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On the soft-gluon resummation in top quark pair production at hadron colliders

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ABSTRACT

We uncover a contribution to the NLO/NLL threshold resummed total cross section for top quark pair production at hadron colliders, which has not been taken into account in earlier literature. We derive this contribution – the difference between the singlet and octet hard (matching) coefficients – in exact analytic form. The numerical impact of our findings on the Sudakov resummed cross section turns out to be large, and comparable in size to the current estimates for the theoretical uncertainty of the total cross section. A rough estimate points toward a few percent decrease of the latter at the LHC.

1. Introduction

Improving the theoretical accuracy of the total cross section for top quark pair production is a major goal, given the importance of top physics, the excellent data-taking ability of the Tevatron and the imminent start of the LHC. Yet, relatively few theoretical calculations for this observable have been done so far. The most important input are the next to leading (NLO) numerical calculations of [1–3], where the NLO correction with accuracy better than 1% was derived. The first analytic NLO calculation of this observable [4] confirmed these results and their estimates of the numerical uncertainties. By demonstrating the appearance of a priori unexpected analytic structures, that work also clarified why theoretical progress in top production was hampered for so long. While work towards the derivation of the NNLO corrections to the total top quark pair production cross section is underway [5–11] more theoretical effort will be needed before the NNLO result becomes available for phenomenological analysis.

In view of the lack of improved fixed order calculations in the last ten years or so, the only other source of refinement in the theoretical predictions for the total top quark pair production cross section was based on the so-called soft-gluon resummation [12–17] (related phenomenological analyses can be found in [18–20]). The basic idea behind the resummation approach is to utilize our ability to predict certain logarithmic terms to all orders in the strong coupling expansion to gain an insight of the behavior of the cross section at higher orders. That, in principle, implies better control over the associated theoretical uncertainties.

Heavy flavor pair production at hadron colliders is a prominent application of the global soft-gluon resummation program. The reason is that at least four hard partons are involved in the underlying scattering process. As it is well known (see, for example, Ref. [21]) in the case of scattering of four or more colored partons the color algebra is non-trivial. This, in turn, spoils the simple exponentiation picture familiar from processes like Drell–Yan and Deep Inelastic Scattering.

As was first established in Ref. [22], the one-loop soft anomalous dimension matrix that controls the non-trivial soft-gluon correlations in this process diagonalizes in the singlet/octet basis in the kinematical configuration in question. This is a very important result that is the basis for the simplified exponentiation of the next-to-leading soft logarithms (NLL) in this process.

Utilizing the above results, the exponentiation formula for the total inclusive top quark pair production at hadron colliders beyond the leading logs was given in Ref. [17]. The singlet/octet diagonalization mentioned above implies that the soft function Δ is just a sum $\Delta = \Delta_{\mathbf{1}} + \Delta_{\mathbf{8}}$ of two standard Sudakov-type exponents, which separately describe the exponentiation of the singlet (resp. octet) color channels. Each one of the two exponents is controlled by its own set of anomalous dimensions

$$\ln \Delta_{ij,\mathbf{I}}(N) = \int_0^1 dz \, \frac{z^{N-1}-1}{1-z} \left(\int_{\mu^2}^{4m^2(1-z)^2} \frac{dq^2}{q^2} A_{ij}\big(\alpha_s(q^2)\big) + D_{ij,\mathbf{I}}\big(\alpha_s\big(4m^2(1-z)^2\big)\big) \right). \tag{1}$$

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The indices ij refer to the partons in the initial state: $ij = (q\bar{q}, gg)$, $\mathbf{I} = \mathbf{1}, \mathbf{8}$ and N is the Mellin moment dual to the kinematical variable $\rho = 4m^2/s$ (with s being the partonic invariant mass). The Mellin transform is defined as $f(N) = \int_0^1 \rho^{N-1} f(\rho)\, d\rho$. The anomalous dimensions $A_{ij} = A_i + A_j$ describe soft-collinear initial state radiation. They have a standard expansion in powers of the running strong coupling, see e.g. Ref. [17], and are known in QCD through three loops [23,24]. Contrary to A_{ij}, the anomalous dimensions $D_{ij,\mathbf{I}}$ control wide angle soft radiation and depend both on the initial and final states. They are a priori unknown, but one linear combination of $D_{ij,\mathbf{1}}$ and $D_{ij,\mathbf{8}}$ can be fixed from existing results for the threshold expansion of the total cross-section [1,2,4] at NLO. To fix uniquely both anomalous dimensions at the same order of perturbative expansion, the authors of Ref. [17] used heuristic arguments, namely, that the Sudakov exponent for the color singlet channel is identical to the one known from processes like Drell–Yan and Higgs production. With the help of an explicit calculation, in this Letter we are able to directly confirm that assumption through NLO/NLL.

