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Output Regulation of Nonlinear Singularly Perturbed Systems

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Abstract—In this paper, the state feedback regulator problem of nonlinear singularly perturbed systems is discussed. It is shown that, under standard assumptions, this problem is solvable if and only if a certain nonlinear partial differential equation is solvable. Once this equation is solvable, a feedback law which solves the problem can easily be constructed. The developed control law is applied to a nonlinear chemical process.

Keywords—Output regulation, Singular perturbations, State feedback, Poisson stability, Chemical process.

1. INTRODUCTION

One of the most important problems in control theory is that of controlling a fixed plant in order to have its output tracking (or rejecting) reference (or disturbance) signals produced by some external generator (the exosystem). For linear cases, this problem has been treated by several authors (for instance, [1–4]). In particular, in [1] Francis has shown that the solvability of a multivariable linear regulator problem corresponds to the solvability of a system of two linear matrix equations. Recently, several authors have developed the corresponding problem for nonlinear cases (see [5–9]). Especially, in [8] an extension of the results established by Francis to nonlinear systems has shown that the nonlinear regulator problem is solvable if and only if a certain nonlinear partial differential equation is solvable.

In this paper, we show how the results established by Isidori and Byrnes can be extended to a class of singularly perturbed systems.

Nonlinear singularly perturbed systems arise naturally in a wide variety of engineering applications, representative examples include catalytic continuous stirred-tank reactors [10], biochemical

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reactors [11], fluidized catalytic crackers [12], flexible mechanical systems [13], electromechanical networks [14], etc. For such systems, the problem of having the output tracking (or rejecting) reference (or disturbance) signals produced by some external generator is also of importance.

Based on the method provided by singular perturbation theory, see [15] or [16], the original model can be decomposed into two lower-order models representing the slow and fast dynamics. Then, the asymptotic properties of the original model can be inferred from the knowledge of the behavior of the lower-order models.

This paper is organized as follows: the problem statement is presented in Section 2. In Section 3, we construct, under some assumption, a reference control law. Section 4 shows that this reference control law can solve the problem if only if a certain nonlinear partial differential equation is solvable. The developed method is applied to a chemical process in Section 5.

2. PROBLEM STATEMENT

Throughout this paper, the following system is considered:

$$\begin{aligned} \dot{x} &= f_1(x) + Q_1(x)z + g_1(x)u + p(x)w, \\ \epsilon\dot{z} &= f_2(x) + Q_2(x)z + g_2(x)u, \\ \dot{w} &= s(w), \\ e &= h(x) + q(w). \end{aligned} \tag{1}$$

The first two equations of this system describe a plant, with states x, defined in a neighborhood X of the origin of R^n, and z, defined in a neighborhood Z of the origin of R^p, input $u \in R$, subject to the effect of a disturbance represented by the vector field $p(x)w$, and a small positive parameter ϵ. The fourth equation defines the error $e \in R$ between the plant output $h(x)$ and the reference signal $q(w)$. The third equation describes an autonomous system, the so-called exosystem, defined in a neighborhood W of the origin of R^s, which models the class of disturbance and reference signals taken into consideration. It is assumed that $f_1(0) = 0$, $f_2(0) = 0$, $s(0) = 0$, $h(0) = 0$, $q(0) = 0$, and that the vector fields and functions involved are smooth.

The state feedback regulator problem seeks a state feedback controller of the form

$$u = \alpha(x, z, w), \qquad \alpha(0,0,0) = 0. \tag{2}$$

The corresponding closed-loop system becomes

$$\begin{aligned} \dot{x} &= f_1(x) + Q_1(x)z + g_1(x)\alpha(x,z,w) + p(x)w, \\ \epsilon\dot{z} &= f_2(x) + Q_2(x)z + g_2(x)\alpha(x,z,w), \\ \dot{w} &= s(w). \end{aligned} \tag{3}$$

The state feedback regulator problem may be formally described as the following.

