{1-\tilde{U}(\lambda+a)}{1-\tilde{U}(\lambda+a)\tilde{G}(\lambda)},$$
$$M \equiv \frac{aD_b}{\lambda+a},$$

and note that $0 < M < 1$.

Similarly, the expected numbers of rollback recoveries caused by network congestion, of handoff and of successful transmissions until communication errors occur are defined as follows,

$$M_R(m) = \frac{1-M^m}{M^m}, \qquad (m = 1, 2, \dots), \tag{13}$$
$$M_H(m) = \left[\frac{1-M^m}{(1-M)M^m}\right]D_a, \qquad (m = 1, 2, \dots), \tag{14}$$
$$M_S(m) = \left(\frac{M}{1-M}\right)\left[1 - M^m - mM^{m-1}(1-M)\right], \qquad (m = 1, 2, \dots). \tag{15}$$

3. OPTIMAL POLICY

We introduce the cost of successful transmissions until communication errors occur. Then, we obtain the expected cost C_I per unit of time and C_{II} per unit of transmission number, and discuss optimal policies which minimize them.

3.1. Optimal Policy I

Let c_1 be the cost for system operation, c_2, the cost for handoff, c_3, the cost for rollback recovery of communication errors, and c_4, the cost for successful transmissions until communication errors occur. Then, we give the expected cost per unit of time until the next checkpoint as

$$C_{\mathrm{I}}(m) \equiv \frac{c_1 + c_2 M_H(m) + [c_3 + c_4 M_S(m)]\, M_R(m)}{\ell_{2,S}(m)}, \qquad (m = 1, 2, \dots). \tag{16}$$

We seek an optimal checkpoint interval m_{I}^* which minimizes $C_{\mathrm{I}}(m)$ in (16) for $c_3 \geq c_2 > c_1$. From the inequality $C_{\mathrm{I}}(m+1) - C_{\mathrm{I}}(m) \geq 0$, we have

$$m\left(1 - M^m\right)\left(1 - M^{m+1}\right) \geq \frac{c_1}{c_4} \qquad (m = 1, 2, \dots). \tag{17}$$

Note that an optimal m_{I}^* does not depend on c_2 and c_3.

Denoting the left-hand side of (17) by $L_{\mathrm{I}}(m)$, $L_{\mathrm{I}}(1) = (1-M)^2(1+M)$ and $L_{\mathrm{I}}(\infty) \equiv \lim_{m\to\infty} L_{\mathrm{I}}(m) = \infty$, and hence, there exists a finite m_{I}^* $(1 \leq m_{\mathrm{I}}^* < \infty)$ which satisfies (16). Further, we have

$$L_{\mathrm{I}}(m) - L_{\mathrm{I}}(m-1) = \left(1 - M^m\right)\left[mM^{m-1}\left(1 - M^2\right) + \left(1 - M^{m-1}\right)\right] > 0.$$

Therefore, we have the following optimal policy.

(i) If $L_{\mathrm{I}}(1) < c_1/c_4$, then there exists a finite and unique $m_{\mathrm{I}}^*(> 1)$ which satisfies (17).

(ii) If $L_{\mathrm{I}}(1) \geq c_1/c_4$, then $m_{\mathrm{I}}^* = 1$.

3.2. Optimal Policy II

Under the same assumptions as Policy I, we give the expected cost per unit of transmission number until the next checkpoint as

$$C_{\mathrm{II}}(m) \equiv \frac{c_1 + c_2 M_H(m) + [c_3 + c_4 M_S(m)] M_R(m)}{m} \qquad (m = 1, 2, \dots). \tag{19}$$

We seek an optimal checkpoint interval m_{II}^* which minimizes $C_{\mathrm{II}}(m)$ in (19), for $c_3 \geq c_2 > c_1$. From the inequality $C_{\mathrm{II}}(m+1) - C_{\mathrm{II}}(m) \geq 0$, we have

$$\begin{aligned} \frac{1}{M^m}\Biggl\{\left[m\left(\frac{1-M}{M}\right) - \left(1 - M^m\right)\right]\left[\frac{D_a}{1-M}c_2 + c_3\right]\frac{1}{c_4} + m\left(1 - M^{2m}\right) \\ -m^2(1-M)M^{2m} - \frac{M\left(1-M^m\right)^2}{1-M}\Biggr\} \geq \frac{c_1}{c_4} \qquad (m = 1, 2, \dots). \end{aligned} \tag{20}$$