Besides the soft Sudakov exponents $\Delta_{ij,\mathbf{I}}$, color dependence is also present in the so-called hard coefficients that we discuss next. In the singlet/octet basis one expresses the Sudakov total cross-section for top quark pair production at hadron colliders within NLL accuracy as

$$\sigma_{ij}^{\mathrm{TOT}}(N) = \sigma_{ij,\mathbf{1}}(N) + \sigma_{ij,\mathbf{8}}(N), \tag{2}$$

where the two terms are given by

$$\sigma_{ij,\mathbf{I}}(N) = \sigma_{ij,\mathbf{I}}^{\mathrm{Born}}(N)\sigma_{ij,\mathbf{I}}^{\mathrm{H}}\Delta_{ij,\mathbf{I}}(N). \tag{3}$$

The hard coefficients $\sigma_{ij,\mathbf{I}}^{\mathrm{H}}$ are process dependent and once the leading order correction $\sigma_{ij,\mathbf{I}}^{\mathrm{Born}}(N)$ has been factored out, they contain only N-independent constant terms. The coefficients $\sigma_{ij,\mathbf{I}}^{\mathrm{H}}$ are uniquely defined by the condition that in the limit $N \to \infty$ the Sudakov exponents $\Delta_{ij,\mathbf{I}}(N)$ contain only powers of $\ln(N)$. As usual, the hard coefficients are extracted from a fixed order calculation. Their derivation through NLO is the main goal of this article.

In Eq. (3) we omitted contributions from Coulomb terms, i.e. terms that in ρ-space behave as $\sim \alpha_s^n/\beta^k$, where $\beta = \sqrt{1-\rho}$ is the small velocity of the quark pair. The Coulomb terms represent an effect distinct from the soft-gluon logs considered in this article. These corrections have been analyzed in Refs. [17,25,26] with the conclusion that they have an impact only in the immediate vicinity of the threshold. We refer to these references for further details.

Next we explain the origin of the color index I in the hard functions appearing in Eq. (3). The easiest way to see why it should be present is to recall the basic factorization property of gauge amplitudes [27,28]. Keeping explicit only information about the color, the factorization relation for any n-particle amplitude M reads

$$M_I = J \cdot S_{IJ} \cdot H_J. \tag{4}$$

In the equation above S_{IJ} is the soft function mentioned above, I, J are color indices and H is the so-called hard function, which is finite. While the structure of the color diagonal jet function J and the soft function S can be made quite transparent based on general process-independent arguments [29–32], the form of the process dependent hard function H_J can only be obtained from a direct, process specific calculation. The matrix structure in Eq. (4) naturally translates into differential or fully integrated over the phase space cross sections. An explicit example for that procedure can be found in Ref. [33].

The color dependence of the hard coefficients $\sigma_{ij,\mathbf{I}}^{\mathrm{H}}$ was not available to any of the previous studies of soft-gluon resummation for the total inclusive cross section in hadronic collisions. The main goal of this article is to complete this gap in the literature by deriving the *exact* coefficients from a dedicated fixed order calculation, thus allowing a consistent NLO/NLL calculation and soft-gluon resummation for this observable. It is also a prerequisite for any attempt for going beyond the current NLL accuracy level.

In the original Ref. [17] these coefficients were approximated with the numerically known, color averaged coefficient taken from the calculations of Refs. [1,2]. Such an approximation is formally correct if one restricts oneself only to the resummation of the NLL soft logs since, as far as the towers of logs are concerned, these matching coefficients contribute starting from NNLL. On the other side, such an approximate choice is also correct to NLO, since by construction it reproduces the fixed order NLO results for the color summed cross section in Eq. (2). Nevertheless, one expects that the specific choice does have a numerical impact on the resummed cross section. This is easy to see with the help of the following argument: the total Sudakov cross section is a linear combination of two all-order exponents (see Eq. (2)) with coefficients proportional to the hard coefficients $\sigma_{ij,\mathbf{I}}^{\mathrm{H}}$. Therefore, a modification of the two coefficients in such a way that their color-averaged contribution is kept fixed, results in a change of the weight these two exponents (singlet/octet) carry.