STATE FEEDBACK REGULATOR PROBLEM. Find, if possible, $\alpha(x,z,w)$ such that

(1) the equilibrium (x,z)=(0,0) of

$$\begin{aligned} \dot{x} &= f_1(x) + Q_1(x)z + g_1(x)\alpha(x,z,0), \\ \epsilon\dot{z} &= f_2(x) + Q_2(x)z + g_2(x)\alpha(x,z,0), \end{aligned}$$

is exponentially stable;

(2) there exists a neighborhood $U \subset X \times Z \times W$ of $(0,0,0)$ such that, for each initial condition $(x(0), z(0), w(0)) \in U$, the solution of (3) satisfies

$$\lim_{t\to\infty \epsilon\to 0} (h(x(t)) + q(w(t))) = 0.$$

3. CONTROL LAW DESIGN

In this section, a reference state feedback regulator is constructed. Two basic assumptions are needed. The first one is as required in [8], and the second one states a stability requirement on the fast subsystem as in [11].

ASSUMPTION H1. *w=0 is a stable equilibrium of the exosystem, and there exists a neighborhood $\hat{W} \subset W$ of the origin with the property that each initial condition $w(0) \in \hat{W}$ is Poisson stable.*

ASSUMPTION H2. *The pair $[Q_2(x), g_2(x)]$ is stabilizable uniformly in $x \in X$.*

Then, just as in [11], we will initially consider control laws of the form

$$u = \tilde{u} + k^{\top}(x)z, \tag{4}$$

where $\tilde{u}$ is an auxiliary input and $k^{\top}(x)$ is a vector field on R^n such that the matrix $Q_2(x) + g_2(x)k^{\top}(x)$ is Hurwitz uniformly in $x \in X$.

Under a control law of the form of (4) system (1) takes the form

$$\begin{aligned} \dot{x} &= f_1(x) + \left[Q_1(x) + g_1(x)k^{\top}(x)\right] z + g_1(x)\tilde{u} + p(x)w, \\ \epsilon\dot{z} &= f_2(x) + \left[Q_2(x) + g_2(x)k^{\top}(x)\right] z + g_2(x)\tilde{u}, \\ \dot{w} &= s(w). \end{aligned} \tag{5}$$

Performing a standard two-time-scale decomposition, one can easily show that the fast subsystem is given by

$$\frac{dz}{d\tau} = f_2(x) + \left[Q_2(x) + g_2(x)k^{\top}(x)\right] z + g_2(x)\tilde{u}, \tag{6}$$

where the fast time-scale

$$\tau = \frac{t}{\epsilon},$$

while the corresponding reduced system or slow subsystem takes the form

$$\begin{aligned} \dot{x} &= \tilde{F}(x) + \tilde{G}(x)\tilde{u} + p(x)w, \\ \dot{w} &= s(w), \\ e &= h(x) + q(w), \end{aligned} \tag{7}$$

where

$$\begin{aligned} \tilde{F}(x) &= f_1(x) - \left[Q_1(x) + g_1(x)k^{\top}(x)\right]\left[Q_2(x) + g_2(x)k^{\top}(x)\right]^{-1} f_2(x), \\ \tilde{G}(x) &= g_1(x) - \left[Q_1(x) + g_1(x)k^{\top}(x)\right]\left[Q_2(x) + g_2(x)k^{\top}(x)\right]^{-1} g_2(x). \end{aligned} \tag{8}$$

Motivated by the satisfaction of a large number of practical applications, including the chemical reactor example shown later, we have the following assumption.

ASSUMPTION H3. *There exists an integer r and a set of coordinates*

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_r \\ \eta_1 \\ \vdots \\ \eta_{n-r} \end{pmatrix} = \Phi(x) = \begin{pmatrix} h(x) \\ L_{\tilde{F}}h(x) \\ \vdots \\ L_{\tilde{F}}^{r-1}h(x) \\ \phi_1(x) \\ \vdots \\ \phi_{n-r}(x) \end{pmatrix} \tag{9}$$

such that the reduced system (7) takes the form

$$
\begin{aligned}
\dot{\xi}_1 &= \xi_2, \\
&\vdots \\
\dot{\xi}_{r-1} &= \xi_r, \\
\dot{\xi}_r &= L_{\tilde{F}}^r h\left(\Phi^{-1}(\xi,\eta)\right) + L_{\tilde{G}} L_{\tilde{F}}^{r-1} h\left(\Phi^{-1}(\xi,\eta)\right)\tilde{u} + L_p L_{\tilde{F}}^{r-1} h\left(\Phi^{-1}(\xi,\eta)\right) w, \\
\dot{\eta}_1 &= \Psi_1(\xi,\eta), \\
&\vdots \\
\dot{\eta}_{n-r} &= \Psi_{n-r}(\xi,\eta), \\
\dot{w} &= s(w), \\
e &= \xi_1 + q(w),
\end{aligned}
\tag{10}
$$

where $L_{\tilde{G}} L_{\tilde{F}}^{r-1} h(x) \neq 0$ *for all* $x \in X$.