Denoting the left-hand side of (20) by $L_{\mathrm{II}}(m)$,

$$L_{\mathrm{II}}(1) = \left(\frac{1-M}{M}\right)\left\{\left(\frac{1-M}{M}\right)\left(\frac{D_a}{1-M}c_2 + c_3\right)\frac{1}{c_4} + \left(1 - M^2\right)\right\}, \tag{21}$$

and $L_{\mathrm{II}}(\infty) \equiv \lim_{m\to\infty} L_{\mathrm{II}}(m) = \infty$, and hence, there exist a finite m_{II}^* $(1 \leq m_{\mathrm{II}}^* < \infty)$ which satisfies (20). Further, we have

$$\begin{aligned} L_{\mathrm{II}}(m) - L_{\mathrm{II}}(m-1) = \frac{m(1-M)}{M^m}\Biggl\{\left(1 - M^{2m-1}\right) + m(1-M)M^{2m-1} \\ +\frac{1}{c_4}\left[\frac{D_a}{M}c_2 + \left(\frac{1-M}{M}\right)c_3\right]\Biggr\} > 0. \end{aligned} \tag{22}$$

Therefore, we have the following optimal policy.

(i) If $L_{\mathrm{II}}(1) < c_1/c_4$, then there exists a finite and unique $m_{\mathrm{II}}^*(> 1)$ which satisfies (20).

(ii) If $L_{\mathrm{II}}(1) \geq c_1/c_4$, then $m_{\mathrm{II}}^* = 1$.

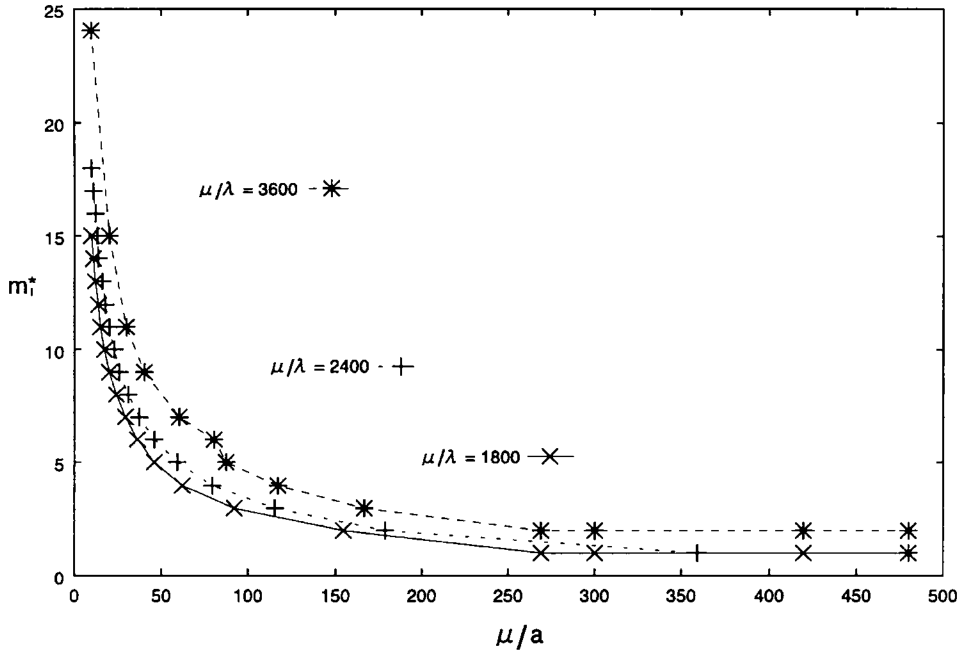

Figure 3. Numerical values of optimal number m_{I}^* for μ/a and μ/λ when $c_2/c_1 = 2$, $c_3/c_1 = 5$, $c_4/c_1 = 10$, $\mu/u = 120$.