The effect of this modification should be much less pronounced at the Tevatron compared to the LHC since there the main production mechanism for top-pair production is through light quark pair annihilation. It is known that for the $q\bar{q}$ production through NLO/NLL only color octet contributes, and in this case the hard coefficient has always been known with high accuracy.

One final comment regarding the hard coefficients $\sigma_{gg,\mathbf{I}}^{\mathrm{H}}$. As was established in Ref. [4] and also investigated in Ref. [26], the "constant" term in the color averaged gg cross-section extracted from [1] differs by around 7% from the exact value. It is reasonable to suspect that this correction might be quite sizable. We detail our findings in the next section.

2. Results

To derive the hard coefficients $\sigma_{ij,\mathbf{I}}^{\mathrm{H}}$ we follow the technique described in Ref. [4]. The main idea is to work directly with cut diagrams (as in the optical theorem) instead of performing explicit integrations over the phase space. Since cut diagrams are very similar to normal Feynman integrals, it is possible to derive integration-by-parts identities linking integrals with different powers of the numerators and denominators. These can in turn be solved for a relatively small set of masters. Using such a reduction, we can obtain a system of differential equations for the master integrals themselves. The equations can be solved analytically, mostly in terms of harmonic polylogarithms. The only required modification with respect to our original publication [4] consists in the need to insert suitable color projection operators in order to separate the contributions where the heavy pair is in a singlet/octet state. To this end we define the singlet state as

$$|\mathbf{1}, i, j\rangle = \frac{1}{\sqrt{N}}\delta_{ij}|\mathbf{1}\rangle, \tag{5}$$

where i and j are the color indices of the quark and anti-quark respectively. The projection onto the singlet state $|\mathbf{1}\rangle$, can now be performed with the help of the following simple modification of the color generators adjacent to the top/anti-top lines from both sides of the cut (see Fig. 1 for definition of the color indices)

$$T^{a_1}_{i,k_1} T^{a_2}_{k_2,j} T^{b_1}_{l_1,i} T^{b_2}_{j,l_2} \longrightarrow \frac{1}{N_c} T^{a_1}_{i,k_1} T^{a_2}_{k_2,i} T^{b_1}_{l_1,j} T^{b_2}_{j,l_2}. \tag{6}$$

The remaining contribution is simply attributed to the color octet state.

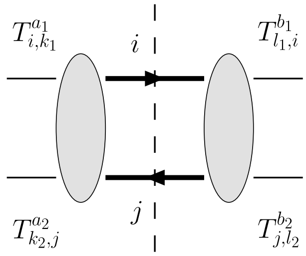

Fig. 1. Color indices in a cut graph for top quark pair production, necessary to define the color projection onto singlet/octet states Eq. (6). The dashed line represents the cut through the top line with color index i, and the anti-top line with color index j.

It is interesting to note that the color separation is well defined only in the vicinity of the threshold. Further away, both contributions become separately divergent. This is not really surprising, since the octet state will have a tendency to attract radiated gluons and hadronize into a singlet. The necessity to combine both singlet and octet contributions in order to obtain finite cross sections is only visible starting from $\mathcal{O}(\beta^3)$, and does not affect our discussion of soft-gluon effects.

Once the color separation has been accomplished, the result needs to be expanded around threshold. The resulting expressions (keeping also the Coulomb terms) read

$$\sigma_{q\bar{q},\mathbf{1}}(\beta) = \alpha_s^3 \times \mathcal{O}(\beta^3), \tag{7}$$