The following assumption poses our stability requirement on the reduced system of (7).

ASSUMPTION H4. *The zero dynamic of the reduced system (7)*

$$
\begin{aligned}
\dot{\eta}_1 &= \Psi_1(0,\eta) \\
&\vdots \\
\dot{\eta}_{n-r} &= \Psi_{n-r}(0,\eta)
\end{aligned}
\tag{11}
$$

is locally exponentially stable.

With these assumptions, the reference control law can be easily constructed as follows

$$
u = \alpha(x,z,w) = -\left[\beta_r L_{\tilde{G}} L_{\tilde{F}}^{r-1} h(x)\right]^{-1} \left[\sum_{k=0}^{r} \beta_k L_{\tilde{F}}^k h(x) + \beta_r L_p L_{\tilde{F}}^{r-1} h(x) w\right] + k^{\top}(x) z, \tag{12}
$$

where the feedback gain $k^{\top}(x)$ is such that the matrix $Q_2(x) + g_2(x) k^{\top}(x)$ is Hurwitz uniformly in $x \in X$, and β_k are parameters chosen so that the polynomial $\beta_r s^r + \beta_{r-1} s^{r-1} + \cdots + \beta_1 s + \beta_0 = 0$ is Hurwitz.

It is clear that the auxiliary input takes the form

$$
\alpha(x,0,w) = -\left[\beta_r L_{\tilde{G}} L_{\tilde{F}}^{r-1} h(x)\right]^{-1} \left[\sum_{k=0}^{r} \beta_k L_{\tilde{F}}^k h(x) + \beta_r L_p L_{\tilde{F}}^{r-1} h(x) w\right] \tag{13}
$$

and under which, the x-subsystem of the closed-loop system of the reduced system (7) is exponentially sable in spite of w.

In the following section, it will be shown that the reference control law can solve the state feedback regulator problem if a nonlinear partial differential equation is solvable.

4. THE MAIN RESULT

In this section, the main result of this paper is given in the form of a theorem.

THEOREM 1. *Under Assumptions H1–H4, the state feedback regulator problem of the singularly perturbed system (1) can be solved by the control law* $u = \alpha(x,z,w)$, *in the form of (12), if and only if there exists a* $C^k (k \geq 2)$ *mapping* $x = \pi(w)$ *with* $\pi(0) = 0$, *defined in a neighborhood* $W^0 \subset W$ *of 0, satisfying the conditions*

$$
\begin{aligned}
\frac{\partial \pi}{\partial w} s(w) &= \tilde{F}(\pi(w)) + \tilde{G}(\pi(w)) c(w) + p(\pi(w)) w, \\
h(\pi(w)) + q(w) &= 0,
\end{aligned}
\tag{14}
$$

where

$$c(w) = -\left[\beta_r L_{\tilde{G}} L_{\tilde{F}}^{r-1} h(\pi(w))\right]^{-1} \left[\sum_{k=0}^{r} \beta_k L_{\tilde{F}}^{k} h(\pi(w)) + \beta_r L_p L_{\tilde{F}}^{r-1} h(\pi(w)) w\right].$$

PROOF. When $w = 0$, system (1) takes the form

$$\begin{aligned} \dot{x} &= f_1(x) + Q_1(x)z + g_1(x)u, \\ \epsilon\dot{z} &= f_2(x) + Q_2(x)z + g_2(x)u, \\ e &= h(x), \end{aligned} \tag{15}$$

and the control law (12) becomes

$$u = \alpha(x,z,0) = -\left[\beta_r L_{\tilde{G}} L_{\tilde{F}}^{r-1} h(x)\right]^{-1} \left[\sum_{k=0}^{r} \beta_k L_{\tilde{F}}^{k} h(x)\right] + k^{\top}(x) z.$$

Then, as a direct result of [11], the equilibrium $(0,0)$ of the closed-loop system

$$\begin{aligned} \dot{x} &= f_1(x) + Q_1(x)z + g_1(x)\alpha(x,z,0), \\ \epsilon\dot{z} &= f_2(x) + Q_2(x)z + g_2(x)\alpha(x,z,0), \end{aligned} \tag{16}$$

is exponentially stable, i.e., the first objective of the state feedback regulator problem is achieved.