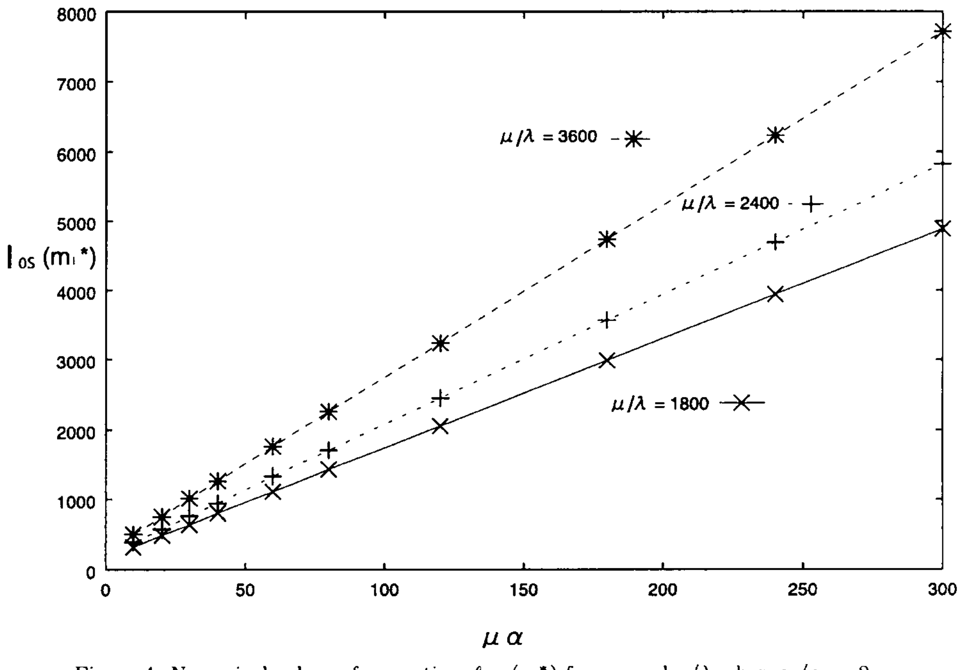

Figure 4. Numerical values of mean time $\ell_{2,S}(m_{\mathrm{I}}^*)$ for $\mu\alpha$ and μ/λ when $c_2/c_1 = 2$, $c_3/c_1 = 5$, $c_4/c_1 = 10$, $\mu/a = 10$, $\mu/u = 120$, $\mu/\beta = 30$.

4. NUMERICAL EXAMPLES AND REMARKS

We compute numerically optimal checkpoint intervals m_{I}^* and m_{II}^* which satisfy (17) and (20), respectively. It is assumed that handoff is caused by random factors of a mobile station and occurs according to an exponential distribution, i.e., $U(t) \equiv 1 - e^{-ut}$. The time required for handoff has an exponential distribution $(1 - e^{-\mu t})$ $(0 < \mu < \infty)$. The request for transmissions

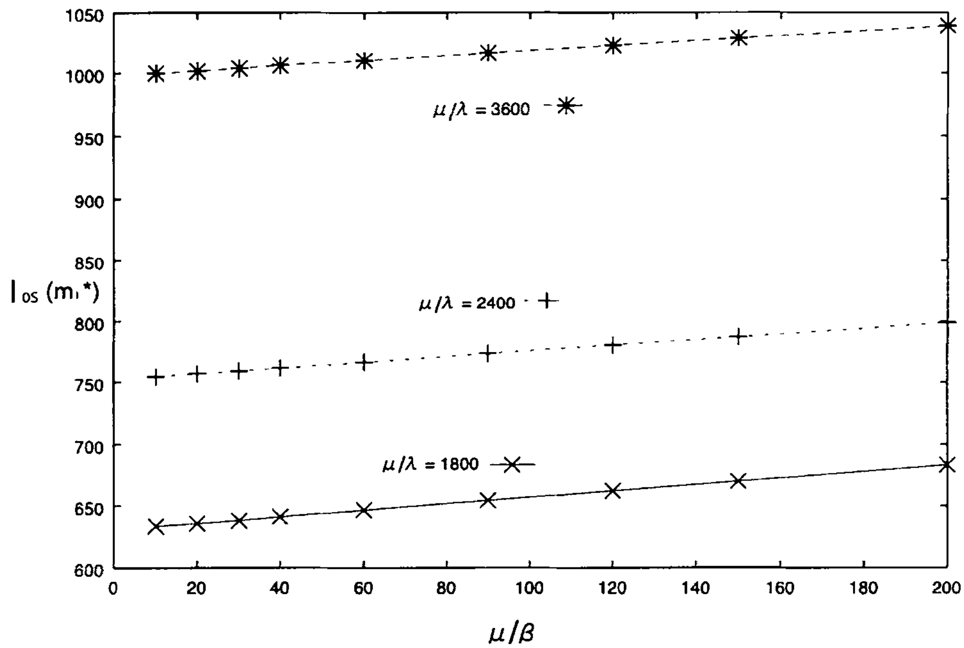

Figure 5. Numerical values of mean time $\ell_{2,S}(m_I^*)$ for μ/λ and μ/β when $c_2/c_1 = 2$, $c_3/c_1 = 5$, $c_4/c_1 = 10$, $\mu/a = 10$, $\mu/u = 120$.

between MS and BS_i is constant, i.e.,

$$A(t) \equiv \begin{cases} 1 & : \quad t \geq \alpha, \\ 0 & : \quad t < \alpha. \end{cases}$$

Suppose that the mean time $1/\mu$ of handoff is a unit of time, the mean time of network congestion is $1/\lambda = 1800 \sim 3600$, the mean time required for transmissions is $1/a = 10 \sim 480$, the mean time of handoff occurrence is $1/u = 30 \sim 1800$, the mean time until the congestion clears up is $1/\beta = 10 \sim 200$, the constant time is $w = 30$, the mean time required for rollback recovery is $v = 30$ and the mean time for transmission is $\alpha = 30$. Further, we introduce the following costs. The cost for system operation is $c_1 = 1$, the cost for handoff is $c_2/c_1 = 1, 2, 5$, the cost for rollback recovery is $c_3/c_1 = 5, 20, 50$, and the cost for retransmission is $c_4/c_1 = 10 \sim 50$.