$$\begin{aligned}\sigma_{q\bar{q},\mathbf{8}}(\beta) = \sigma^{\mathrm{Born}}_{q\bar{q},\mathbf{8}}(\beta)\Bigg\{1 + \frac{\alpha_s}{\pi}\Bigg[&\left(C_F - \frac{C_A}{2}\right)\frac{\pi^2}{2\beta} + 8C_F\log^2\beta \\ &+ \big(C_F(-16 + 24\log 2) - 2C_A\big)\log\beta \\ &+ C_F\left(8 - \frac{\pi^2}{3} - 21\log 2 + 16\log^2 2\right) \\ &+ C_A\left(\frac{77}{9} - \frac{\pi^2}{4} - 5\log 2\right) + n_l\left(-\frac{5}{9} + \frac{2\log 2}{3}\right) - \frac{8}{9} \\ &+ \log\left(\frac{\mu^2}{m^2}\right)\left(-4C_F\log\beta + C_F\left(\frac{5}{2} - 4\log 2\right)\right. \\ &\left. + \frac{11}{6}C_A - \frac{n_l+1}{3}\right) + \mathcal{O}(\beta)\Bigg] + \mathcal{O}(\alpha_s^2)\Bigg\},\end{aligned} \tag{8}$$

$$\begin{aligned}\sigma_{gg,\mathbf{1}}(\beta) = \sigma^{\mathrm{Born}}_{gg,\mathbf{1}}(\beta)\Bigg\{1 + \frac{\alpha_s}{\pi}\Bigg[&C_F\frac{\pi^2}{2\beta} + 8C_A\log^2\beta \\ &+ C_A(-16 + 24\log 2)\log\beta + C_F\left(-5 + \frac{\pi^2}{4}\right) \\ &+ C_A\left(17 - \frac{7\pi^2}{12} - 24\log 2 + 16\log^2 2\right) \\ &+ \log\left(\frac{\mu^2}{m^2}\right)\left(-4C_A\log\beta + C_A(4 - 4\log 2) - \frac{1}{3}\right) \\ &+ \mathcal{O}(\beta)\Bigg] + \mathcal{O}(\alpha_s^2)\Bigg\},\end{aligned} \tag{9}$$

$$\begin{aligned}\sigma_{gg,\mathbf{8}}(\beta) = \sigma^{\mathrm{Born}}_{gg,\mathbf{8}}(\beta)\Bigg\{1 + \frac{\alpha_s}{\pi}\Bigg[&\left(C_F - \frac{C_A}{2}\right)\frac{\pi^2}{2\beta} + 8C_A\log^2\beta \\ &+ C_A(-18 + 24\log 2)\log\beta + C_F\left(-5 + \frac{\pi^2}{4}\right) \\ &+ C_A\left(21 - \frac{17\pi^2}{24} - 26\log 2 + 16\log^2 2\right) \\ &+ \log\left(\frac{\mu^2}{m^2}\right)\left(-4C_A\log\beta + C_A(4 - 4\log 2) - \frac{1}{3}\right) \\ &+ \mathcal{O}(\beta)\Bigg] + \mathcal{O}(\alpha_s^2)\Bigg\},\end{aligned} \tag{10}$$

where

$$\sigma^{\mathrm{Born}}_{q\bar{q},\mathbf{1}}(\beta) = 0, \tag{11}$$

$$\sigma^{\mathrm{Born}}_{q\bar{q},\mathbf{8}}(\beta) = \frac{\pi\alpha_s^2}{8m^2}\frac{(N_c^2-1)}{N_c^2}\beta + \mathcal{O}(\beta^3), \tag{12}$$

$$\sigma^{\mathrm{Born}}_{gg,\mathbf{1}}(\beta) = \frac{\pi\alpha_s^2}{4m^2}\frac{1}{N_c(N_c^2-1)}\beta + \mathcal{O}(\beta^3), \tag{13}$$

$$\sigma^{\mathrm{Born}}_{gg,\mathbf{8}}(\beta) = \frac{\pi\alpha_s^2}{8m^2}\frac{N_c^2-4}{N_c(N_c^2-1)}\beta + \mathcal{O}(\beta^3). \tag{14}$$

The coupling α_s is the renormalized $\overline{\mathrm{MS}}$ coupling evaluated at scale μ^2 and running with $n_f = n_l + 1$ active flavors. We follow the definitions and conventions from Ref. [4]. The relation between the coupling running with n_l and $n_l + 1$ flavors can also be found there.

The results in Eqs. (7)–(10) are in agreement with the ones extracted in Ref. [26] from calculations of quarkonium production at hadron colliders [34,35].