In the third section, we have noted that, under the control law (12), the x-subsystem of the closed-loop system of the reduced system (7) is exponentially stable in spite of w. Combining (6) and (13), the closed-loop fast system takes the form

$$\frac{d\bar{\eta}}{d\tau} = \left[Q_2(x) + g_2(x)k^{\top}(x)\right]\bar{\eta}, \tag{17}$$

where $\bar{\eta} = z - \bar{\xi}$, and

$$\bar{\xi} = -\left[Q_2(x) + g_2(x)k^{\top}(x)\right]^{-1} \left\{ f_2(x) - g_2(x)\left[\beta_r L_{\tilde{G}} L_{\tilde{F}}^{r-1} h(x)\right]^{-1} \left[\sum_{k=0}^{r} \beta_k L_{\tilde{F}}^{k} h(x) + \beta_r L_p L_{\tilde{F}}^{r-1} h(x) w\right]\right\}.$$

Since the matrix $Q_2(x) + g_2(x)k^{\top}(x)$ is Hurwitz, the fast dynamics of the closed-loop system possess an exponentially stable equilibrium manifold $\bar{\xi}$. Utilizing these two stability properties, it can be shown [16, Theorem 8.4]

$$x(t) = \bar{x}(t) + O(\epsilon),$$

where $x(t)$ and $\bar{x}(t)$ the solutions of the x-subsystems of the closed-loop systems of (1) and (7) under the control law (12), respectively.

Then, the analyticity of the scalar field $h(x)$ and the boundedness of the trajectories $x(t)$ and $\bar{x}(t)$ directly imply that

$$\lim_{\epsilon\to 0}(h(x(t)) + q(w(t))) = h(\bar{x}(t)) + q(w(t)),$$

i.e.,

$$\lim_{t\to\infty,\epsilon\to 0}(h(x(t)) + q(w(t))) = \lim_{t\to\infty} h(\bar{x}(t)) + q(w(t)).$$

Now, it can be seen clearly that the sate feedback regulator problem of the singularly perturbed system (1) can be solved by the control law (12) if and only if the state feedback regulator problem of the reduced system (7) can be solved by the control law (13), and the result follows directly from Lemma 1 of [8].

5. APPLICATION

Consider the cascade of the two CSTR shown in Figure 1, where the following autocatalytic reaction [11] takes place:

$$A + B \rightarrow 2B,$$

where A is a reactant, B is the autocatalytic species, followed by the zero-order side reaction

$$B \rightarrow C,$$

where C is the undesired product. The species A is assumed to be in excess in the two reactors, while the inlet streams consist of autocatalytic species B of concentration C_{B0}. Some kind of inactive gas produced by the exosystem is filled in the reactor to make the liquid in the reactor mixed perfectly. Under the assumptions

(a) uniform temperature in the reactor,
(b) constant volume of the liquid in the reactor, and
(c) constant density and heat capacity of the reacting liquid

the material and energy balances that describe the dynamical behavior of the system take the following form:

$$\begin{aligned}
V_1 \frac{dC_{B_1}}{dt} &= F_1 C_{B0} - F_1 C_{B1} + k_{10} \exp\left(\frac{-E_1}{RT_1}\right) C_{B1} V_1 - k_0 \exp\left(\frac{-E_0}{RT_1}\right) V_1, \\
\frac{dT_1}{dt} &= \frac{F_1}{V_1}(T_{B0} - T_1) + \frac{\bar{Q}_1}{\rho_m c_{pm} V_1} + \frac{(-\Delta H_{r1})}{\rho_m c_{pm}} k_{10} \exp\left(\frac{-E_1}{RT_1}\right) C_{B1} \\
&\quad + \frac{(-\Delta H_{r_0})}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E_0}{RT_1}\right), \\
V_2 \frac{dC_{B_2}}{dt} &= F_1 C_{B1} + F_2 C_{B0} - F_3 C_{B0} + k_{10} \exp\left(\frac{-E_1}{RT_2}\right) C_{B2} V_2 - k_0 \exp\left(\frac{-E_0}{RT_2}\right) V_2 + k\omega, \\
\frac{dT_2}{dt} &= \frac{F_2}{V_2} T_{B0} + \frac{F_1}{V_2} T_1 - \frac{F_3}{V_2} T_2 + \frac{\bar{Q}_2}{\rho_m c_{pm} V_2} + \frac{(-\Delta H_{r1})}{\rho_m c_{pm}} k_{10} \exp\left(\frac{-E_1}{RT_2}\right) C_{B2} \\
&\quad + \frac{(-\Delta H_{r_0})}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E_0}{RT_2}\right), \\
\dot{\omega} &= s(\omega),
\end{aligned} \tag{18}$$

