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NOTE

FUNDAMENTAL CIRCUITS AND A CHARACTERIZATION OF BINARY MATROIDS

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Let M be a matroid on a finite set E. Consider a basis B of M. For $e \in E\backslash B$ we denote by $C(B; e)$ the unique circuit contained in $B \cup \{e\}$, called the *fundamental circuit* of e with respect to B.

If M is binary given any circuit C of M we have clearly

$$C = \underset{e \in C\backslash B}{\Delta} C(B; e)$$

(where Δ denotes the symmetric difference). Conversely if this property holds for all bases B and circuits C M is binary ([1, Th. 20], [2, Chap. 10.1, Th. 3]). Our purpose in the present note is to give a simple proof of the following stronger statement:

Proposition. *Let M be a matroid on a finite set E. Suppose that for some base B of M, we have $C = \Delta_{e \in C\backslash B} C(B; e)$ for every circuit C of M. Then M is binary.*

Proof. Let V be the subspace of $CG(2)^E$ generated by the fundamental circuits $C(B; e)$ $e \in C\backslash B$ and M' be the matroid having for circuits the non-zero vectors of V minimal with respect to inclusion (as usual for subspaces of $CG(2)^E$ we identify a vector and its support).

By hypothesis every circuit C of M is a vector of V, hence is a (disjoint) union of circuits of M' ([1, Th. 19], [2, Chap. 10.1, Th. 3]). Thus the identity function on E is a strong map from M onto M'. Now Dim $V = |E\backslash B|$ hence rk $(M') = |B| =$ rk (M). Since M and M' are related by a strong map, it follows that $M = M'$ (cf. [2, Chap. 17.4]).

References

[1] R. Von Randow, Introduction to the Theory of Matroids, Lecture Notes in Economics and Mathematical Systems, No. 109 (Springer, Berlin–New York, 1975).
[2] D.J.A. Welsh, Matroid Theory (Academic Press, London–New York, 1976).