For applications to soft-gluon resummation the above results are also needed in Mellin space. One can easily switch between the two representations of the fixed order threshold expansion and the relevant formulas can be found, for example, in Ref. [18]. To further simplify the expressions, we have effectively absorbed the Euler constant into the soft function by switching to a modified Mellin moment $\overline{N} = N\exp(\gamma_E)$. After performing the Mellin transformation we can extract the exact expressions for the hard coefficients $\sigma^{\mathrm{H}}_{q\bar{q},\mathbf{1}}$ as defined in Eq. (3) by keeping only the non-$\log(\overline{N})$ terms. The results read

$$\sigma^{\mathrm{H}}_{q\bar{q},\mathbf{1}} = \mathcal{O}(\alpha_s^2), \tag{15}$$

$$\begin{aligned}\sigma^{\mathrm{H}}_{q\bar{q},\mathbf{8}} = 1 + \frac{\alpha_s}{\pi}\Bigg[&C_F\left(-8 + \frac{2\pi^2}{3} + 3\log 2\right) \\ &+ C_A\left(\frac{59}{9} - \frac{\pi^2}{4} - 3\log 2\right) \\ &+ n_l\left(-\frac{5}{9} + \frac{2\log 2}{3}\right) - \frac{8}{9} + \log\left(\frac{\mu^2}{m^2}\right)\left(-\frac{3}{2}C_F\right. \\ &\left. + \frac{11}{6}C_A - \frac{n_l+1}{3}\right)\Bigg] + \mathcal{O}(\alpha_s^2),\end{aligned} \tag{16}$$

$$\begin{aligned}\sigma^{\mathrm{H}}_{gg,\mathbf{1}} = 1 + \frac{\alpha_s}{\pi}\Bigg[&C_F\left(-5 + \frac{\pi^2}{4}\right) + C_A\left(1 + \frac{5\pi^2}{12}\right) \\ &- \frac{1}{3}\log\left(\frac{\mu^2}{m^2}\right)\Bigg] + \mathcal{O}(\alpha_s^2),\end{aligned} \tag{17}$$

$$\begin{aligned}\sigma^{\mathrm{H}}_{gg,\mathbf{8}} = 1 + \frac{\alpha_s}{\pi}\Bigg[&C_F\left(-5 + \frac{\pi^2}{4}\right) + C_A\left(3 + \frac{7\pi^2}{24}\right) \\ &- \frac{1}{3}\log\left(\frac{\mu^2}{m^2}\right)\Bigg] + \mathcal{O}(\alpha_s^2).\end{aligned} \tag{18}$$

The coefficients equations (15)–(18) are the main result from the present work. Due to the lack of a singlet contribution through that order in perturbation theory (see however the discussion at the end of Section 3), $\sigma^{\mathrm{H}}_{q\bar{q},\mathbf{8}}$ coincides with the known expressions for the color averaged cross section in the literature [1,2,4]. On the other hand, the coefficients for the gg reaction are new. They disagree with the corresponding coefficients presented in Ref. [18]. Such disagreement is not surprising given the fact that the color dependence of the hard coefficients has not been taken into account in the attempt made in that reference to exponentiate the NNLL soft logs.

Next we comment on the properties of the results above. The vanishing of the LO singlet contribution in the $q\bar{q}$ is well known, and is due to the fact that the LO reaction is mediated by an s-channel gluon. Since the vanishing is not due to kinematics but due to color effects, one expects that at higher orders this property may no longer be true. At NLO the virtual corrections are again zero due to the sandwiching of the one-loop amplitude with the projected tree-level diagram. On the other hand, the square of the one-gluon real emission diagrams is not identically zero. Our direct calculation establishes that the color singlet contribution in the $q\bar{q}$ reaction at NLO is suppressed by a factor of β^2 relative to the color averaged tree-level contributions and is thus subleading. Such leading behavior is due to the absence of Coulomb singularities and stronger suppression from the three-particle phase space.

A rather striking feature of the exact color coefficients is that their color dependence is “standard”, i.e. they are simply polynomials in C_F, C_A, etc. This is to be contrasted to the color averaged coefficients used in the earlier literature where color factors $\sim 1/(N_c^2-2)$ appear.