where C_{B1}, T_1 and C_{B2}, T_2 denote the temperatures and the concentrations of the autocatalytic species in the first and second reactor, C_{B0} and T_{B0} denote the inlet temperature and concentration of the species B, F_3 is the outlet flowrate in the second reactor, $\bar{Q}_1, \bar{Q}_2$ denote the heat inputs to the reactors, and k_{10}, k_0, E_1, E_0, ΔH_1, and ΔH_0 denote the preexponential constants, the activation energies and the enthalpies of the two reactors, respectively. ω denotes some kind of inactive gas.

The control objective is tracking the concentration the autocatalytic species B in the second reactor by manipulating the inlet inactive gas ω. In order to decrease the effect of the side reaction, that is minimize the production of the species C, the liquid hold-up of the first reactor is smaller than the liquid hold-up of the second reactor. Defining the parameter ϵ as

$$\epsilon = \frac{V_1}{V_2}$$

and setting

$$u = C_{B0} - C_{B0_s}, \qquad x_1 = T_1, \qquad x_2 = C_{B2}, \qquad x_3 = T_2, \qquad z_1 = C_{B1}, \qquad y = x_2,$$

the original set of equations can be put in form (1) with

$$f_1(x) = \begin{pmatrix} \frac{F_1}{V_1}(T_{B0} - x_1) + \frac{\bar{Q}_1}{\rho_m c_{pm} V_1} + \frac{(-\Delta H_{r_0})}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E_0}{Rx_1}\right) \\ \frac{1}{V_2}\left(F_2 C_{B0_s} - F_3 x_2 + k_{10}\exp\left(\frac{-E_1}{Rx_3}\right) x_2 V_2 - k_0 \exp\left(\frac{-E_0}{Rx_3}\right) V_2\right) \\ \frac{F_2}{V_2} T_{B0} + \frac{F_1}{V_2} x_1 - \frac{F_3}{V_2} x_3 + \frac{\bar{Q}_2}{\rho_m c_{pm} V_2} + \frac{(-\Delta H_{r1})}{\rho_m c_{pm}} k_{10} \exp\left(\frac{-E_1}{Rx_3}\right) x_2 \\ + \frac{(-\Delta H_{r0})}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E_0}{Rx_3}\right) \end{pmatrix},$$

$$Q_1(x) = \begin{pmatrix} \frac{(-\Delta H_{r_1})}{\rho_m c_{pm}} k_{10}\exp\left(\frac{-E_1}{Rx_1}\right) \\ \frac{F_1}{V_2} \\ 0 \end{pmatrix}, \quad g_1(x) = \begin{pmatrix} 0 \\ \frac{F_2}{V_2} \\ 0 \end{pmatrix}, \quad p(x) = \begin{pmatrix} 0 \\ \frac{k}{V_2} \\ 0 \end{pmatrix},$$

$$f_2(x) = \left(\frac{F_1}{V_2} C_{B0_s} - \frac{k_0 \exp(-E_0/Rx_1) V_1}{V_2}\right), \qquad Q_2(x) = \left(-\frac{F_1}{V_2} + \frac{k_{10}\exp(-E_1/Rx_1)V_1}{V_2}\right),$$

$$g_2(x) = \left(\frac{F_1}{V_2}\right), \qquad h(x) = (x_2).$$

The values of the system parameters are given in [11]. one can easily see that for these operating conditions the matrix $Q_2(x)$ is invertible, and the fast dynamics of the system are unstable. Therefore, the controller of Theorem 1 was employed. Moreover, it was verified that the zero dynamics of the reduced system are exponentially stable.