Figure 3 shows m_{I}^* for μ/a and μ/λ when $c_2/c_1 = 2$, $c_3/c_1 = 5$, $c_4/c_1 = 10$, $\mu/u = 120$. Figure 4 shows $\ell_{0,S}(m_{\mathrm{I}}^*)$ for $\mu\alpha$ and μ/λ when $c_2/c_1 = 2$, $c_3/c_1 = 5$, $c_4/c_1 = 10$, $\mu/a = 10$, $\mu/u = 120$, $\mu/\beta = 30$. Figure 5 shows $\ell_{0S}(m_{\mathrm{I}}^*)$ for μ/β and μ/λ when $c_2/c_1 = 2$, $c_3/c_1 = 5$, $c_4/c_1 = 10$, $\mu/a = 10$, $\mu/u = 120$. Since m_{II}^* in Figures 3–5 changes similarly to m_{I}^* for each parameter, we omit numerical examples.

Figure 3 indicates that m_{I}^* decreases with μ/a and increases with μ/λ. Similarly, from Figure 4, $\ell(m_{\mathrm{I}}^*)$ increases with μ/λ and $\mu\alpha$. Moreover, from Figure 5, $\ell(m_{\mathrm{I}}^*)$ increases with μ/β.

Table 1 gives optimal checkpoint intervals m_{II}^*. This indicates that m_{II}^* decreases with c_2/c_1 and c_4/c_1. Further, when c_4/c_1 is large, m_{II}^* depends relatively little on μ/u and becomes constant. Moreover, optimal m_{II}^* depends relatively little on c_3/c_1. Similarly, m_{II}^* decreases with μ/u. Further, when λ and μ/a are large, m_{II}^* depends relatively little on c_2/c_1, and c_3/c_1 and becomes constant.

5. CONCLUSIONS

We considered the reliability of a mobile communication system with network congestion by adopting the recovery schemes of checkpoint and rollback. We derived the mean time to take the next checkpoint and the expected number of rollback recoveries, handoff, and successful

Table 1. Optimal numbers m^*_{II} to minimize $C_{\mathrm{II}}(m)$.

$\frac{c_2}{c_1}$	$\frac{c_3}{c_1}$	$\frac{c_4}{c_1}$	$\frac{\mu}{u}$	$\frac{\mu}{\lambda}=1800$, $\frac{\mu}{a}$ = 30	60	120	$\frac{\mu}{\lambda}=3600$, $\frac{\mu}{a}$ = 30	60	120
2	2	10	30	5	3	1	8	4	2
			60	6	3	2	9	5	3
			300	7	4	2	11	6	4
			600	7	4	2	11	7	4
			1800	7	4	2	11	7	4
		50	30	3	2	1	5	3	2
			60	4	2	1	6	3	2
			300	4	2	1	6	4	2
			600	4	2	1	6	4	2
			1800	4	2	1	6	4	2
	5	10	30	5	3	1	8	4	2
			60	6	3	2	9	5	3
			300	7	4	2	11	6	4
			600	7	4	2	11	7	4
			1800	7	4	2	11	7	4
		50	30	3	2	1	5	3	2
			60	4	2	1	6	3	2
			300	4	2	1	6	4	2
			600	4	2	1	6	4	2
			1800	4	2	1	6	4	2
5	5	10	30	4	2	1	6	3	1
			60	5	2	1	7	4	2
			300	6	4	2	10	6	3
			600	6	4	2	10	6	4
			1800	7	4	2	11	7	4
		50	30	3	1	1	5	2	1
			60	3	2	1	5	3	1
			300	4	2	1	6	4	2
			600	4	2	1	6	4	2
			1800	4	2	1	6	4	2

transmissions until communication errors occur. Further, we discussed analytically the optimal cost-minimizing checkpoint intervals.

From the numerical examples, we showed that the optimal checkpoint interval increases with time until the congestion disappears, and decreases with the frequencies of communication errors, handoff, the time required for transmissions, and the processing time handoff. Further, it decreases with the rate of costs for handoff, rollback recoveries, and for retransmissions of the message after rollback recovery. Moreover, we discovered that the optimal interval reaches a mostly fixed value relatively independent from the frequency of handoff when the processing time for handoff is large. The optimal checkpoint interval which minimizes $C_I(m)$ changes similarly to that which minimizes $C_{\mathrm{II}}(m)$ for any parameters.

Improvement and evaluation of the reliability of mobile communication systems is important from practical viewpoints because of greatly developed day after day usage rapidly spreading throughout various parts of the world.

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