It is very interesting to try to estimate the size of the numerical effect of the new terms in the gg reaction derived here and in Ref. [4]. To that end we calculate the hard corrections $\sigma^{\mathrm{H}}_{ij,\mathbf{I}}$ and compare them to their counterparts from Ref. [17]. In the calculation we take $N_c=3$, $\mu^2=m^2=m^2_{\mathrm{top}}$, $\alpha_s(n_f=n_l+1)\approx 0.108$ and we restore the dependence of γ_{E} as explained above. We get the following results

$$\sigma_{gg}^{\mathrm{H(BCMN)}} = 1+\frac{\alpha_s}{\pi}14.39+\mathcal{O}(\alpha_s^2), \tag{19}$$

$$\sigma_{gg}^{\mathrm{H(BCMN)}}\big|_{C_3\mathrm{exact}} = 1+\frac{\alpha_s}{\pi}12.04+\mathcal{O}(\alpha_s^2), \tag{20}$$

$$\sigma_{gg,\mathbf{1}}^{\mathrm{H}} = 1+\frac{\alpha_s}{\pi}9.16+\mathcal{O}(\alpha_s^2), \tag{21}$$

$$\sigma_{gg,\mathbf{8}}^{\mathrm{H}} = 1+\frac{\alpha_s}{\pi}13.19+\mathcal{O}(\alpha_s^2). \tag{22}$$

The function $\sigma_{gg}^{\mathrm{H(BCMN)}}$ is defined as in Ref. [17]. The function $\sigma_{gg}^{\mathrm{H(BCMN)}}|_{C_3\mathrm{exact}}$ has the same functional form as the one in Eq. (19), but the value of the constant C_3 has been modified from $C_3=37.23$ (as derived from Ref. [1] and as applied in Ref. [17]) to its exact value $C_3=34.88$ (as derived in Ref. [4]).

Dividing Eqs. (21), (22) by Eq. (19) and recalling Eq. (3), we see that the effect of the color dependence in the hard coefficients is a decrease of the singlet (resp. octet) Sudakov cross section by 12% (resp. 3%) compared to the ones in Ref. [17].

This is a large effect. Indeed, the shifts we observe in the Sudakov factor are as large in size as the present conservative estimate [19] of the theoretical uncertainty of the total top pair production cross section and are significantly larger than the total cross section uncertainty estimate in Ref. [18]. A detailed phenomenological investigation of the results derived here will require a dedicated analysis.

It is also interesting to demonstrate the impact purely due to the numerical uncertainty in the constant C_3. To that end we consider the shift with respect to the results in Ref. [17]. Dividing Eq. (20) by Eq. (19) one can easily see that its effect is to decrease the hard function σ_{gg}^{H} (and thus the whole Sudakov cross section) by 5%. This is also a very significant effect given that its origin is pure numerics.

Another view of the impact of our results can be obtained by looking at the cross section expanded to NNLO, similarly to what has been done in [18]. Ignoring terms coming from Coulomb enhancement, and cutting the logarithmic expansion at $\log^2\beta$ (see discussion in Section 3), the result presented in Eq. (21) of Ref. [18] for the NNLO contribution reads

$$\sigma_{gg}^{(2)} = \sigma_{gg}^{\mathrm{Born}}(\beta)\big(4608\log^4\beta+1894.9\log^3\beta - 3.4811\log^2\beta+\mathcal{O}(\log\beta)\big), \tag{23}$$

where the expansion parameter has been taken to be $\alpha_s/(4\pi)$

$$\sigma_{gg}(\beta) = \sigma_{gg}^{\mathrm{Born}}(\beta)+\frac{\alpha_s}{4\pi}\sigma_{gg}^{(1)}+\left(\frac{\alpha_s}{4\pi}\right)^2\sigma_{gg}^{(2)}+\mathcal{O}(\alpha_s^3). \tag{24}$$

It turns out that the coefficient of $\log^2\beta$ exhibits an accidental cancellation and is in fact given by

$$-14306.9505+384C_3. \tag{25}$$

Inserting the exact value of C_3 derived in [4], the coefficient of $\log^2\beta$ in Eq. (23) changes to

$$-912.35, \tag{26}$$

i.e. the change of only 7% in the value of C_3 results in a magnification by a factor of about 260 of the coefficient of the quadratic log of $\sigma_{gg}(\beta)$ at NNLO.