Setting $\epsilon = 0$, the representation of the open-loop reduced system of form (7) can be easily obtained with

$$\tilde{F}(x) = \begin{pmatrix} \frac{F_1}{V_1}(T_{B0} - x_1) + \frac{\bar{Q}_1}{\rho_m c_{pm} V_1} + \frac{(-\Delta H_{r_0})}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E_0}{Rx_1}\right) \\ + \frac{(-\Delta H_{r1})}{2\rho_m c_{pm}} k_{10}\exp\left(\frac{-E_1}{Rx_1}\right) \times \left(F_1 - k_{10}\exp\left(\frac{-E_1}{Rx_1}\right) V_1\right)^{-1} \\ \left(F_1 C_{B0_s} - k_0 \exp\left(\frac{-E_0}{Rx_1}\right) V_1\right) \\ \frac{F_2}{V_2} C_{B0_s} - \frac{F_3}{V_2} x_2 + k_{10}\exp\left(\frac{-E_1}{Rx_3}\right) x_2 - k_0 \exp\left(\frac{-E_0}{Rx_3}\right) + \frac{F_1}{2V_2} \\ \times \left(F_1 - k_{10}\exp\left(\frac{-E_1}{Rx_1}\right) V_1\right)^{-1} \left(F_1 C_{B0_s} - k_0 \exp\left(\frac{-E_0}{Rx_1}\right) V_1\right) \\ + \frac{3F_2}{2F_1 V_2}\left(F_1 C_{B0_s} - k_0 \exp\left(\frac{-E_0}{Rx_1}\right) V_1\right) \\ \frac{F_2}{V_2} T_{B0} + \frac{F_1}{V_2} x_1 - \frac{F_3}{V_2} x_3 + \frac{\bar{Q}_2}{\rho_m c_{pm} V_2} + \frac{(-\Delta H_{r1})}{\rho_m c_{pm}} k_{10}\exp\left(\frac{-E_1}{Rx_3}\right) x_2 \\ + \frac{(-\Delta H_{r0})}{\rho_m c_{pm}} k_0 \exp\left(\frac{-E_0}{Rx_3}\right) \end{pmatrix},$$

$$\tilde{G}(x) = \begin{pmatrix} \frac{(-\Delta H_{r1})}{2\rho_m c_{pm}} k_{10}\exp\left(\frac{-E_1}{Rx_1}\right) \times \left(F_1 - k_{10}\exp\left(\frac{-E_1}{Rx_1}\right) V_1\right)^{-1} F_1 \\ \frac{5F_2}{2V_2} + \frac{F_1}{2V_2}\left(F_1 - k_{10}\exp\left(\frac{-E_1}{Rx_1}\right) V_1\right)^{-1} F_1 \\ 0 \end{pmatrix}.$$

It can be easily verified that the relative order is $r = 1$ and the controller of Theorem 1 takes the form

$$u = \alpha(x,z,w) = -\left[\beta_1 L_{\tilde{G}} h(x)\right]^{-1} \left[\sum_{k=0}^{1} \beta_k L_{\tilde{F}}^k h(x) + \beta_1 L_p h(x) w\right] + k^\top(x) z.$$

The nonlinear gain $k^\top(x)$ was chosen as

$$k^\top(x) = -\frac{3.0}{F_1}\left(-F_1 + k_{10}\exp\left(\frac{-E_1}{Rx_1}\right)V_1\right)$$

to place the eigenvalue of the matrix $Q_2(x) + g_2(x)k^\top(x)$ in the open left-half of the complex plane.

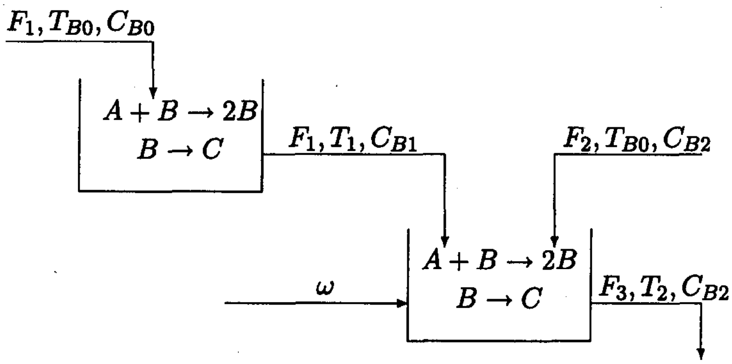

Figure 1. A cascade of two CSTRs.

6. CONCLUSION

This article addresses the state feedback regulator problem of nonlinear singularly perturbed systems. It has been shown that, using the standard two-time-scale decomposition, a state feedback regulator can be easily constructed under some reasonable assumptions, and the regulator can solve the problem if and only if a certain nonlinear partial differential equation is solvable. The proposed control methodology is illustrated with a nonlinear chemical process with time time-scale multiplicity.

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