3. Summary and implications beyond NLO/NLL

In this work we demonstrate that separate color singlet/color octet hard (matching) coefficients need to be introduced in the Sudakov total top quark pair production cross section. With the help of a dedicated fixed order calculation we derive these coefficients in analytic form. The difference between the hard coefficients for the two color states has not been considered in earlier soft-gluon resummation literature. We estimate the effect of these new contributions showing that they decrease the Sudakov total cross section by 12% in the singlet and by 3% in the octet channel. These shifts are large when compared to the current conservative estimate of the uncertainty on the total top quark pair production cross section.

The effect of these new contributions on the total top production cross section will be somewhat reduced due to the subtraction of the $\mathcal{O}(\alpha_s)$ contributions from the Sudakov cross section (see Ref. [17] for detailed description of the NLO/NLL matching procedure). However, we have also demonstrated that the new corrections modify the terms in the Sudakov cross section at order $\mathcal{O}(\alpha_s^2)$ by a large amount. Therefore, a few percent effect on the total top production cross section at the LHC can easily be anticipated. The exact size of the impact of our findings can only be obtained from a detailed phenomenological analysis.

With the results derived here and in Ref. [4] the NLO/NLL program for the total top quark pair production cross section at hadron colliders is now completed. The precision requirements for this crucial observable for the LHC program are very high and mandate improved theoretical precision. In principle, to achieve that one has to go beyond the current NLO/NLL level of accuracy.

Significant progress has already been made towards the direct fixed order calculation of the top-pair production cross section at NNLO [4–11]. In the near future, improvements can be expected from the careful analysis of the new results reported here and in Ref. [4]. The natural step beyond that is to try to promote the resummation formalism to the NNLL level. We discuss in the following how this can be done. Before we proceed, we would like to make a comment concerning Ref. [18] where such an attempt has already been made. The comparison with the direct exact calculation reported here shows that the one-loop hard coefficients used in that reference are incorrect. Since this discrepancy starts at the level of NLO/NLL it will clearly also affect their predictions for the NNLL terms. For that reason we have excluded the single log terms from our discussion around Eq. (23).

As we emphasized in our previous discussion, the basis [22] of the threshold exponentiation for the top pair cross section is the singlet/octet diagonalization of the soft anomalous dimension matrix in this special kinematics. Therefore, the extension of the resummation formalism to the NNLL level, requires to first verify if the corresponding two-loop *massive* anomalous dimension diagonalizes in a similar manner. As of writing of this Letter there exists no such an indication in the literature. In fact, the only known [36] property of the two-loop massive anomalous dimension matrix is that it should differ from the corresponding massless one (known through two-loops from Refs. [31,37]) by terms vanishing in the massless limit as powers of the mass. Clearly this information is insufficient to determine its behavior near threshold.

The next open problem in the NNLL resummation program would then be the derivation of the two-loop anomalous dimension $D_{ij,\mathbf{8}}$ appearing in Eq. (1). Arguments about its value are given in Ref. [18]. In the present work, based on an NLO fixed order calculation, we make no statement about it. That would be a subject for future investigation.

The contributions from Coulomb singularities through two-loops are known from other processes, and have been summarized in Ref. [18].

Finally, we turn our attention to the hard coefficients $\sigma^{\mathrm{H}}_{ij,\mathbf{I}}$ from Eq. (3). The complete set at NLO has been presented in this work. The corresponding two-loop corrections can only be extracted from a future two-loop calculation of the top-production cross-section near threshold. Clearly, this is a very demanding task. Moreover, there might be an a priori non-vanishing contribution from the square of the one-loop virtual diagrams to $\sigma_{q\bar{q},\mathbf{1}}$ starting from order α_s^4 (due to Coulomb enhancements and weaker suppression from the two-particle phase space). If indeed nonzero, it will contribute to a tower of NNLL soft logs and might have a numerical impact on the Tevatron predictions. Such a possibility has not been investigated so far in the literature.

As a final comment we would like to point to our discussion in Ref. [4], where we have argued about the rather limited phenomenological value of truncating the all order exponentiation to derive partial NNLO (or higher) terms. Specific examples can be found in Fig. 3 of Ref. [38] and in Ref. [4].

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References

[1] P. Nason, S. Dawson, R.K. Ellis, Nucl. Phys. B 303 (1988) 607.
[2] W. Beenakker, et al., Phys. Rev. D 40 (1989) 54.
[3] W. Bernreuther, et al., Nucl. Phys. B 690 (2004) 81, hep-ph/0403035.
[4] M. Czakon, A. Mitov, arXiv:0811.4119 [hep-ph], 2008.
[5] M. Czakon, A. Mitov, S. Moch, Nucl. Phys. B 798 (2008) 210, arXiv:0707.4139 [hep-ph].
[6] M. Czakon, A. Mitov, S. Moch, Phys. Lett. B 651 (2007) 147, arXiv:0705.1975 [hep-ph].
[7] M. Czakon, Phys. Lett. B 664 (2008) 307, arXiv:0803.1400 [hep-ph].
[8] J.G. Korner, Z. Merebashvili, M. Rogal, Phys. Rev. D 77 (2008) 094011, arXiv:0802.0106 [hep-ph].
[9] B. Kniehl, et al., Phys. Rev. D 78 (2008) 094013, arXiv:0809.3980 [hep-ph].
[10] R. Bonciani, et al., JHEP 0807 (2008) 129, arXiv:0806.2301 [hep-ph].
[11] C. Anastasiou, S.M. Aybat, arXiv:0809.1355 [hep-ph], 2008.
[12] G. Sterman, Nucl. Phys. B 281 (1987) 310.
[13] S. Catani, L. Trentadue, Nucl. Phys. B 327 (1989) 323.
[14] S. Catani, L. Trentadue, Nucl. Phys. B 353 (1991) 183.
[15] E. Laenen, J. Smith, W.L. van Neerven, Phys. Lett. B 321 (1994) 254, hep-ph/9310233.
[16] S. Catani, et al., Nucl. Phys. B 478 (1996) 273, hep-ph/9604351.
[17] R. Bonciani, et al., Nucl. Phys. B 529 (1998) 424, hep-ph/9801375.
[18] S. Moch, P. Uwer, Phys. Rev. D 78 (2008) 034003, arXiv:0804.1476 [hep-ph].
[19] M. Cacciari, et al., JHEP 0809 (2008) 127, arXiv:0804.2800 [hep-ph].
[20] N. Kidonakis, R. Vogt, Phys. Rev. D 78 (2008) 074005, arXiv:0805.3844 [hep-ph].
[21] R. Bonciani, et al., Phys. Lett. B 575 (2003) 268, hep-ph/0307035.
[22] N. Kidonakis, G. Sterman, Nucl. Phys. B 505 (1997) 321, hep-ph/9705234.
[23] S. Moch, J.A.M. Vermaseren, A. Vogt, Nucl. Phys. B 688 (2004) 101, hep-ph/0403192.
[24] A. Vogt, S. Moch, J.A.M. Vermaseren, Nucl. Phys. B 691 (2004) 129, hep-ph/0404111.
[25] S. Catani, et al., Phys. Lett. B 378 (1996) 329, hep-ph/9602208.
[26] K. Hagiwara, Y. Sumino, H. Yokoya, Phys. Lett. B 666 (2008) 71, arXiv:0804.1014 [hep-ph].
[27] R. Akhoury, Phys. Rev. D 19 (1979) 1250.
[28] A. Sen, Phys. Rev. D 28 (1983) 860.
[29] N. Kidonakis, G. Oderda, G. Sterman, Nucl. Phys. B 531 (1998) 365, hep-ph/9803241.
[30] G. Sterman, M.E. Tejeda-Yeomans, Phys. Lett. B 552 (2003) 48, hep-ph/0210130.
[31] S. Mert Aybat, L.J. Dixon, G. Sterman, Phys. Rev. D 74 (2006) 074004, hep-ph/0607309.
[32] A. Mitov, G. Sterman, in preparation.
[33] N. Kidonakis, et al., Phys. Rev. D 64 (2001) 114001, hep-ph/0105041.
[34] J.H. Kühn, E. Mirkes, Phys. Rev. D 48 (1993) 179, hep-ph/9301204.
[35] A. Petrelli, et al., Nucl. Phys. B 514 (1998) 245, hep-ph/9707223.
[36] A. Mitov, S. Moch, JHEP 0705 (2007) 001, hep-ph/0612149.
[37] S. Mert Aybat, L.J. Dixon, G. Sterman, Phys. Rev. Lett. 97 (2006) 072001, hep-ph/0606254.
[38] K. Melnikov, A. Mitov, Phys. Lett. B 620 (2005) 69, hep-ph/0505097.